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Adaptive Recombination Weighting Schemes Applied to the CMA-ES

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MASTER'S THESIS

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Abstract

The *strategy parameters* of an *Evolutionary Strategy (ES)* govern the responsibility for determining how it searches for improved solutions in the search space. The values of the parameters are assigned at the start of the optimization process and some of them are updated in each iteration. The values of the parameters are determined by a set of formulas, which have empirically proved to work well.

A specific ES, called the *Covariance Matrix Adaptation Evolutionary Strategy (CMA-ES)* is equipped with a covariance matrix $\boldsymbol{C}$ that adapt its sampling distribution to the search space $\mathbb{R}^n$ by means of information retrieved from the evaluation process, allowing it to convergence faster to the optimum on a large set of optimization problems than ordinary ESs. In each iteration the distribution samples λ offspring solutions and a $\left(\mu \overset{+}{,} \lambda\right)$ selection scheme selects μ solutions with the most optimal objective fitness function value.

The recombination weights are a set of values that govern the responsibility of defining in each iteration the mean $\boldsymbol{m}$ of the sampling distribution from the aforementioned set of μ solutions. For each of the μ solutions there is a corresponding recombination weight value, which indicate the amount of information used from the solution to which it is associated to construct the mean $\boldsymbol{m}$. Currently, these values are defined once at the beginning of the search process and remain static during it. In this document, the possibility of making them adaptive to the search space is investigated.

Three methods are proposed, allowing to make the recombination weights adaptive to the search space by means of information retrieved from evaluation of the offspring solutions that are sampled in each iteration. The designs have been applied to both a CMA-ES with (μ, λ)- and $(\mu + \lambda)$- selection scheme. The performance of them have been compared with a canonical CMA-ES with a corresponding selection scheme. This performance assessment has been done on the *Black Box Optimization Benchmark (BBOB)*. The results of the CMA-ESs with the non-adaptive recombination schemes have been used as a baseline to compare the results of the methods that we proposed.

The results show that on a set of problems with single objective functions all

algorithms obtain similar results. However, the results show that on multimodal functions the heuristics with adaptive recombination weights are able to obtain better convergence results than the one with static recombination weights. We have hypothesised that it was caused by that the offspring sampled by heuristics with adaptive recombination weight have a higher variance. We have conducted two additional experiments that measure how solutions are sampled, that confirmed this hypothesis.

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Chapter 1

Introduction

Optimization heuristics equipped with self-adaptive distribution parameters have the ability to adapt themselves to the search space. The *Covariance Matrix Adaptation Evolutionary Strategy* also known as the (*CMA-ES*), proposed by Hansen and Ostermeier in 1996 in [24], is an example of such an algorithm. It belongs to the class of *Evolutionary Strategies (ES)* [8]. It differs from ordinary ESs in that is equipped with a *covariance matrix* to adapt the sampling distribution to the search space. Empirical tests have shown that for some problems, it is able to converge faster than ordinary ESs [24], to an optimum.

1.1 Problem Statement

In the following text, we discuss the problem statement. The CMA-ES is equipped with multiple parameters that govern the responsibility of determining how it searches for improved solutions in the search space. Several of these parameters have the ability to adapt themselves, while the others remain static. The reader is referred to Chapter 3 for a technical description of the algorithm and its parameters. Beyer and Hansen claimed in [12] that no single set of parameter values exists that is able to yield optimal results on a large set of problems. We consider it unlikely that such a set is found, because the set of optimal parameters values is problem dependent and could also change during the optimization process.

General rules have been developed that select the values of these parameters, which have empirically proven to work well on a large set of problems, as has been described for example in [23] and in [25]. Nevertheless, situations can arise where using these rules might yield sub-optimal performance convergence results. Therefore, by making one of the parameters adaptive to the search space that is currently static, we consider it possible that the convergence speed can be improved for a subset of problems.

Currently, at the beginning of the optimization process, all so-called strategy parameters are initialized. At the end of each iteration, only the ones that are responsible for adapting the covariance matrix remain are updated. The recombination weights also remain static. These weights govern the responsibility of defining the mean of the sampling distribution, which is later on used to create new solutions. In this thesis we propose three adaptive recombination weighting schemes, that aim to make the original recombination weights adaptive to the search space, similar as to the covariance matrix.

1.2 Document Structure

The structure of the thesis is as follows. In Chapter 2, elementary theory of single objective optimization is described. The designs that we propose are applied on the heuristic the CMA-ES. In Chapter 3, we provide a technical description of the heuristic. Furthermore, we conducted a literature review about recombination weighting schemes applied to the CMA-ES. Finally, we elaborate the designs of the adaptive recombination weighting schemes.

Finally, in Chapter 4, we describe our experimental configuration, experimental methodology, results and conclusions. Furthermore, we compare the performance of the proof-of-concept that we have developed with that of a regular CMA-ES on *Black Box Optimization Benchmark (BBOB)*, which is considered the standard for assessing the convergence speed or CPU time of optimization algorithms on either a set of noisy- or non-noisy functions [27, 18, 16, 4]. Finally, we have a general discussion about our findings.

Chapter 2

Single Objective Optimization

In mathematical optimization we aim to find an optimal solution for one or more so-called objective functions. These functions can be constrained or unconstrained. A distinction is made between single and multi objective optimization. In the former, a single function is optimized and in the latter multiple ones are optimized. A simple example of a single objective optimization problem is the booking of a holiday that is as low priced as possible without exceeding certain constraints. This means that the price of the holiday, is the objective function that needs to be optimized. Furthermore, an example of an multi-objective optimization problem is the cooking of a dinner for your friends for which the price needs to be minimized and the healthiness need to be maximized. For this problem, healthiness and price are the objective functions that needs to be maximized and minimized respectively.

Optimization problems can be divided into sub-classed based on the domain of the objective function that need to be optimized. Table 2.1 lists an overview of this. Optimization problems are a type of decision problems. The complexity class **NP** is a class that describes certain decision problems. Problems that belong to this class have an answer that is *yes* and solutions can deterministically be verified. A problem is in **NP-hard** if every other problem in **NP** can be reduced to this problem in polynomial time. A particular problem is in **NP-complete** when it is both in **NP-hard** and in **NP**. Finally, a problem that is solvable in polynomial time belongs to the complexity class **P**. Figure 2.1 illustrates two inclusions diagrams. One that illustrates the assumption that $\mathbf{P} \neq \mathbf{NP}$ and another one that assumes that $\mathbf{P} = \mathbf{NP}$.

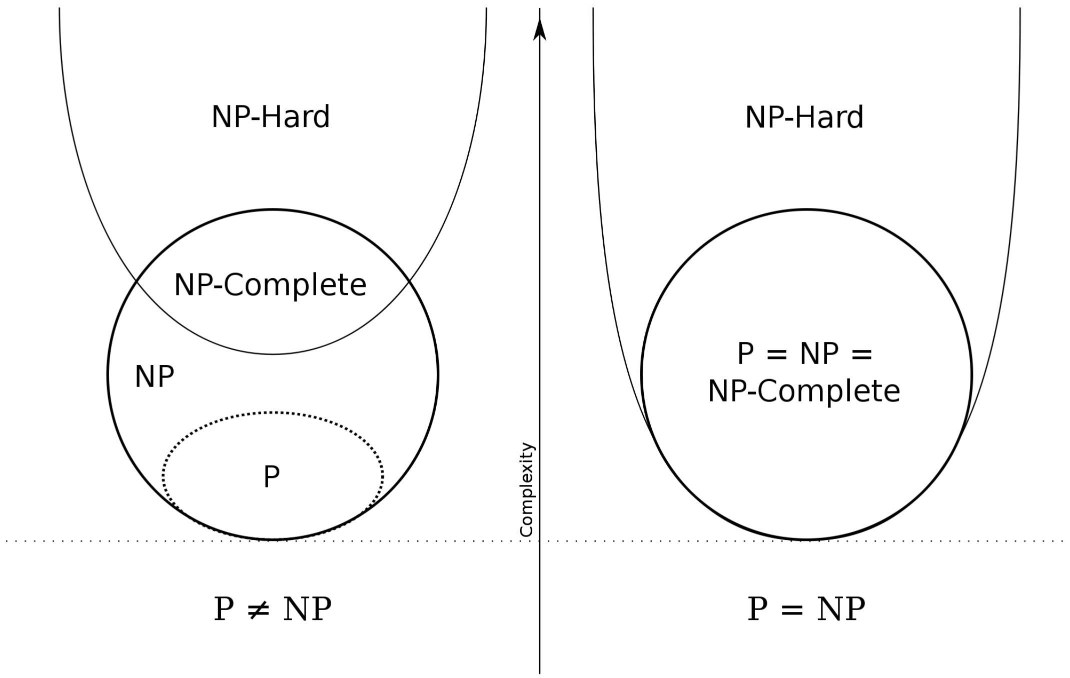

Figure 2.1: An inclusion diagram of the complexity classes **NP**, **NP-hard**, **NP-complete** and **P**. On the left the assumption that **P** ≠ **NP**. On the right the assumption that **P** = **NP**.

Next, we explain terminology used in the field of single objective optimization. Problems that are solved in this field can be defined as the mapping from a *search space* to an *objective space* $f : \mathbb{S} \subseteq \mathbb{R}^n \rightarrow \mathbb{R}$. Here, the variable n indicates the dimensionality of the search space. An objective function value is associated to each unique element in $\mathbb{S}$.

A single objective function f is optimized by finding a *global* or *local optimizer* of f. A solution $\boldsymbol{x}^* = (x_1^*, \ldots, x_n^*)$, $\boldsymbol{x}^* \in \mathbb{S}$ is considered a global *minimizer* for f, if and only if:

$$\forall \boldsymbol{x} \in \mathbb{S} : f(\boldsymbol{x}) \geq f(\boldsymbol{x}^*) .$$

A solution $\boldsymbol{x}^* = (x_1^*, \cdots, x_n^*)$, $\boldsymbol{x}^* \in \mathbb{R}^n$ is considered a global *maximizer* for f, if and only if:

$$\forall \boldsymbol{x} \in \mathbb{S} : f(\boldsymbol{x}) \leq f(\boldsymbol{x}^*) .$$

Next, a solution $\hat{\boldsymbol{x}} = (\hat{x}_1, \ldots, \hat{x}_n)$, $\hat{\boldsymbol{x}} \in \mathbb{S}$ is considered a local minimizer for f, if it holds that:

$$\exists \epsilon > 0 \ \forall \boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x} - \hat{\boldsymbol{x}}\| < \epsilon \Rightarrow f(\boldsymbol{x}) \geq f(\hat{\boldsymbol{x}}) .$$

Sub-class	Search-space	Degree of non-linearity
Linear Programming	$\mathbb{R}^n$	linear
Quadratic Programming	$\mathbb{R}^n$	quadratic
Non-linear programming	$\mathbb{R}^n$	non-linear
Integer Programming	$\mathbb{Z}^n$	arbitrary
Integer Linear Programming	$\mathbb{Z}^n$	linear
Mixed Integer Linear Programming	$\mathbb{R} \times \mathbb{Z}^n$	linear
Mixed Integer Non-linear Programming	$\mathbb{R} \times \mathbb{Z}^n$	non-linear
Continuous unconstrained Optimization	$\mathbb{R}^n$	non-linear

Table 2.1: An overview of different sub-classes of optimization problem. They are divided into sub-classes based on the type of search space we are searching for the optima.

Then, a solution $\hat{\boldsymbol{x}} = (\hat{x}_1, \ldots, \hat{x}_n)$, $\hat{\boldsymbol{x}} \in \mathbb{S}$ is considered a local maximizer for f, if it holds that:

$$\exists \epsilon > 0 \ \forall \boldsymbol{x} \in \mathbb{R}^n : \|\boldsymbol{x} - \hat{\boldsymbol{x}}\| < \epsilon \Rightarrow f(\boldsymbol{x}) \leq f(\hat{\boldsymbol{x}}).$$

Constraints can be added to a problem and thereby restricting the possible set of solutions that are deemed useful. They can be added by means of equality and inequality constraints which are expressed respectively as $g(\boldsymbol{x}) \geq 0$ and $h(\boldsymbol{x}) = 0$. Solutions that satisfy the constraints are called *feasible* and those which do not satisfy the criteria are called *infeasible*. It holds for unconstrained problems that $\mathbb{S} = \mathbb{R}^n$.

2.1 Mathematical Preliminaries

In single objective optimization we start with an initial point and iteratively search for a local optimum. Next, we describe the fundamental mathematical theory behind mathematical optimization.

Theorem 2.1.1 ([34]) *(Mean value theorem) Let f be differentiable on (a, b) and continuous on $[a, b]$ where $a < b$. Then, $\exists c \in [a, b]$ such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.*

We can rewrite the mean value theorem equation as the form:

$$f(b) = f(a) + f'(c)(b - a).$$

In case it holds that:

$$0 = f(b) = f(a) + f'(c)(b - a),$$

then $f(b)$ is the minimum in the interval $[a, b]$.

Theorem 2.1.2 ([35]) *(Taylor's theorem). Assume that f is $(n+1)$-times differentiable, and $P_n(x)$ is the degree n Taylor approximation of f with centre c. Then if x is any value, there exists some value b between x and c such that*

$$f(x) = P_n(x) + \frac{f^{n+1}(b)}{(n+1)!}(x-c)^{n+1}$$

In case $n = 0$, then the Taylor's theorem equals the mean value theorem:

$$\begin{aligned} f(x) &= P_n(x) + \frac{f^{n+1}(b)}{(n+1)!}(x-c)^{n+1} \\ &= P_0(x) + \frac{f^{(0+1)}(b)}{(0+1)!}(x-c)^1 \\ &= P_0(x) + \frac{f'(b)}{(1)}(x-c) \\ &= P_0(x) + f'(b)(x-c). \end{aligned}$$

We will now explain how Taylor's theorem and the CMA-ES are related. The CMA-ES quadratically approximates functions, implying a Taylor expansion where $n = 2$. Given the convex-quadratic objective function $f_{\boldsymbol{H}} : \boldsymbol{x} \leftarrow \frac{1}{2}\boldsymbol{x}^T\boldsymbol{H}\boldsymbol{x}$, where $\boldsymbol{H}$ expresses the Hessian matrix $\boldsymbol{H} \in \mathbb{R}^{n \times n}$ [26]. Given the sampling distribution $\mathcal{N}(\boldsymbol{m}, \boldsymbol{C})$. with mean $\boldsymbol{m}$ and covariance $\boldsymbol{C}$. The matrices $\boldsymbol{C}$ and $\boldsymbol{H}$ are related by setting $\boldsymbol{C} = \boldsymbol{H}^{-1}$ on $f_{\boldsymbol{H}}$, which is equivalent to optimizing the isotropic function $f_{\text{sphere}} = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{x} = \frac{1}{2}\sum_i \boldsymbol{x}_i^2$ for $\boldsymbol{C} = \boldsymbol{I}$ and $\boldsymbol{H} = \boldsymbol{I}$. The previous equalities means that on a convex-quadratic objective function, setting the covariance matrix of the search distribution to the inverse Hessian matrix, is equivalent to rescaling the ellipsoid function into a spherical one. Note, that we assume that the optimal covariance matrix equals the inverse Hessian matrix, up to a constant factor.

Next, we describe the necessary and sufficient conditions to guarantee the existence of a local optimal solution for a differentiable and unconstrained function. A necessary condition for a solution to be local optimal is that it needs to be a stationary point, which is expressed as:

Definition 1. (Stationary point). Given a differentiable function $f : \mathbb{R} \mapsto \mathbb{R}$. Then a point $x \in \mathbb{R}^n$ is a stationary point of f if $f'(x) = 0$.

Then, a sufficient condition to guarantee that a local optimal solution exists for a differentiable and unconstrained function is that ∇f needs to be positive semi-definite. Which is defined as follows.

Definition 2. *(Positive semi-definite). A function for which holds $f(0) = 0$ and $f(x) \geq 0$ is called positive semi-definite.*

Theorem 3. *(Guarantee of a Local Optimal Point) Given an unconstrained continuous function f a point $\boldsymbol{x} \in \mathbb{S}$ is a local optimal if and only if it holds that $\nabla f(\boldsymbol{x}) = 0$ and $\nabla^2 f(\boldsymbol{x})$ is positive (semi-)definite.*

A technique known as the *gradient descent method*, allows to iteratively approximate the maximum point by stepwise adding the gradient to an additional point $\boldsymbol{x}_0$ until one is converged to the optima. The reason that it works is because the gradient vectors point in the direction of the maximum and are perpendicular to the contour lines. Given the function $f(x, y) = -(x^2 + y^2)$, then on the left hand of Figure 2.1, we see a plot of the function and on the right hand side of it we see a contour plot combined with gradient vectors of the same function represented. The gradient vectors are perpendicular to the contour lines and point in the direction of the maximum.

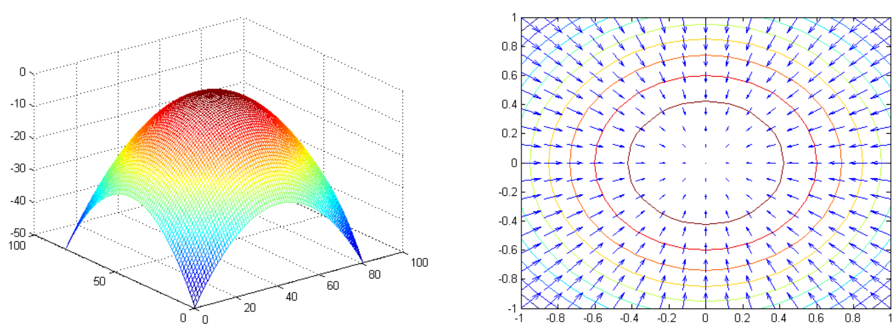

Figure 2.1: On the left, a 2D plot of the inverse of the Sphere function. On the right, a contour plot of the same function.

Gradient-based search heuristics converge faster than stochastic-based heuristics. However, the prior type of heuristics have certain disadvantages. Namely, the fail to perform certain optimization tasks. Figure 2.2, shows to plots of Ackley's and Rosenbrock's function, that are two examples of functions that are difficult to solve. Instead, a stochastic-based heuristic can be used to solve the problems more optimal. In the following text we briefly discuss these heuristics.

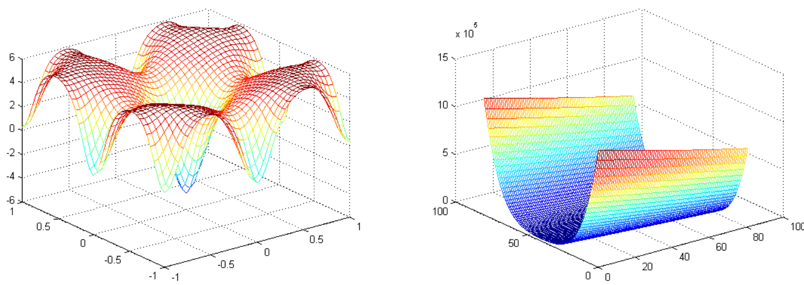

Figure 2.2: On the left A plot of Ackley's function. On the right plot of Rosenbrock's.

Chapter 3

The CMA-ES

This chapter is structured as follows. In Chapter 3.1, we provide an introduction to Evolutionary Algorithms (EAs), the class of heuristics to which it belongs to. Since the CMA-ES was proposed by Hansen in 1996 in [24] many improvements have been made. In Chapter 3.2, we provide an excerpt of notable improvements that have been made to the original CMA-ES. In Chapter 3.3, we provide a technical explanation of the CMA-ES, by means of an algorithm in pseudo-code and a description of each step. We aim to propose adaptive recombination weighting schemes. In Chapter 3.4, is about our recombination weighting schemes. In Chapter 3.4.1., we made a summary about existing research about recombination weighting schemes of the CMA-ES. Based on this we were able to make a decide which type of scheme we could use to make adaptive. Then in Chapter 3.4.2., we propose three adaptive recombination weighting schemes.

3.1 Evolutionary Algorithms

EAs are a class of stochastic and nature-inspired optimization algorithms, which can be utilized to solve single- or multi-objective optimization problems. Figure 3.1 illustrates the class of EAs and its subclasses. The subclass to which an EA belongs depends on the type of problem it is able to solve. For example, Genetic Algorithms are typically applied to solving integer optimization problems and Evolutionary Strategies (ESs) are mostly applied to solving real valued optimization problems and in some cases to mixed-integer spaces. The subject of the thesis, the CMA-ES, belongs to the sub-class of ESs.

EAs are optimization algorithms inspired by nature. ESs are conceptually

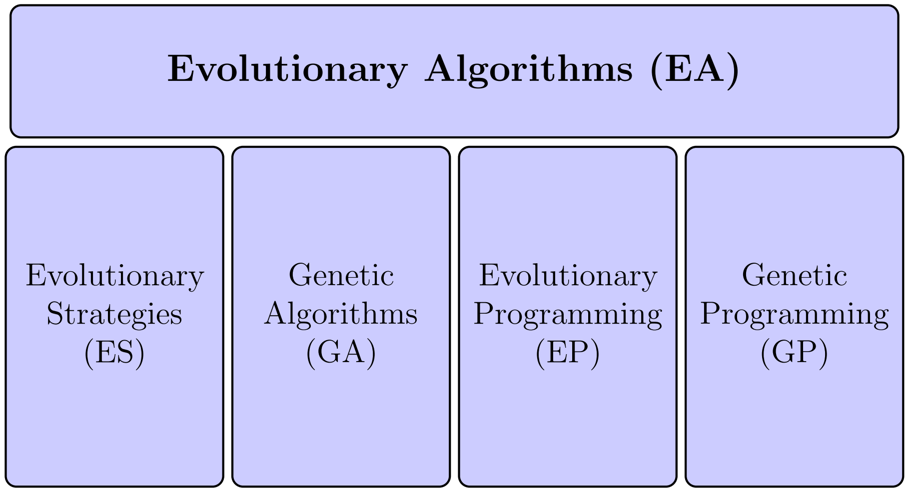

Figure 3.1: An overview of subclasses of Evolutionary Algorithms.

inspired by repair genes, which repair genetically harmful parts from a set of genes . In June 1964, an empirical test on a 2D joint plate in turbulent air flow showed that a simple randomized heuristic was able to yield better results than a univariate discrete gradient-oriented strategy. This heuristic was later on used for other experiments such as the design of a 3D nozzle. The earlier versions of the ES where not concerned with finding minima or maxima from a given function like nowadays. Instead, they were designed as a set of rules by using stepwise variable adjustments of driving a flexible system in an optimal state [11].

The simplest form of an ES is known as the $(1+1)$-ES, using only a single parent and a single offspring. It resembles the concept of *survival of the fittest,* in which the least fittest leaves the evolutionary pool and the most optimal one is able to reproduce itself. Later on, Rechenberg proposed in his dissertation two new types of ES, namely (μ, λ)- and $(\mu + \lambda)$-ES. Here, μ and λ represent the number of parents and the offspring solutions that are sampled, for which it holds that $1 \leq \lambda \leq \mu$.

- The $(\mu + \lambda)$-ES selects the optimal solutions from both the set of parent and offspring solutions, resulting in that a decrease in the quality of the solutions is not permitted. Hence, the the optimal solution of the current evolution cycle is at least as good as the optimal solution of the previous evolution cycle. This configuration could lead to getting stuck in a local optimum .
- The (μ, λ)- ES selects the set of optimal solutions only from the set of offspring solutions. The parent solutions are discarded irrespective of their quality. This permits the possibility of overcoming getting stuck in local optima.

An ES works from a high level as follows. It starts with an initial set of (parent) solutions and iteratively searches for improvements. In the *evolution cycle*, the set of parents solutions is used to sample another set of (offspring) solutions by means of a sampling procedure. To each offspring solution, an objective function value is assigned by evaluating them with an objective function. Thereafter, a subset of solutions is selected to become the parents in the next cycle (or iteration). Depending on the type of strategy used, the subset can be selected from the offspring solutions or both the parent and the offspring solutions. At the end of each iteration, information derived from evaluating the solutions is used to update the parameters. This cycle is repeated until a desired solution is found or a stopping criterion is reached. A pseudo-code description of a general CMA-ES is given in Algorithm 1. We see that in contrast to ordinary ES, that update their parameters at the beginning of each iteration, the CMA-ES does this at the end.

Algorithm 1 A general Description of the CMA-ES

```
1: Initialize strategy parameters, such as μ, λ, and C^0
2: while until stopping criterion is reached do
3:     Sample λ offspring solutions from the μ parent solutions
4:     Evaluate the λ solutions with the objective function f
5:     Apply a (μ, λ) or a (μ + λ) selection scheme to select the μ solutions
   that have the most optimal objective function values. They become the
   parent solutions in the next iteration.
6:     Update strategy parameters
7: end while
8: return optimal solution
```

3.2 Notable Developments of the CMA-ES

Since the CMA-ES was proposed by Hansen in 1996 in [24], much research has been done to improve the algorithm, such as for example, reduce the computation time [29] or extend the spectrum of problems that can be solved by it as described in [33, 6, 32, 28]. In the following text, we briefly describe several of these notable developments.

The factorization of the covariance matrix $\boldsymbol{C}$ is computationally expensive and takes $O(n^3)$ computations. The so-called *Cholesky-update* for a $(1+1)$-CMA-ES proposed by Igel et al. in [29], allows to perform the task in $O(n^2)$ computations. The performance gain is realised by performing the computation on the Cholesky factors, instead of performing the computing on the matrix explicitly. This method has been generalized to $(\mu/\mu_{\mathrm{w}}, \lambda)$-CMA-ES in [33].

The original CMA-ES is a local search optimization algorithm, meaning that only a portion of the space is searched for improvements. In contrast to local search algorithms, global search algorithms explore a larger portion of the space, resulting in that solutions can be found that could be better than the optimal solutions of local search algorithms. In 2009, Hansen proposed in [18] a variant of the CMA-ES that uses two populations. Hence, the name BI-population CMA-ES. The performance of it has been assessed on BBOB. The results show that it is able to solve between 20 and 24 of the 24 functions depending on the dimension of the search space, which were 2, 3, 5, 10, 20 and 40.

A second global search variant of the CMA-ES, is the IPOP-CMA-ES proposed by Auger and Hansen in [6]. It utilizes a restart method, that after each restart increases the size of the population. The performance of it had been assessed by comparing it to the original algorithm on both uni-modal and multi-modal functions. The results show that both perform equally well on uni-modal functions. However, the IPOP-CMA-ES outperforms the original CMA-ES on multi-modal functions.

Situations arise where the evaluation of a single solution with an objective function could take several hours or even longer [32]. In order to decrease the computation time, a so-called meta-model can be constructed. The model is constructed or trained by using data from the original objective function as input. When the model is trained well enough, which is determined by a pre-defined quality criterion, the model can than be used as a substitute of the objective function. Techniques that can be used to construct such a model are for example, Polynomial Models, Kriging Models, Artificial Neural Networks or Support Vector Machines [30]. We refer the reader to multiple surveys, describing developments in this area of research [30, 32, 31].

The original CMA-ES was developed to solve single objective problems. In 2007, Igel et al. proposed in [28] two variants of the CMA-ES that can be applied to solving multi objective optimization problems. The proposed algorithms differ in that they utilize a different selection criterion. The performance of both has been assessed by comparing it to the NSGA-II which is described in [15]. The results show that one of the proposed algorithms outperforms the NSGA-II.

The aforementioned developments are only a few examples of the vast improvements made to the original CMA-ES that was proposed in 1996. Having summarized several notable improvements, we will now give a technical explanation of the CMA-ES.

3.3 Technical Description

We will now provide a technical explanation of the CMA-ES. We will not derive each equation in detail or explain each variable used in the process. We will only limit ourselves to that what is important for the thesis. The reader is referred to a tutorial of Hansen [17] for an elaborate description of the inner workings of the CMA-ES. In each iteration, a fixed number of parent solutions is used to sample a fixed number of offspring solutions, which are expressed respectively as $\lambda = 4 + \lfloor 3 \cdot \log n \rfloor$ and $\mu = \lfloor \frac{\lambda}{2} \rfloor$. Here, the variable n indicates the dimension of the search space $\mathbb{R}^n$. Given the parameters of the sampling distribution, at iteration g: the vector of mean values $\boldsymbol{m}^{(g)} \in \mathbb{R}^n$; the step-size $\sigma^{(g)} \in \mathbb{R}$; the covariance matrix $\boldsymbol{C}^{(g)} \in \mathbb{R}^{n \times n}$. Then the following equation is responsible for sampling a total of λ solutions at iteration $(g+1)$:

$$\boldsymbol{x}_k^{(g+1)} = \boldsymbol{m}^{(g)} + \sigma^{(g)} \cdot \mathcal{N}_k \left(\mathbf{0},\ \boldsymbol{C}^{(g)}\right)$$

for $k = 1, \ldots, \lambda$. Note that we have the following equalities, $\boldsymbol{y}_k = \boldsymbol{BDz}_k \sim (\mathbf{0},\ \boldsymbol{C})$ and $\boldsymbol{z}_k \sim (\mathbf{0},\ \boldsymbol{I})$. Here, indicates a n-dimensional multivariate normal distribution. Given the set of μ optimal solutions expressed as $\boldsymbol{x}_{1:\mu}^{(g+1)} = \left(\boldsymbol{x}_1^{(g+1)}, \ldots, \boldsymbol{x}_\mu^{(g+1)}\right)$, the weighted mean is initialized as $\boldsymbol{m}^{(0)} = \mathbf{0}$ and its value at iteration $(g+1)$ is computed by means of the following expression:

$$\boldsymbol{m}^{(g+1)} = \sum_{i=1}^{\mu} w_i \boldsymbol{x}_{i:\mu}^{(g+1)}.$$

In the previous equation the variable w was used, indicating a set of recombination weights. For each of the μ optimal solutions there exists a single recombination weight. It must hold for the sum of values of w that $\sum_{i=1}^{\mu} w_i = 1$ and the inequalities $w_1 \geq w_2 \geq \ldots \geq w_\mu \geq 0$ must be satisfied. An example of a method to calculate the values of the recombination weights is expressed as:

$$w_i = \ln\left(\frac{\lambda+1}{2}\right) - \ln(i) \text{ for } i = 1, \ldots,\ \mu, \tag{3.3.1}$$

which has been formulated in Hansen [25].

We will now explain how the covariance matrix $\boldsymbol{C}$ is updated, which is later on used to adapt the sampling distribution to the search space. The adaptive recombination weights schemes that we propose in Chapter 3.4.2. is updated in a similar fashion as $\boldsymbol{C}$. The matrix is initialized as $\boldsymbol{C}^{(0)} = \boldsymbol{I}$, where $\boldsymbol{I}$ represents the identity matrix in $\mathbb{R}^{n \times n}$. The covariance matrix $\boldsymbol{C}$ is updated by means of the following equation:

$$\boldsymbol{C}^{(g+1)} = \overbrace{\left(1 - c_1\right) \boldsymbol{C}^{(g)} + c_1 \boldsymbol{p}_c^{(g+1)} \boldsymbol{p}_c^{(g+1)^{\mathrm{T}}}}^{\text{rank-one-update}} + \overbrace{\left(1 - c_\mu\right) c_\mu \sum_{i=1}^{\mu} w_i \boldsymbol{y}_{i:\mu}^{(g+1)} \boldsymbol{y}_{i:\mu}^{(g+1)^{\mathrm{T}}}}^{\text{rank-}\mu\text{-update}},$$

which consists of two parts: the rank-one- and rank-μ-update. The rank-one-update reads as follows:

$$\boldsymbol{C}^{(g+1)} = \left(1 - c_1\right) \boldsymbol{C}^{(g)} + c_1 \boldsymbol{p}_c^{(g+1)} \boldsymbol{p}_c^{(g+1)^{\mathrm{T}}}.$$

In the previous equation, the variable $\boldsymbol{p}_c \in \mathbb{R}^n$ is a so-called *evolution path*, which represents a sequence of solutions. It employs an exponential smoothing term such that it gives a higher priority to more recent information. It is initialized as $\boldsymbol{p}_c^{(0)} = \boldsymbol{0}$ and is updated in iteration g by means of the following expression:

$$\boldsymbol{p}_c^{(g+1)} = \left(1 - c_c\right) \cdot \boldsymbol{p}_c^{(g)} + c_c^u \cdot c_{\mathrm{w}} \boldsymbol{B}^{(g)} \boldsymbol{D}^{(g)} \langle \boldsymbol{z} \rangle_{\mathrm{w}}^{(g+1)}.$$

Here, the variable c_c indicates the rate at which information that is stored in $\boldsymbol{p}_c$ decays. The adaptive recombination weighting schemes that we propose also use the concept of an evolution path, in the sense that a variable is used to store information from previous iterations. The rank-μ-update reads as follows:

$$\boldsymbol{C}^{(g+1)} = \left(1 - c_\mu\right) \boldsymbol{C}^{(g)} + c_\mu \sum_{i=1}^{\mu} w_i \boldsymbol{y}_{i:y}^{(g+1)} \boldsymbol{y}_{i:y}^{(g+1)^{\mathrm{T}}}.$$

3.4 Recombination Weights

In previous text, we provided an explanation of the general CMA-ES as described in [25]. Equation 3.3.1 is an example of a expression to determine the values of w. These are initialized before the simulation of the problem. During the optimization process they remain unchanged. Our objective is to propose multiple designs that allow the weights adapt to the environment. First, we start with a literature review, that summarizes existing studies in this field. Finally, we propose our designs.

3.4.1 Literature Review

The set of recombination weights $w \in \mathbb{R}^{\mu}$, for which it holds that $\sum_{i=1}^{\mu} w_i = 1$, govern the responsibility of redefining the value of the weighted mean $\boldsymbol{m}^{(g)}$ from the μ optimal solutions, in every iteration. In each iteration thereafter, the new mean is used to create new solutions by means of the sampling procedure. For each solution there exists one recombination weight. Each weight variable indicates the amount of information extracted from the solution to which it is related. The broad class of problems that can be solved by this algorithm does not allow the existence of a single set of optimal recombination weights. However, research has been done that investigates the existence of a set of optimal recombination weights for a subset of problems or a specific variant of ES. Next, we summarize several of these studies.

An analytical and empirical prove of the optimal values for the non-adaptive recombination weights of a $(\mu,\ \lambda)$-ES applied to the sphere function: $f = \sum_{i=1}^{n} x_i^2$, was given by Arnold in [1], where n indicates the dimensionality of the search space. Beyer showed in [9], by means of the *sphere model* applied to a $(1+1)$-ES, that the highest possible serial efficiency was 0.202. Arnold showed in his paper, that with using rank-based weighted recombination, he was able to improve the result of Beyer by a factor 2.5.

A variant on the sampling procedure called *mirrored sampling* generates $\lambda/2$ independent and $\lambda/2$ dependent individuals, resulting in an improvement in convergence speed as shown in [5, 13] . Unfortunately, applying both weighted recombination and cumulative step-size adaptation together would cause a bias towards smaller step-sizes, resulting in pre-mature convergence of the algorithm. It happens because, when selecting both an individual and its mirrored counter-part they partially cancel each other out, causing a biases toward smaller step-sizes.

Juan Chen et al. [14] compared different recombination configurations such as for example: global and local intermediate recombination, weighted recombination, selection of the single fittest solution, or no recombination. The configuration that yields the most optimal convergence results is weighted recombination. Furthermore, the one yielding the least favourable performance is the discrete recombination operator. Each configuration used in the experiment, utilizes a set of pre-defined recombination weights, that remained static during the optimization process. Unfortunately, the report does not shed light on the usage of adaptive configurations. However, we are able to conclude that weighted recombination is a configuration that can be used as inspiration for our designs.

3.4.2 Design of Adaptive Recombination Weights

We have concluded in the literature review, that weighted recombination is a method that performs in general well. In the following text, multiple designs are proposed, inspired by weighted recombination, to make them adaptive to the search space, by means of information from the evaluation process. The information obtained from evaluating the solutions, is used to define another set of weights $\boldsymbol{u} \in \mathbb{R}^{\mu}$, which we name *penalty weights*. The method that we use to select the values of the penalty weights is as follows. Solutions which have a better objective function value are assigned a lower penalty value, and those which are worse are assigned a higher penalty. We consider it probable that values selected from a single iteration as susceptible to noise and thereby negatively influencing the convergence. Therefore, the values are chosen from a multiple iterations, which are exponentially smoothed to give a higher priority to more recent information. The idea behind the smoothing is that it removes noise.

After having calculated the values of the penalties, they are then projected on the original set of weights w, resulting in that when the variation of the fitness values of the solutions becomes small, the adaptive weights are roughly equal to the values of w. This happens for example, when the optimum is approached and σ becomes smaller, resulting in a minor variation of the population $\boldsymbol{x}_{1:\mu}^{(g+1)}$. Projecting the penalty weights on the original weights permits the algorithm to adapt itself to the environment when the population has a relatively large variation. Furthermore, it also allows to use the original set of weights when it becomes harder to determine proper penalty weights.

Our design is based on three basic assumptions. The first assumption it that a set of solutions is evaluated by means of one single arbitrary objective function $f(x)$. Then the second assumption is that the μ optimal solutions are identified, resulting in a set of objective values and ranked based on their objective function value, expressed as $\boldsymbol{f} = (f_1, f_2, \ldots, f_\mu)$. Finally, the third assumption is that the optimizer is the null vector $\boldsymbol{x}^* = \boldsymbol{0}, \boldsymbol{x}^* \in \mathbb{R}^n$, but this does not imply that $\boldsymbol{f}^* = \boldsymbol{0}$. After proposing the designs, we describe how the final assumption can be resolved.

Having described the underlying assumptions, we elaborate our designs. Three designs are proposed that each influence the adaptation of the weights with a different magnitude. In our first design we use the squared objective function values to define u. It results in that the adaptive weights have the highest variation of all three designs. The values of the set of penalties u at iteration g is expressed formally as:

$$\boldsymbol{u}^{(g)} = \frac{1}{\sum_{i=1}^{\mu} f_i^2} \left(f_1^2,\ f_2^2, \ldots, f_\mu^2 \right).$$

In design number two, we take the absolute value from the objective function fitness values, resulting in that values of the adaptive recombination weights have a moderate variation. The values are specified formally by the following expression:

$$\boldsymbol{u}^{(g)} = \frac{1}{\sum_{i=1}^{\mu} |f_i|} \left(\|f_1\|,\ \|f_2\|, \ldots, \|f_\mu\| \right).$$

The third and final design utilizes the square root on the objective function values, resulting in that is has the lowest variation of recombination values of all three proposals. The computation of $u^{(g)}$ is formally expressed :

$$\boldsymbol{u}^{(g)} = \frac{1}{\sum_{i=1}^{\mu} \sqrt{|f_i|}} \left(\sqrt{f_1},\ \sqrt{f_2}, \ldots, \sqrt{f_\mu} \right).$$

After computing the values, they are then exponentially smoothed with information from previous iterations. The vector of penalties at iteration g is formally expressed as:

$$\boldsymbol{u}_{\text{smth}}^{(g)} = \alpha \cdot \boldsymbol{u}^{(g)} + (1 - \alpha) \cdot \boldsymbol{u}_{\text{smth}}^{(g-1)\prime},$$

where indicates $\alpha \in [0,\ 1]$ the learning rate of the self-adaptive recombination weights. Next, the resulting penalty vector is normalised in order to guarantee that the entries sum up to one.

$$\boldsymbol{u}_{\text{smth}}^{(g)\prime} = \frac{1}{\sum_{i=1}^{\mu} \boldsymbol{u}'_{\text{smth},\ i}} \cdot \boldsymbol{u}'_{\text{smth},\ i}.$$

Having calculated the penalties, it is projected on $\boldsymbol{p}$ by means of the following expression:

$$\boldsymbol{w}'_i = \boldsymbol{p}_{\text{smth},\ i}^{(g)\prime} / \boldsymbol{w}_j$$

for $i = \mu,\ (\mu - 1), \ldots, 1$ and for $j = 1,\ 2, \ldots, \mu$. Finally, the resulting vector is normalized in order to guarantee that the entries of the new weights sum up to one, which is expressed as:

$$\boldsymbol{w}'' = \frac{1}{\sum_{i=1}^{\mu} \boldsymbol{w}'_i} \boldsymbol{w}'.$$

We mentioned in our assumptions that the optimum should be located at the null vector. This implies that the algorithm is biased towards this point. It is caused by using the absolute value function. This bias can be resolved by, after obtaining the objective function values, ranking them and then assigning a value depending on their rank. Here, the solution with the lowest rank is the most optimal one, and the one with the highest ranking is the least optimal one. Therefore, the higher the ranking, the higher the penalty value assigned.

Chapter 4

Experiments

In this chapter, we discuss the experiments that we have conducted to assess the performance of the adaptive recombination weighting schemes that we proposed in Chapter 3.4.2. In Chapter 4.1, we explain the configuration of the experiments that we performed on BBOB. In Chapter 4.2, we elaborate the results from the BBOB experiments. Here, we will see that schemes that we proposed work well on multi-modal functions. In Chapter 4.3, we investigate why these results are obtained. In Chapter 4.4, we describe subjects for future research. Finally, we conclude in Chapter 4.5.

4.1 BBOB Experimental Configuration

We will next elaborate the experimental configuration used to test the performance of our designs. We utilized the *Black Box Optimization Benchmark* also known as *BBOB* to measure the performance. It consists of two sets of single objective optimization functions, one with and the other without noise, which are described respectively in [22] and in [21]. In this field of science, the benchmark is considered to be the standard of measuring the performance of these type of heuristics. An excerpt of examples where it also has been applied are in [20, 2, 7, 20, 3, 4]. The benchmark allows to define an custom-made configuration to assess the performance. However, we have decided to use the configuration described in [19], because it is the most recent version of the configuration used by others in this field of science.

We have decided to assess the performance on the set of non-noise functions, consisting of 24 single objective functions with a separable, uni-modal, multi-modal or a weak global structure, or a combination of them [21]. The motivation for using this set over the one with noise, is because the functions are relatively easier to solve, which also allows to draw conclusions more easily. The functions needed to be solved on a search space with a dimensionality of 2, 3, 5, 10, 20 and 40. The heuristics, use 15 trials per function for each different dimensionality to reach $f_{\text{target}} = f_{\text{opt}} + \Delta f$. Where f_{opt} and Δf indicate the optimal function value for each trial and the precision which is defined as the difference between the smallest function value and f_{opt}, respectively. In this context, it holds that $\Delta f = 10^{-8}$. Next, we introduce the measure called *expected running time* or *ERT*, which estimates the expected running time to reach f_{target}. Then, the success rate is expressed as p_s, indicating the number of successful trials. When p_s is non-zero, the measure is expressed as follows:

$$
\begin{aligned}
\text{ERT}\left(f_{\text{target}}\right) &= \text{RT}_{\text{S}} + \frac{1-p_s}{p_s}\text{RT}_{\text{US}} \\
&= \frac{p_S \text{RT}_S + \left(1-p_s\right)\text{RT}_{US}}{p_S},
\end{aligned}
$$

where RT_{S} and RT_{US} indicate the denote the average number of function evaluations for successful and unsuccessful trials, respectively.

The methods that we have proposed are tested on both a (μ, λ) and a $(\mu + \lambda)$ strategy. In our design, we introduced a new parameter α indicating the rate at which information decays. The tests are conducted for ten different values of α for each of the six variants. The value of α is equal to $\frac{1}{2^i}$ for $i = 1, 2, \ldots, 10$. Table 4.1 lists these 10 different values and categorises each of the six different variants in a (μ, λ) or $(\mu + \lambda)$ strategy. Figure 4.1 illustrates how we are going to test the performance of all different possible configurations as displayed in 4.1.

The experimental results from this group is displayed in Appendix A in A.6 until **??**. We have limited ourselves to presenting only the *empirical cumulative distributions* of the performance. We have omitted all the other results due to space restrictions and that the figures that we have included show sufficient information to make our point clear.
Having compared the performance of the different configurations of each variant, we selected the best of each variant and compared the best from the (μ, λ)-ESs with a (μ, λ)-CMA-ES and (μ, λ)-ESs a $(\mu + \lambda)$- CMA-ES.

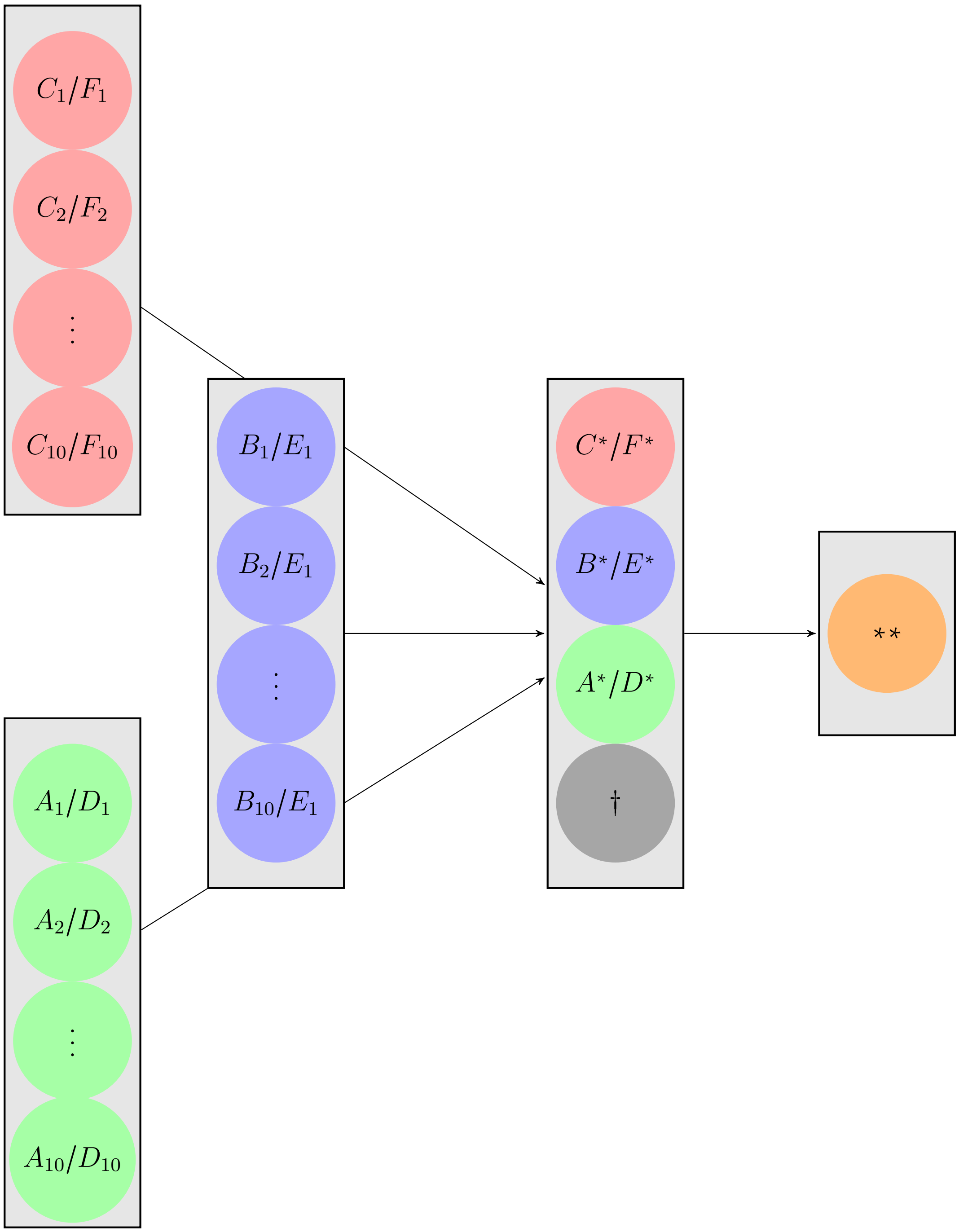

Figure 4.1: An illustration of how the algorithm with the optimal parameters are selected. First, the algorithm with the optimal α is selected for series A, B and C. Then, the optimal one from each series, defined as A^*, B^* and C^* are than compared with the CMA-ES, denoted as †. Then, the optimal algorithm is selected which is represented as $**$ in the diagram.

$(\mu + \lambda)$-ES			(μ, λ)-ES			$\alpha = \frac{1}{2^i} =$
A_1	B_1	C_1	D_1	E_1	F_1	0.5
A_2	B_2	C_2	D_2	E_2	F_2	0.25
A_3	$\boldsymbol{B_3}$	C_3	$\boldsymbol{D_3}$	E_3	F_3	0.125
$\boldsymbol{A_4}$	B_4	$\boldsymbol{C_4}$	D_4	$\boldsymbol{E_4}$	F_4	0.0625
A_5	B_5	C_5	D_5	E_5	F_5	0.03125
A_6	B_6	C_6	D_6	E_6	$\boldsymbol{F_6}$	0.015625
A_7	B_7	C_7	D_7	E_7	F_7	0.0019531
A_8	B_8	C_8	D_8	E_8	F_8	0.0039063
A_9	B_9	C_9	D_9	E_9	F_9	0.0078125
A_{10}	B_{10}	C_{10}	D_{10}	E_{10}	F_{10}	0.00097656

Table 4.1: An overview of the different choices of α for our proposed designs used in the experiment. The entries in the table expressed in boldface are considered the optimal ones and are compared with each other and a variant of the original CMA-ES.

4.2 Experimental Results

We will now discuss the performance of the set of (μ, λ)-strategies. The performance of them on all figures plotted on a log scale is presented in Appendix A in Figure A.57. Then, Figures A.58 until A.99 show the empirical cumulative distributions of the performance of this set. In Figure A.57, we see that on function 7, 13 and 16 algorithm A_1 is significantly faster than the others (excluding the best algorithm provided by the developers of the benchmark). This also holds for function 13. On function 11 and 14 it it slightly faster in the beginning. However, when the dimensionality of the search space increase it will perform similar as to the others. Looking at the results of function 1, 2, 5, 7, 8, 9, 10, 11, 12, 14, 21 and 22 we see that the functions perform similar. There is not a notable difference in performance as with functions 7, 13 and 16.

We continue with discussing the performance of the set of (μ, λ)-strategies. In Appendix A in Figure **??**, the performance of the set on all functions is presented on a log scale. Then, Figures A.58 until A.99 show the empirical cumulative distributions of the performance of this set. In Figure **??** we see for the vast majority of functions that the our designs perform similar as the CMA-ES. However, we see on function 7 E_4 that until dimension 10 it is faster than the others (excluding the best algorithm provided by the developers of the benchmark), however when the dimensionality increases its performance becomes worse than the others. Furthermore, we see that D_3 is performs even better than the most optimal algorithm, which was provided by the benchmark, until dimension 5. When looking at the empirical cumulative distributions we see frequently see that at least one of our designs is faster the original CMA-ES. For functions 1, 2, 5, 8, 9, 10, 11, 12, 13, 14, 21 and 22 the algorithms performs similar. Only the algorithm provided by the benchmark is faster.

In general, the designs that we proposed scale well when the dimensionality of the search space increases. Because, we decided not to increase the size of the number of parents and offspring produced the algorithms where not able to solve a function in all caes. For the (μ, λ)-ESs, it holds that for all dimensions 13 out of 24 functions were solved. For the class of $(\mu + \lambda)$-ESs, it holds that 14 out of 24 functions were solved. The functions on which the their was a clear performance difference are the set of multi-modal functions. In most cases, the algorithms performed similar on the set of uni-modal functions. Based on our conclusion that CMA-ES with adaptive recombination weights perform better on multi-modal functions, we have decided to perform two additional experiments that would explain why that is the case, which is done in Chapter 4.3.

4.3 Additional Experiment: Explanation of the Performance

In the previous paragraph, we assessed the performance of our proposed designs. The results show that adaptive recombination weights are able to reach an optimum faster than the original CMA-ES on the functions 15, 16, 17 and 18 described in [21]. All of them are multi-modal functions. We hypothesised that is was caused that the heuristics with adaptive weights have a higher variance. We have decided to conduct two additional experiments that would verify if this hypothesis is true.

In the first experiment, we measured the variance of the solutions sampled by the mean. The tests where conducted on functions 15, 16, 17 and 18 described in [21] on a 2D search space. The data that we had collected were the average values of data collected over 15 trials. Appendix B shows a sequence of 2D-plots, representing the x-and y-values of the mean and the solutions that are sampled in the search space for each iteration of function 15. We have limited ourselves to only presenting the result of a single function due to space restrictions and because the results give a clear picture of what happens in the other sequences.

In the second experiment, we measured the population variance as described in [10]. It is expressed as:

$$\mathrm{Var}\{\boldsymbol{y}\} = \frac{1}{\mu}\sum_{m=1}^{\mu}\left(\boldsymbol{y}_m - \langle\boldsymbol{y}\rangle\right),$$

where y defined as the offspring produced. Furthermore, the variable $\langle\boldsymbol{y}\rangle$ is defined as the average of the offspring population, which is expressed as:

$$\langle\boldsymbol{y}\rangle = \frac{1}{\mu}\sum_{m=1}^{\mu}\boldsymbol{y}_m.$$

We conducted the experiment on the same functions as in the previous experiment. The data was also collected as the average of 15 trials. In Appendix C, we see the results of this experiment with function 15. The results of this experiment shows that the variance of the designs is higher than that of CMA-ES. The results of the two experiments show that the hypothesis is true, which means that the performance improvement is indeed caused by a higher variance of the population.

4.4 Future Work

We are restricted by time and thus limited to what could have researched and tested. In the following text we suggest multiple ideas for future research.

- The different designs that have been compared with each other, used all the default configuration settings of the canonical CMA-ES. Therefore, our first suggestion is to conduct experiments with non-default parameter settings, such as for example using a lower or higher value for the number of parent and offspring solutions that created in each iteration.

- The designs that we have proposed were tested with the canonical CMA-ES. Many variants have been proposed of the CMA-ES, such as described in Chapter 3.2. Another suggestion for further research is to compare the performance of our designs with that of those variants. It would allow us to get a better view of the value of our contributions.

- We measured the variation of the objective fitness function values of the offspring solution during the optimization process. We noted that the value that we measured during the process varied much. It might be a good idea to adapt the parameters to this, such as lowering the number of solutions created when the variation is low and increase the number of solutions created when it is high. By decreasing the value of the parameter when the variation is low, leads to that fewer offspring are sampled when it is less needed, because the low variation suggest that a saturation is reached about what more can be discovered in this area of the search space. Then, by increasing the number of offspring produced when the variation is high, has objective analyse the area of the search space in more detail, because the high variation suggests that no saturation is reached about what can be discovered more in this area of the search space.

- Based on the measurements of the variation of objective function fitness value values we would also like to suggest to adapt the parameters (such as c_c, c_σ, c_1, c_μ) that are responsible for specifying the rate at which information is decayed, to this information. When a high variation is measured, the information that is stored is more susceptible to noise. This should especially hold for information from less recent iterations.

4.5 Conclusions

The canonical CMA-ES is a heuristic that can be applied to solve single objective optimization problems. Multiple strategy parameters determine how it searches for improvements. Several of these parameters are self-adaptive, while the others remain unchanged. The covariance matrix is one example of parameters that is updated at the end of each iteration. It does so by using information collected from evaluation of solutions.

The weighted mean is one example of parameters that remains unchanged during the updating process. The weighting scheme governs the the responsibility of defining in each evolution cycle a new mean value. It is then used by the sampling procedure to define new solutions. The mean is defined as the weighted average of a number of optimal solutions. For each optimal solution there exists one weight value, that specifies the amount of information that is used from that solution to define the mean. We considered making the weights adaptive to search space, instead of keeping them static, it might be possible to obtain a performance gain. So, the objective of this thesis was to design a method that allows this.

Multiple designs have been proposed to make an adaptive recombination weighting scheme. It is done by defining an additional set of weights, so-called *penalty weights*. As with the original set of weights, there is one penalty weight value for each solution. The values of these weights are defined by assigning a penalty for solutions with a lower objective function value. After computing the penalty weights, a computation is performed on the original set of weights to design the new final set of recombination weights. This computation takes into account that when there the variation between the solutions is too low, to be relevant, the original set of recombination weights are used.

We have performed an elaborate test on BBOB (Black Box Optimization Benchmark). First, for each different design we tested 10 different configurations. We did this for both a (μ, λ)- and $(\mu + \lambda)$-strategy. Then we selected the optimal configuration from each design for each different type of strategy and compared it with the canonical CMA-ES with the same type of strategy (thus (μ, λ)- or (μ, λ)-ES). The results show that in general the canonical CMA-ES performs better. However, as discussed in Chapter 4.2, the designs that we proposed performed very well on multi-model objective functions. We hypothesized that it was caused by that the variance of the recombination weights of our designs must be higher. In order to find out what caused these results, we decided to conduct two additional experiments, the first one measured the variance of the population sampled by the mean and in the second experiment we measured the variance of the population. The results seem to confirm our hypothesis.

Appendices

Appendix A

BBOB Results

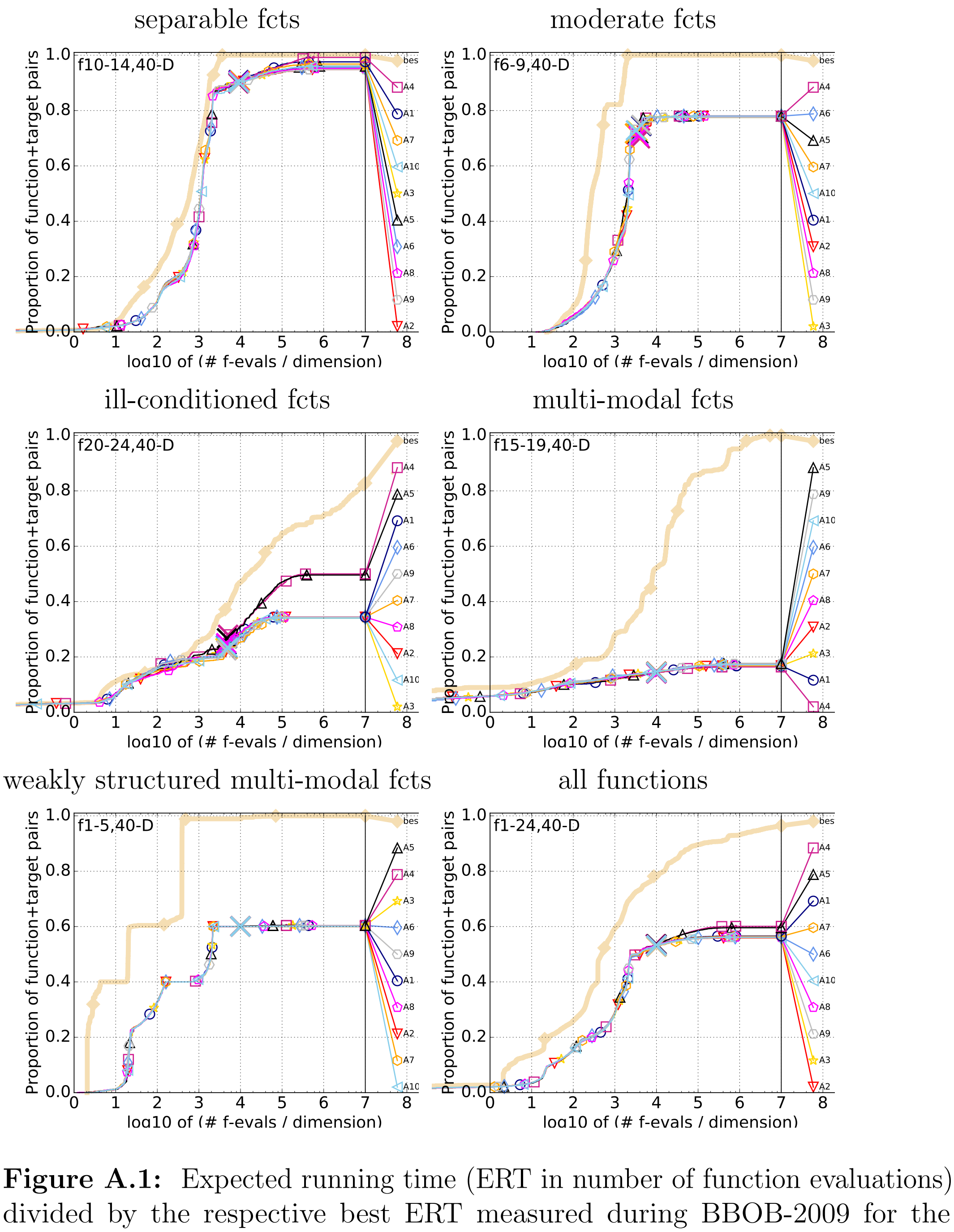

Figure A.1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation A. Measured for all functions on search spaces with a dimensionality of 40.

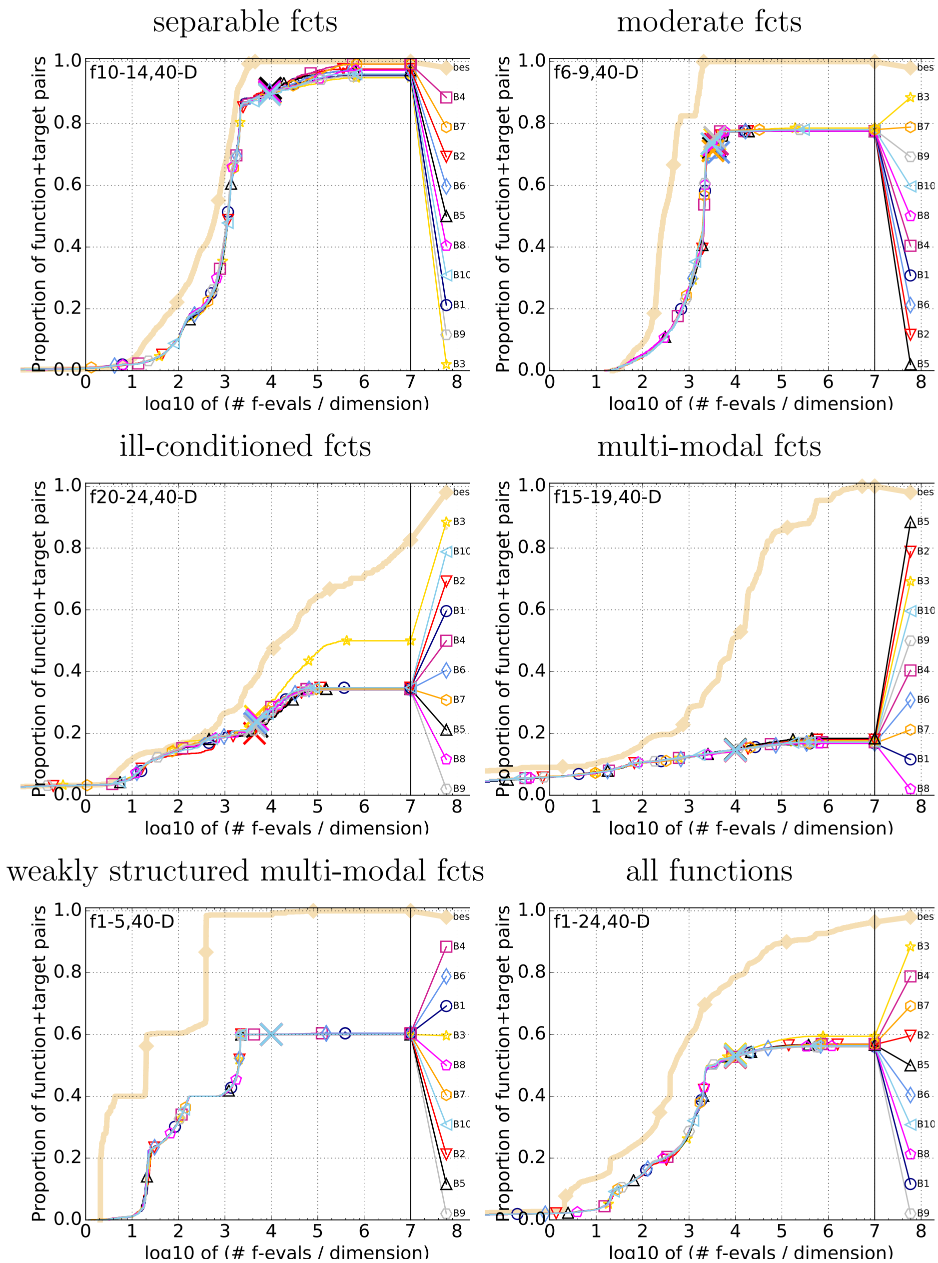

Figure A.2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation *A*. Measured for all functions on search spaces with a dimensionality of 40.

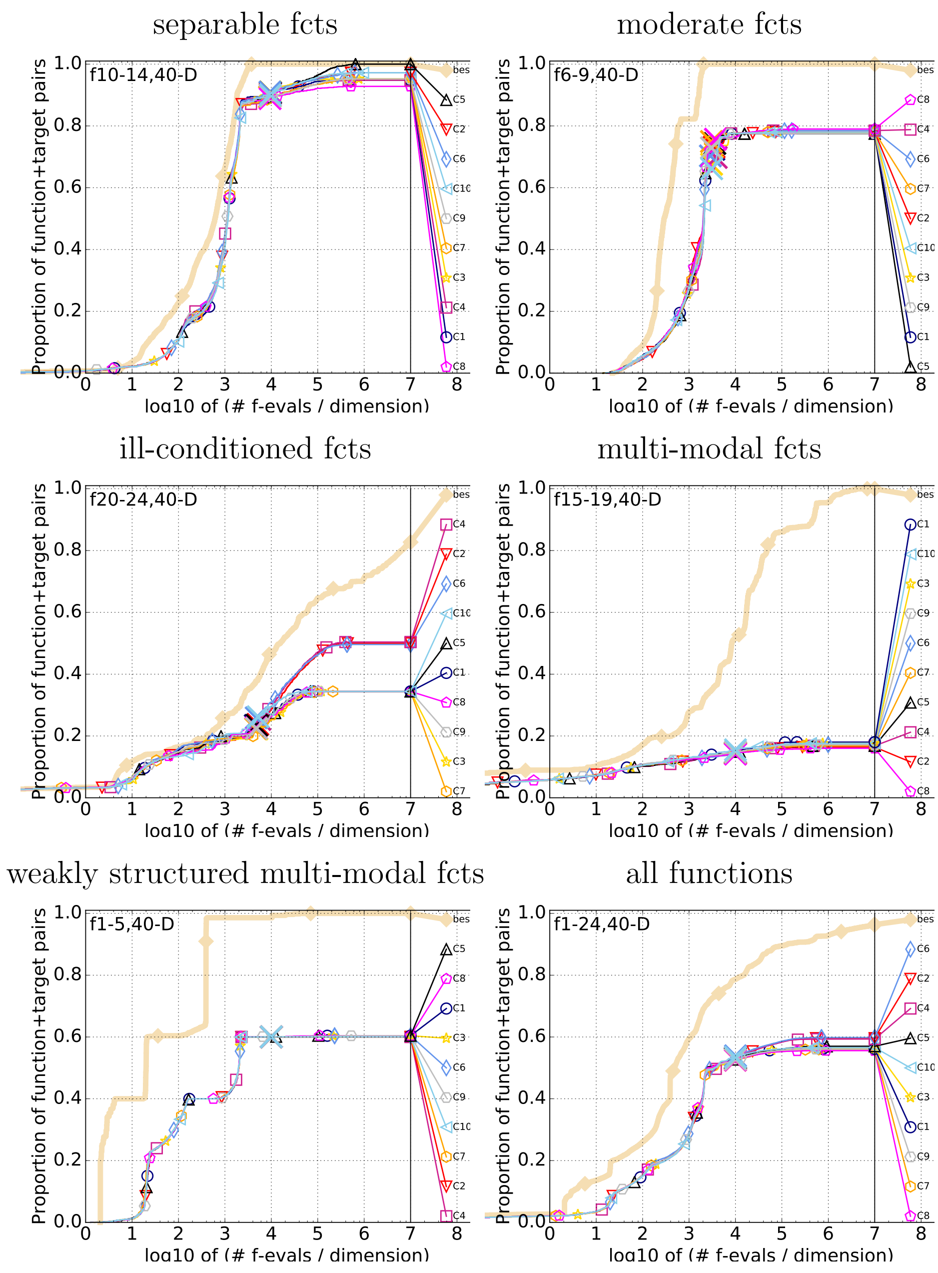

Figure A.3: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation A. Measured for all functions on search spaces with a dimensionality of 40.

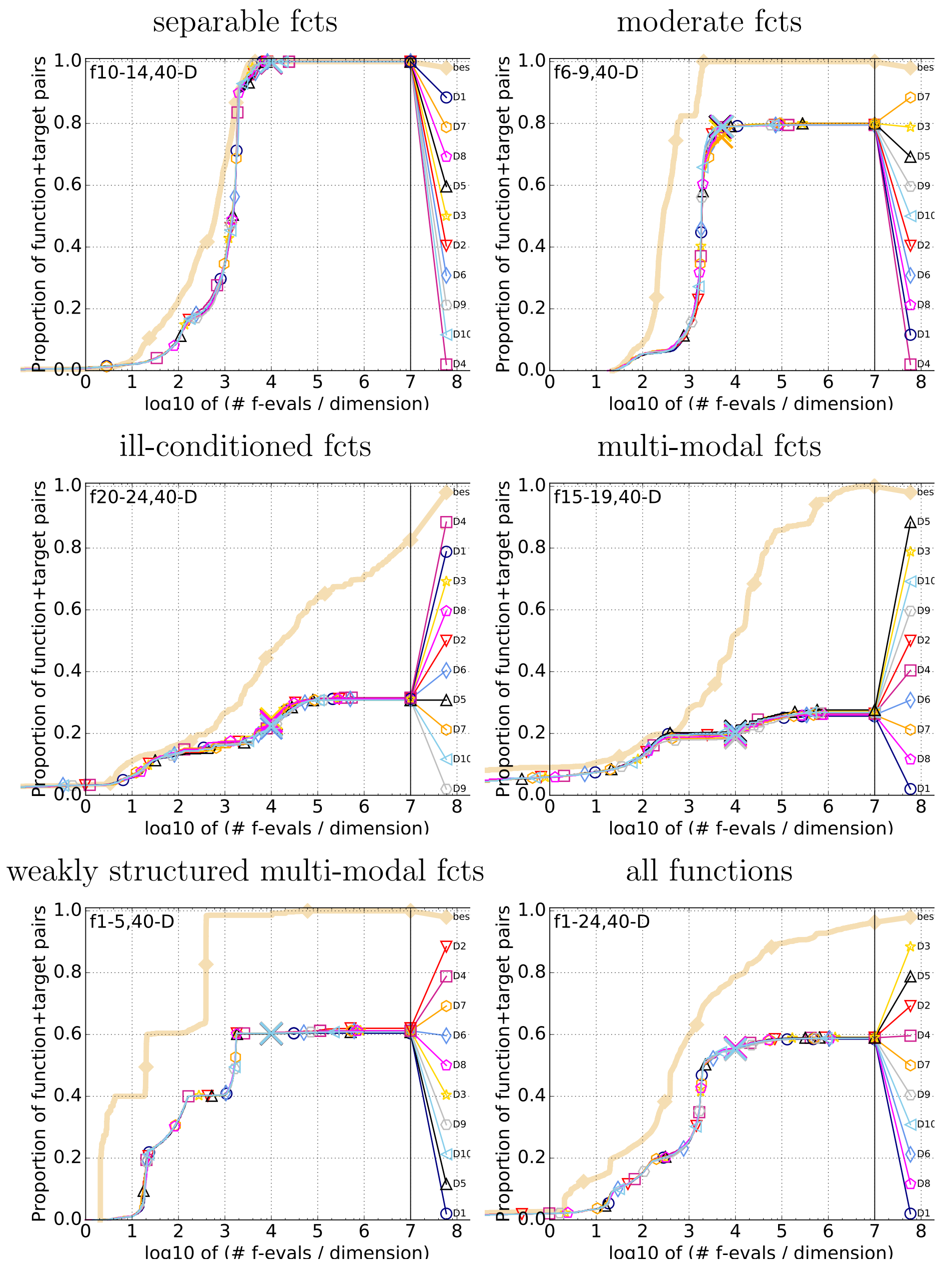

Figure A.4: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation A. Measured for all functions on search spaces with a dimensionality of 40.

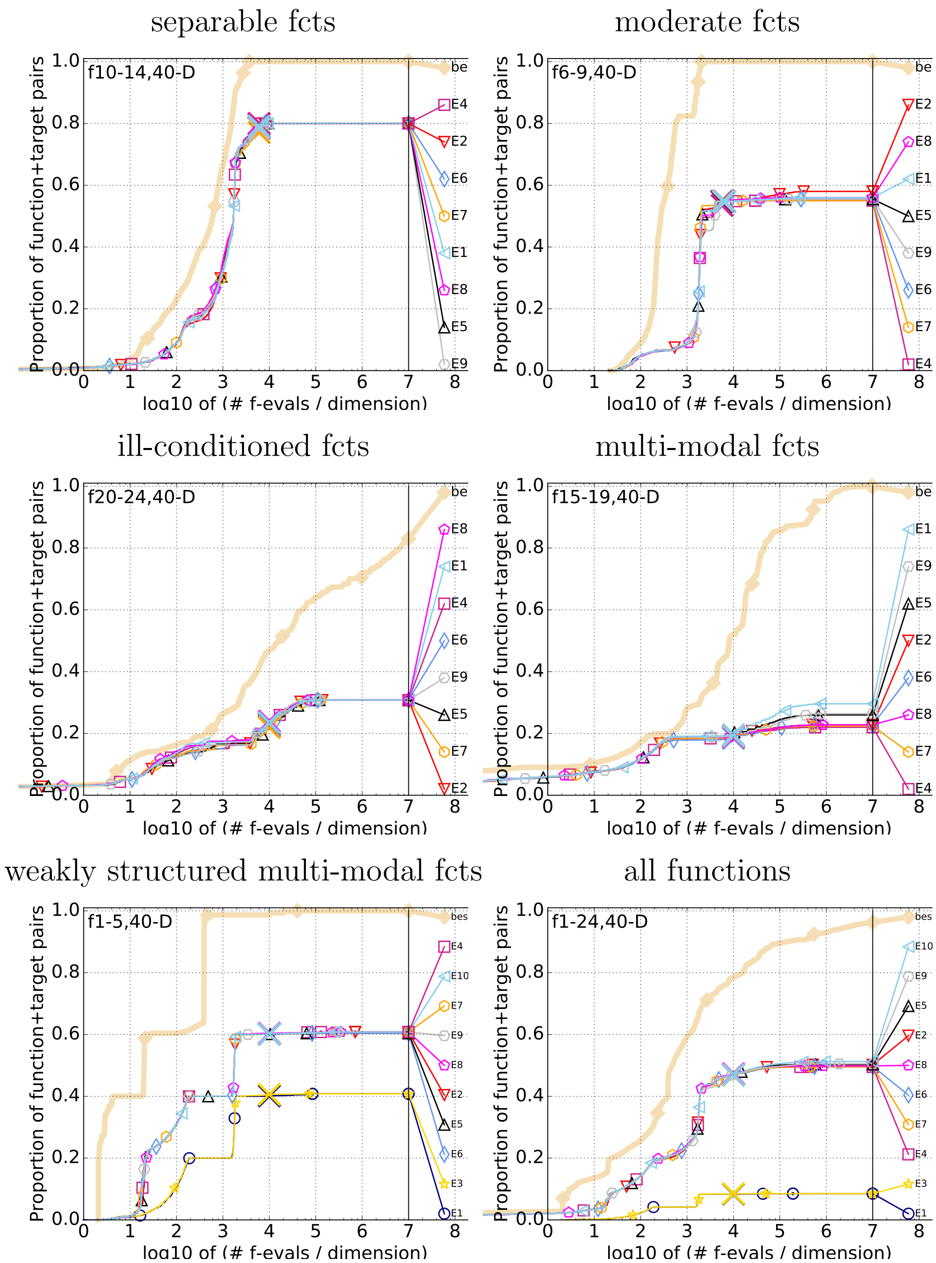

Figure A.5: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation A. Measured for all functions on search spaces with a dimensionality of 40.

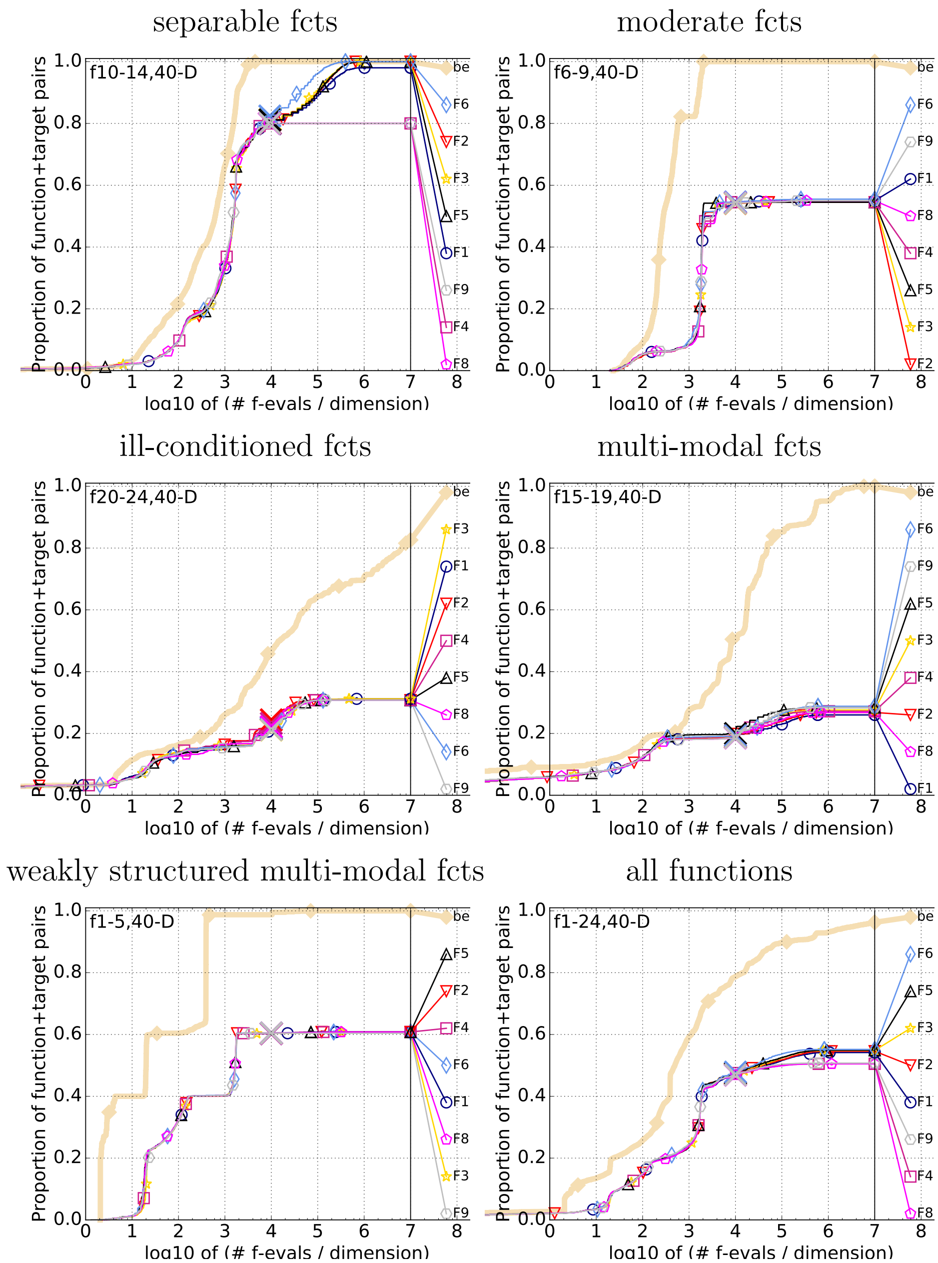

Figure A.6: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 for the different configurations of the implementation A. Measured for all functions on search spaces with a dimensionality of 40.

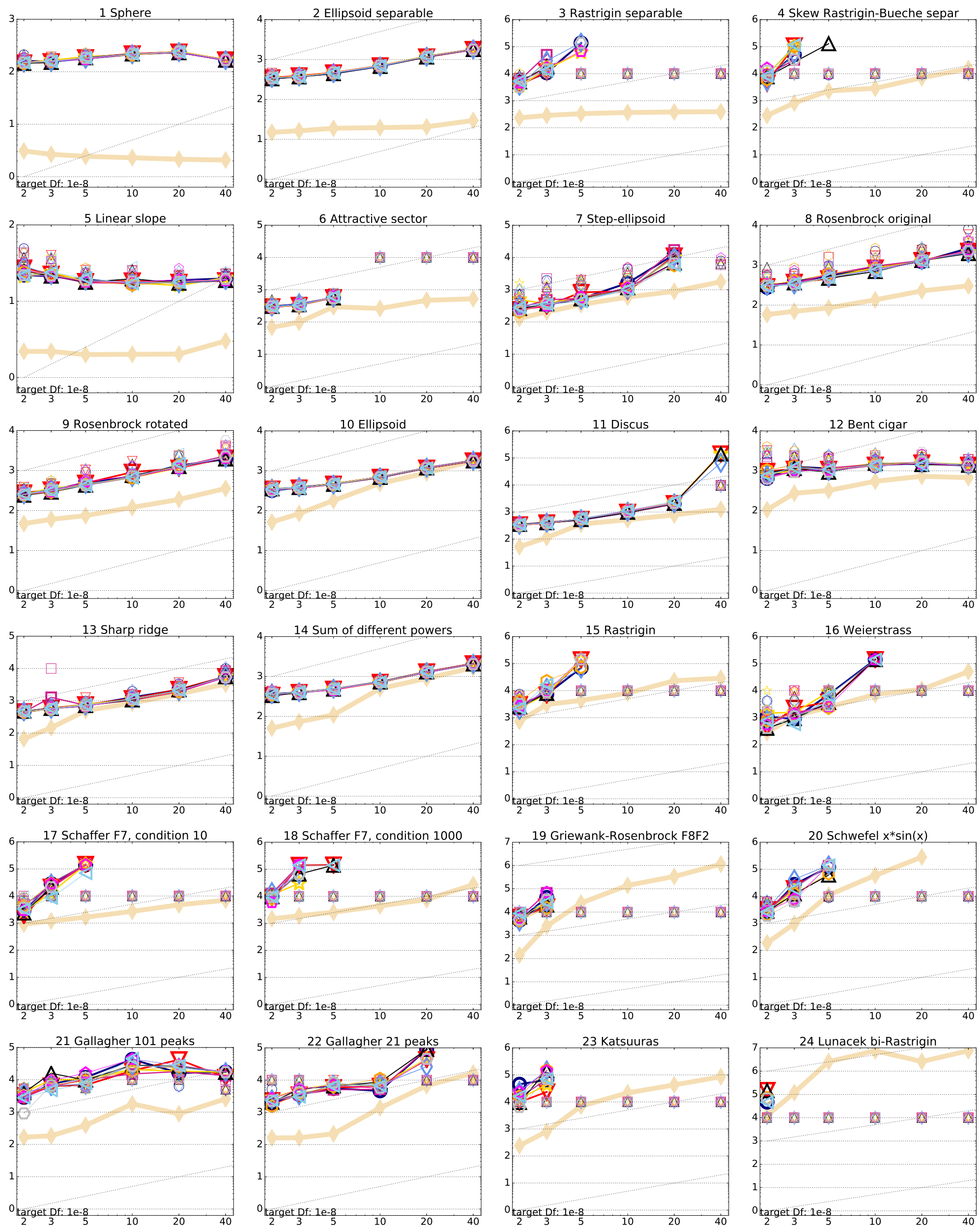

Figure A.7: Expected running time (ERT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○:A1, ▽:A2, ⋆:A3, □:A4, △:A5, ◇:A6, ⬡:A7, ⬠:A8, ⬡:A9, ◁:A10

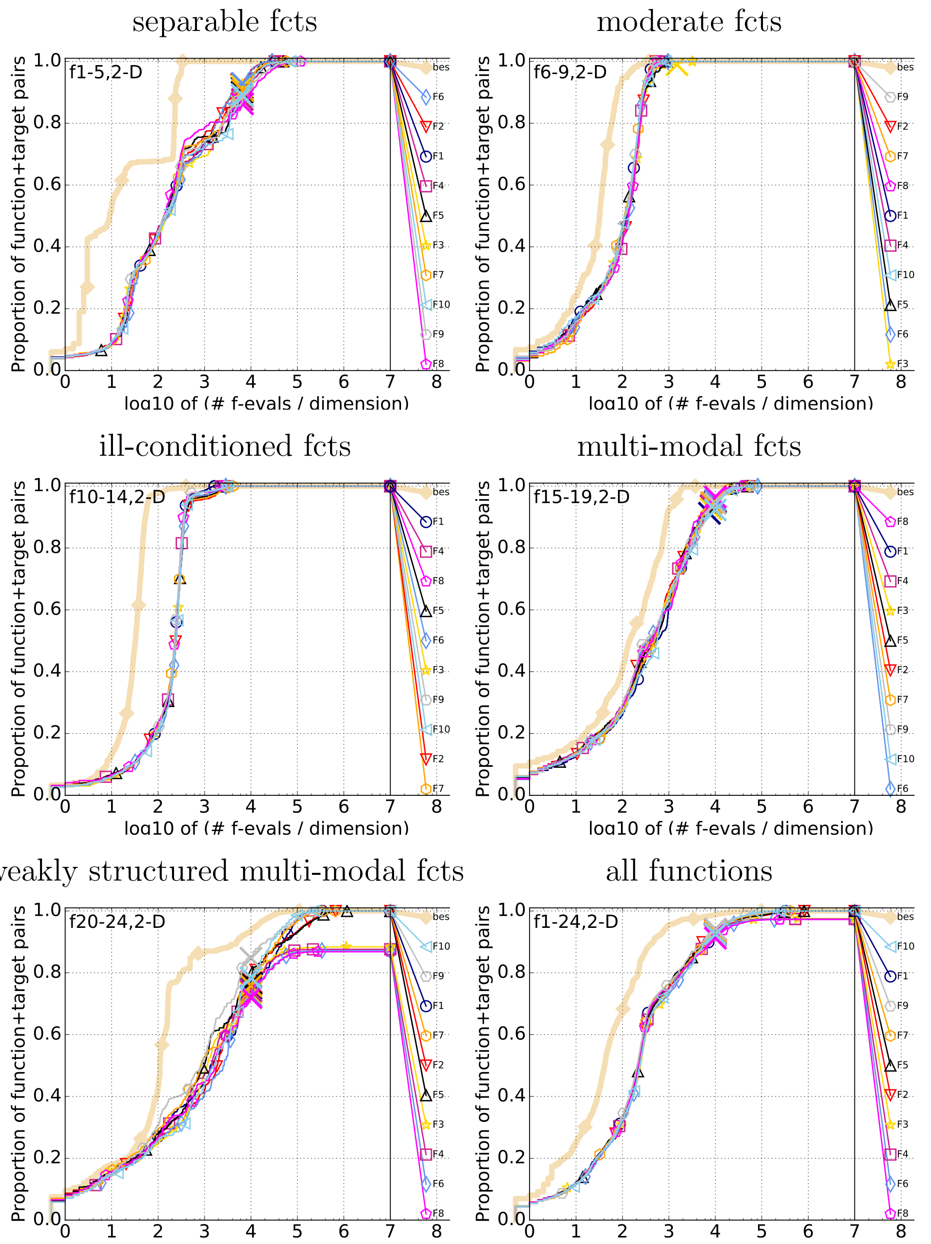

Figure A.8: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

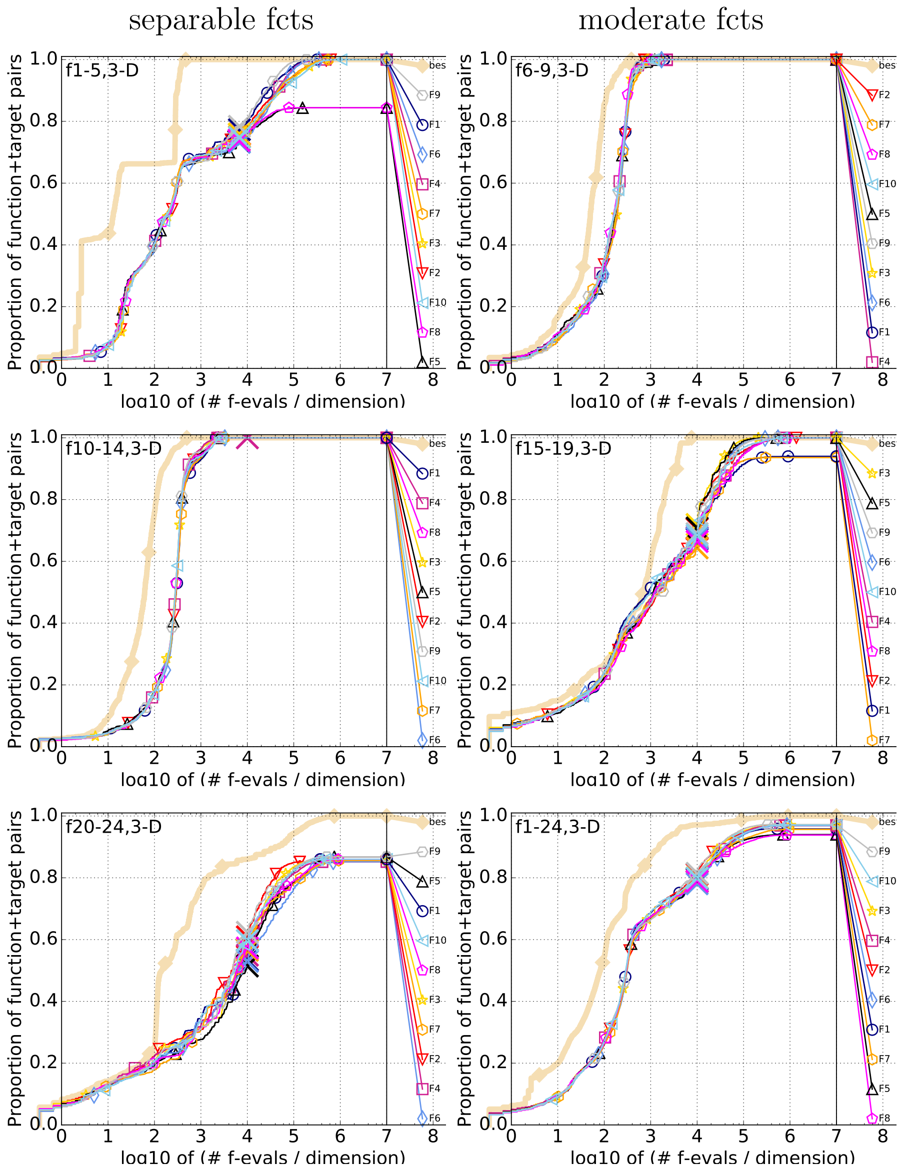

Figure A.9: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

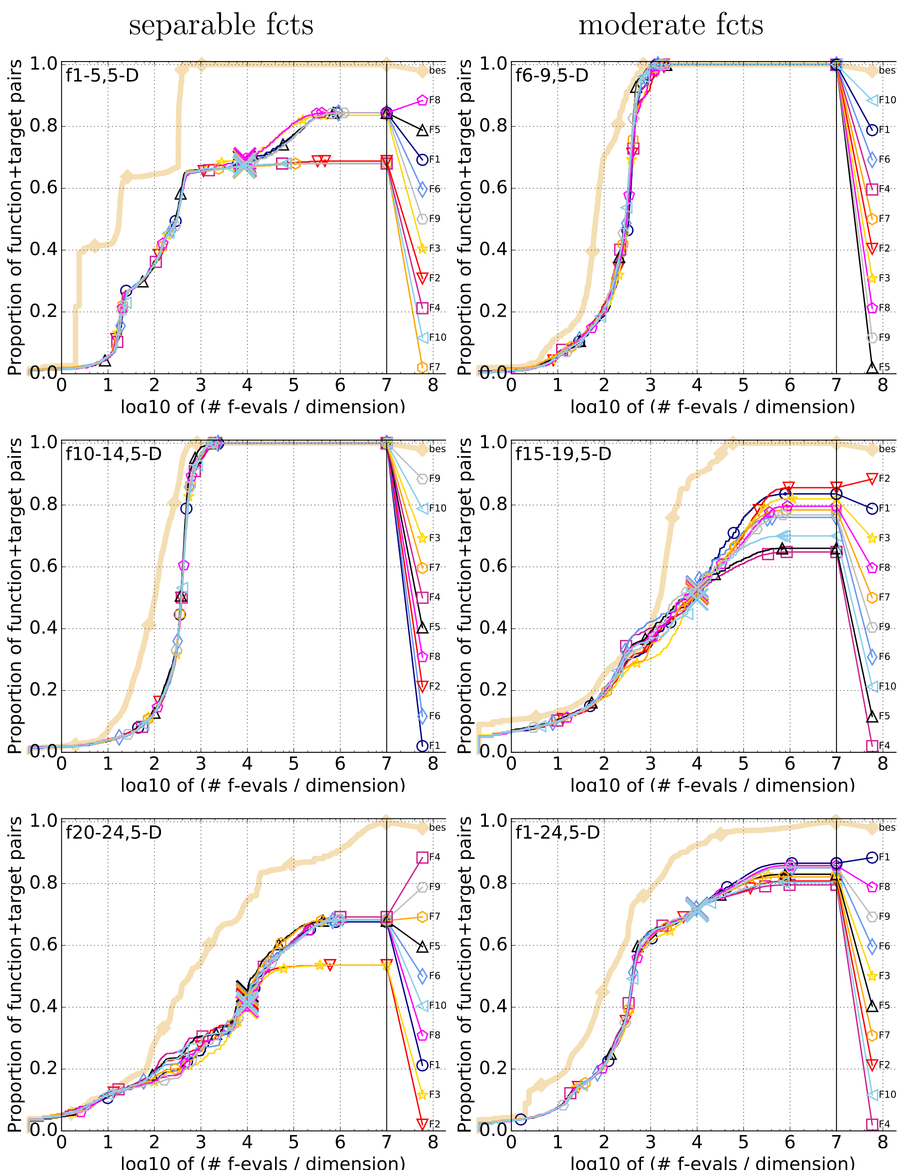

Figure A.10: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

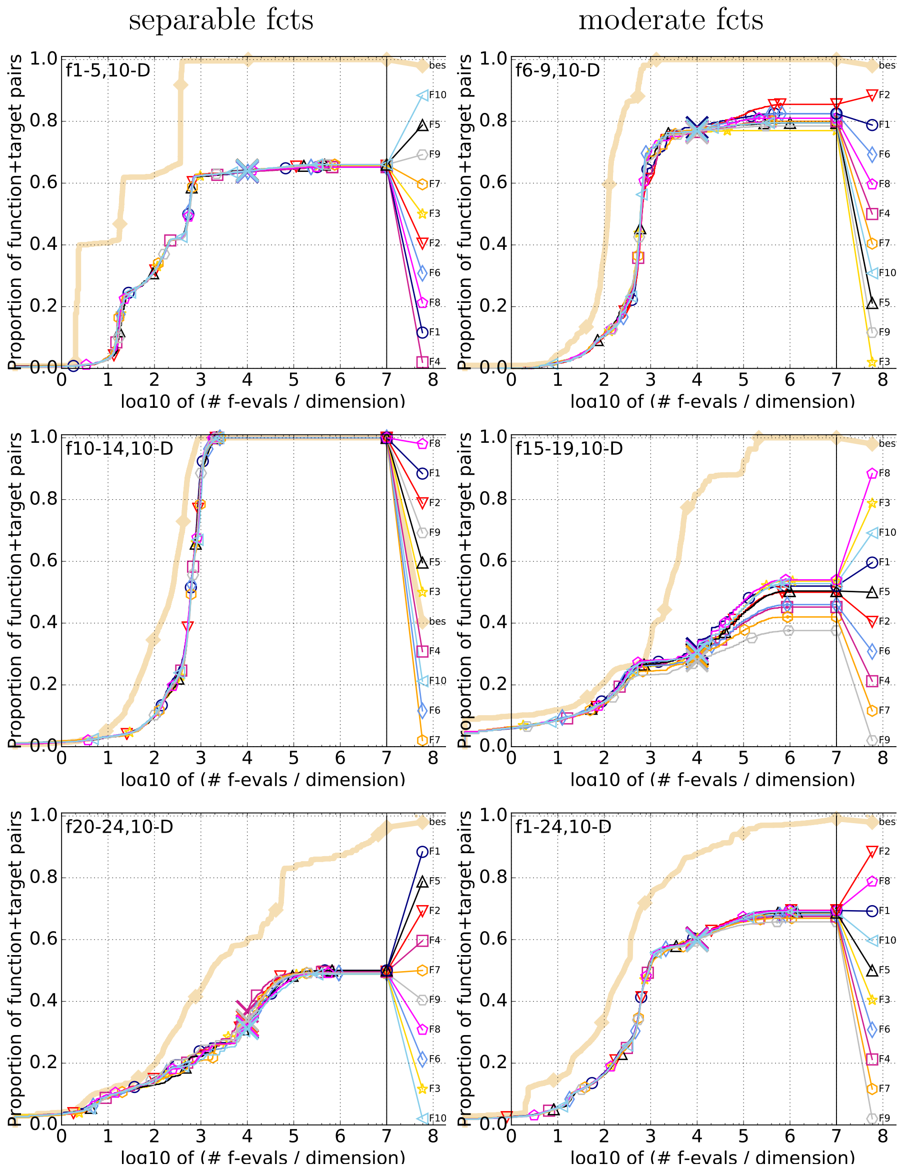

Figure A.11: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

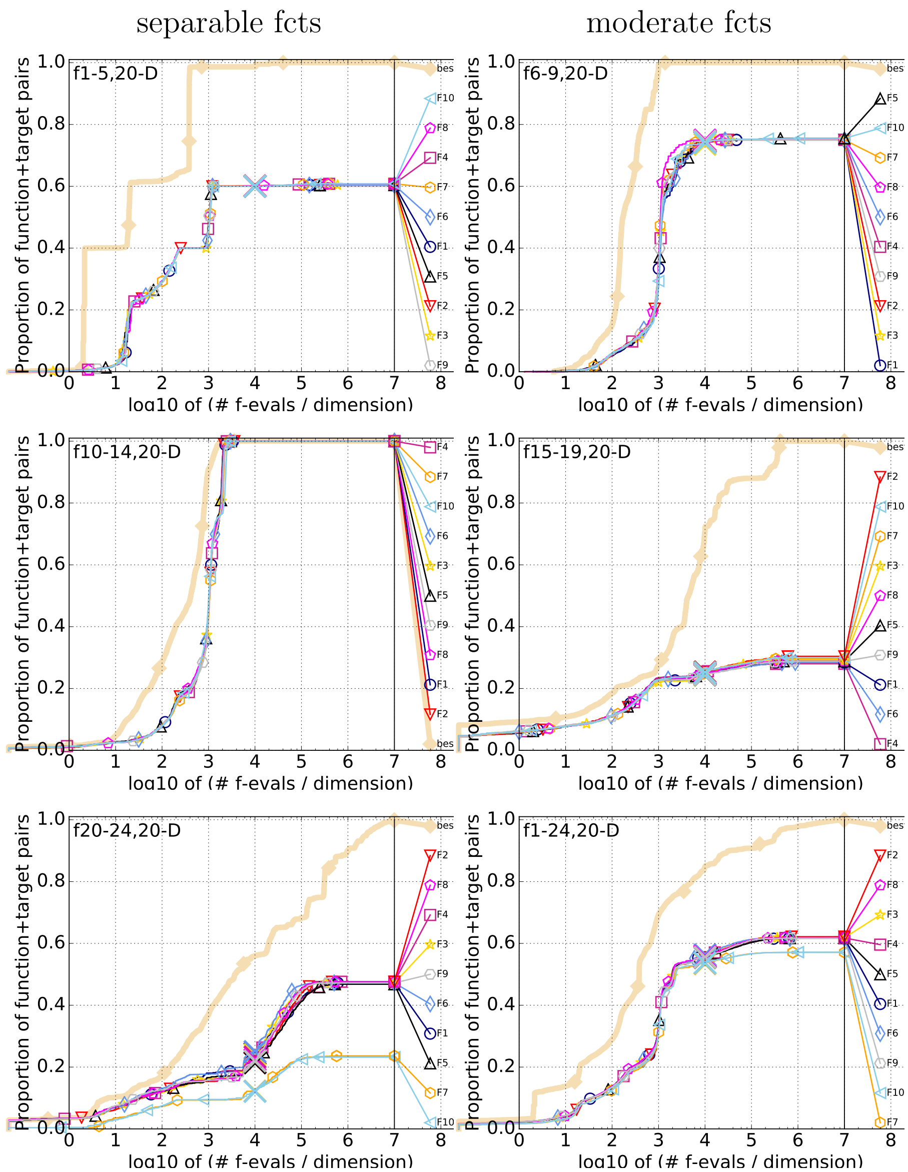

Figure A.12: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

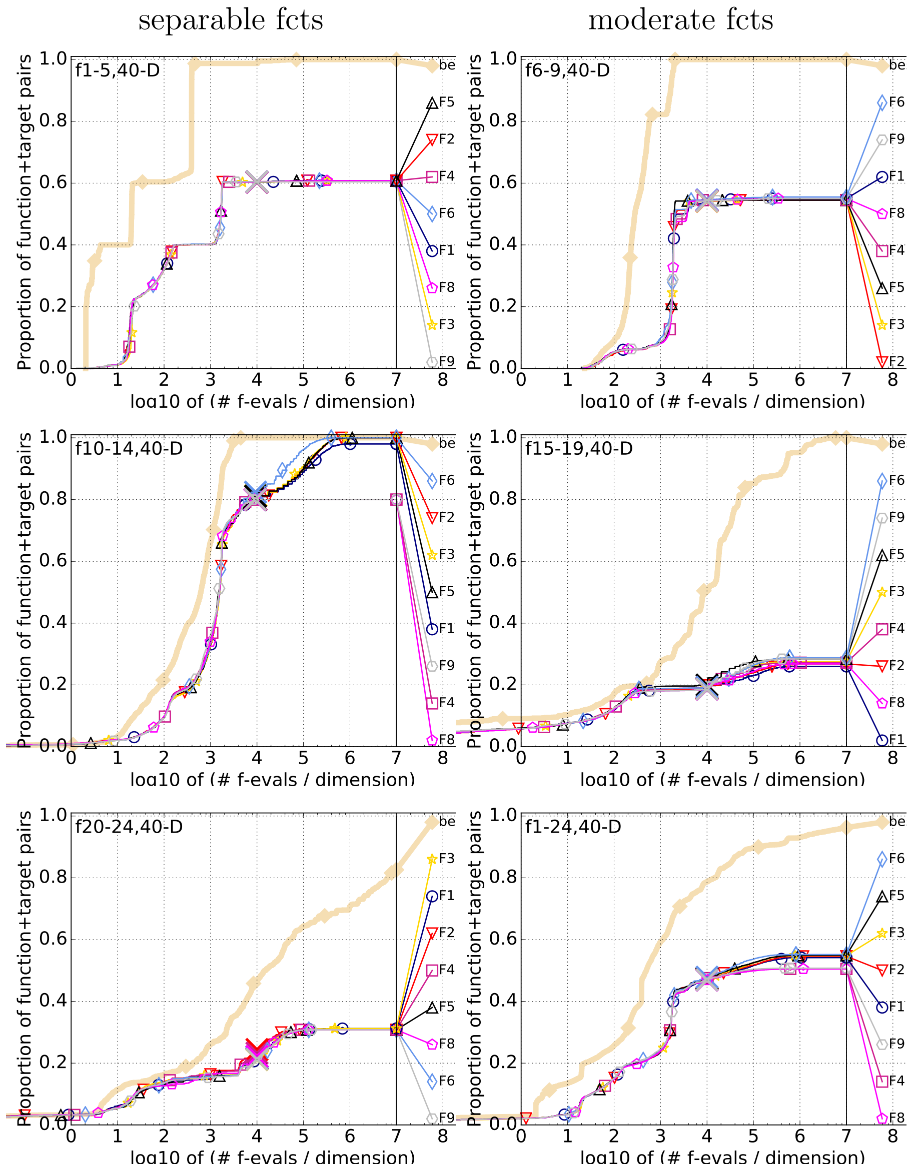

Figure A.13: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

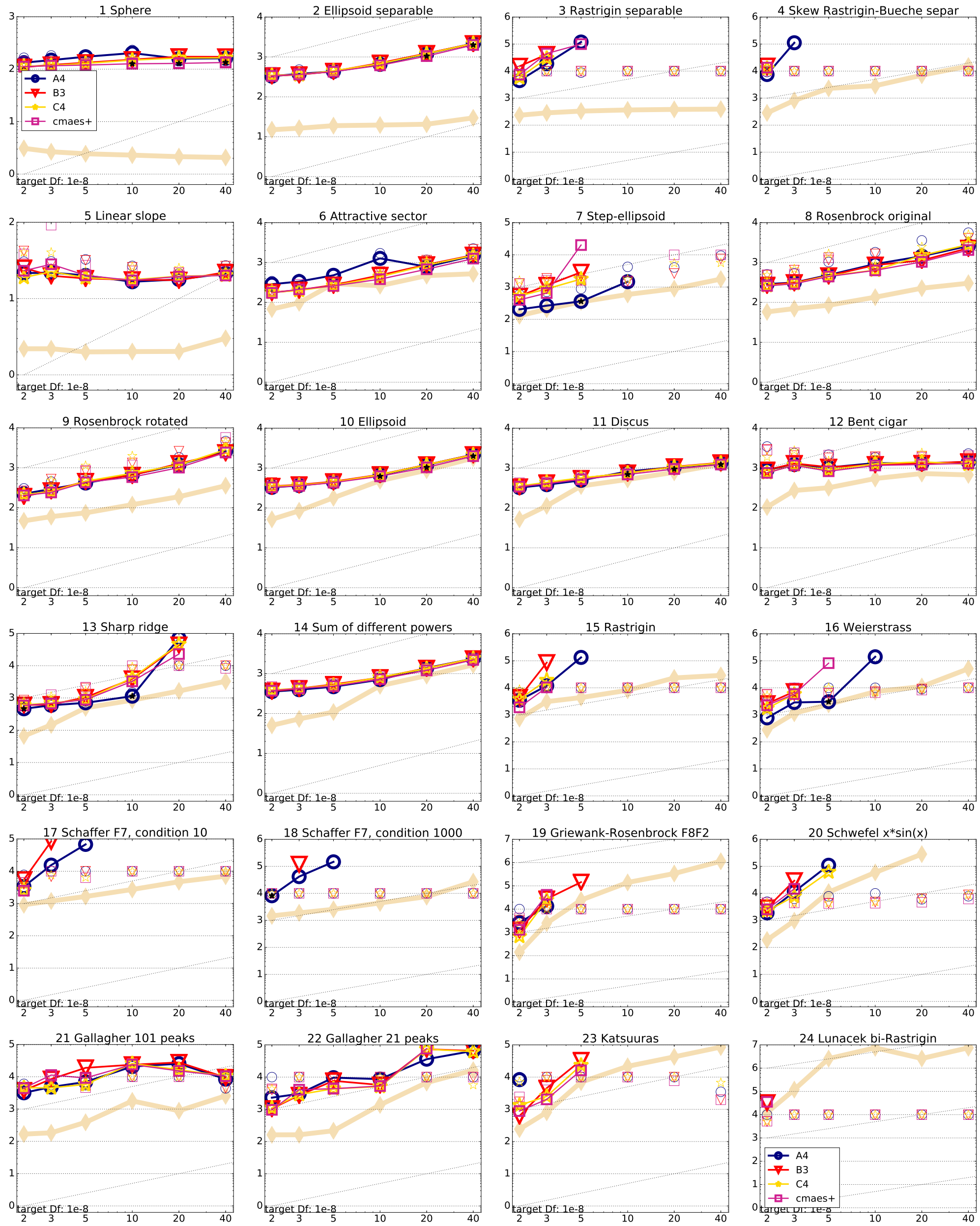

Figure A.14: Expected running time (ERT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○:A1, ▽:A2, ⋆:A3, □:A4, △:A5, ◇:A6, ⬡:A7, ⬠:A8, ⬡:A9, ◁:A10

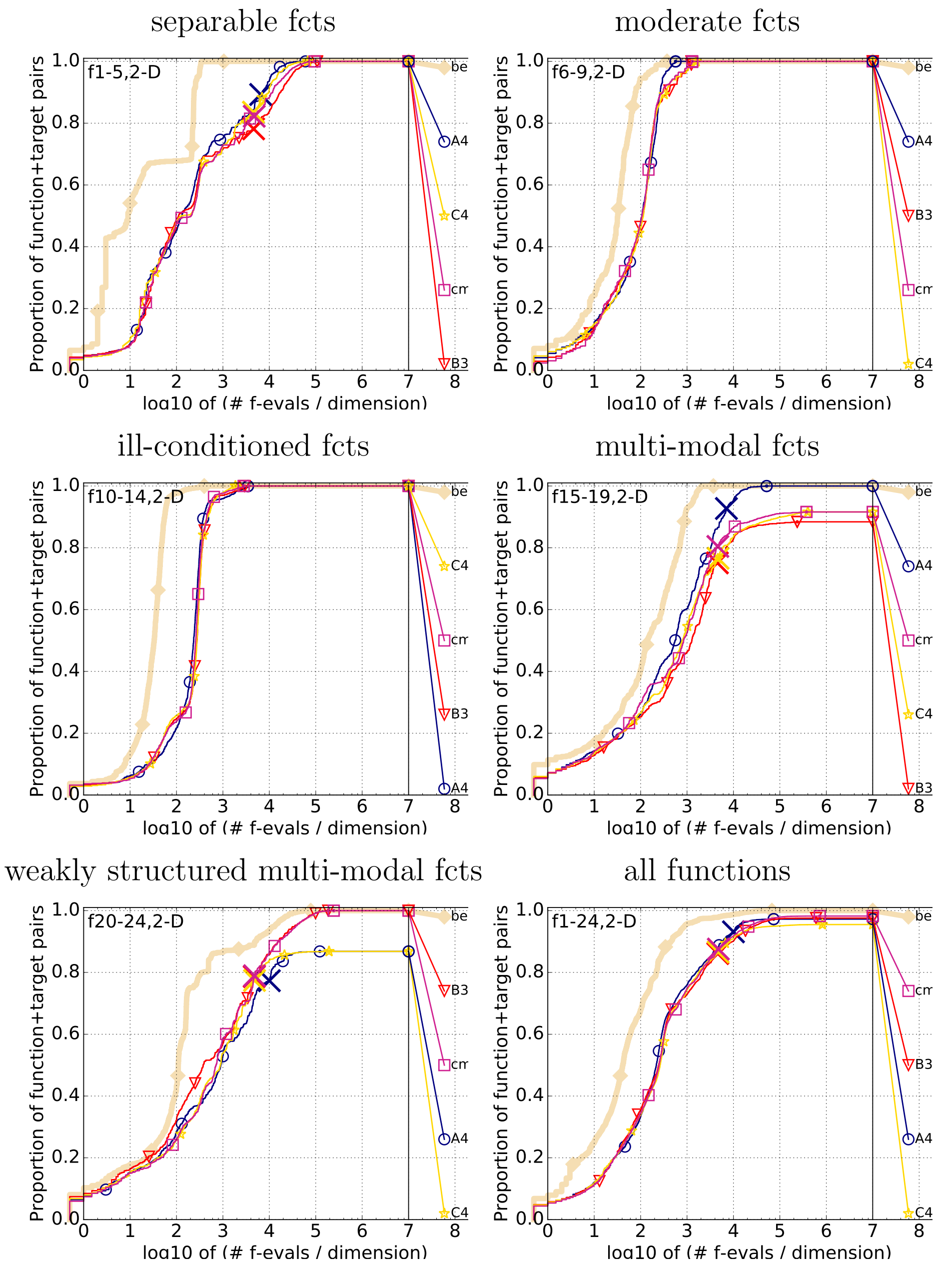

Figure A.15: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

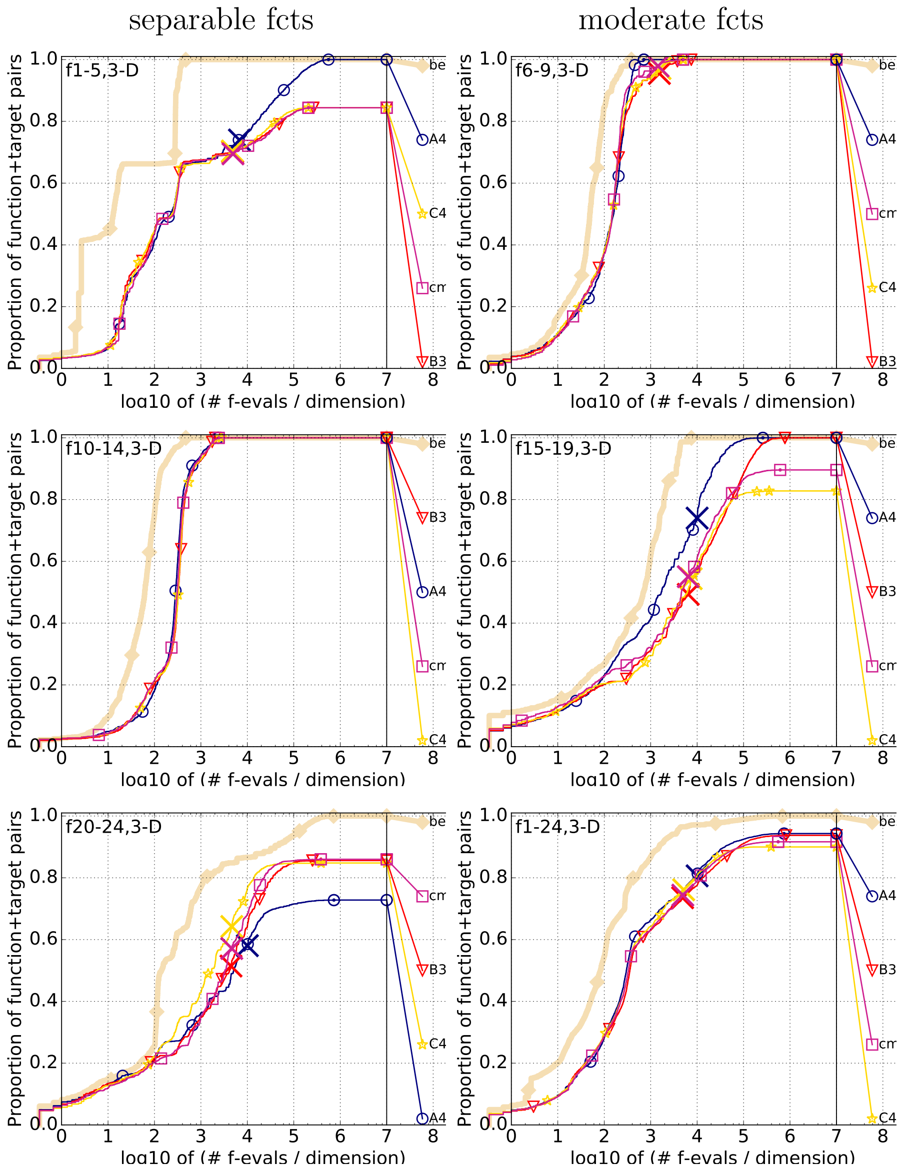

Figure A.16: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

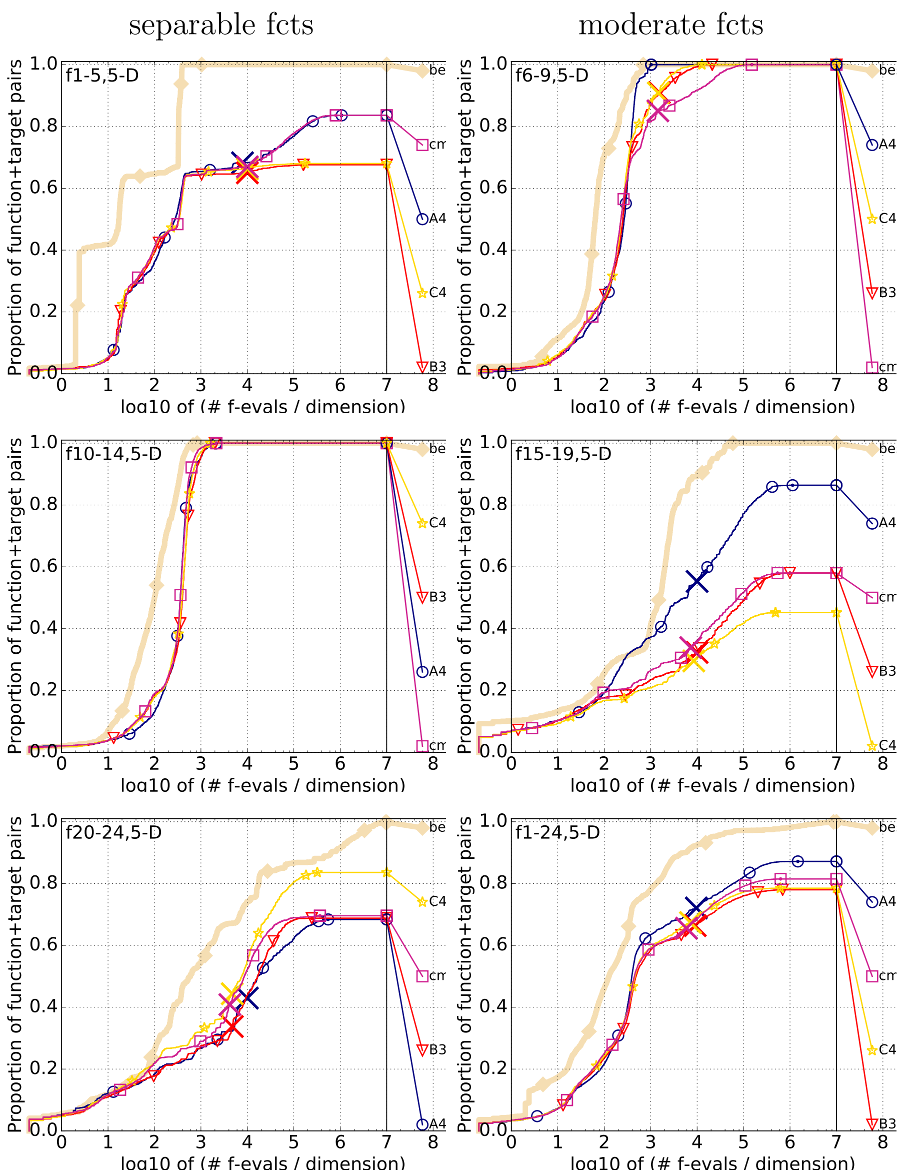

Figure A.17: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

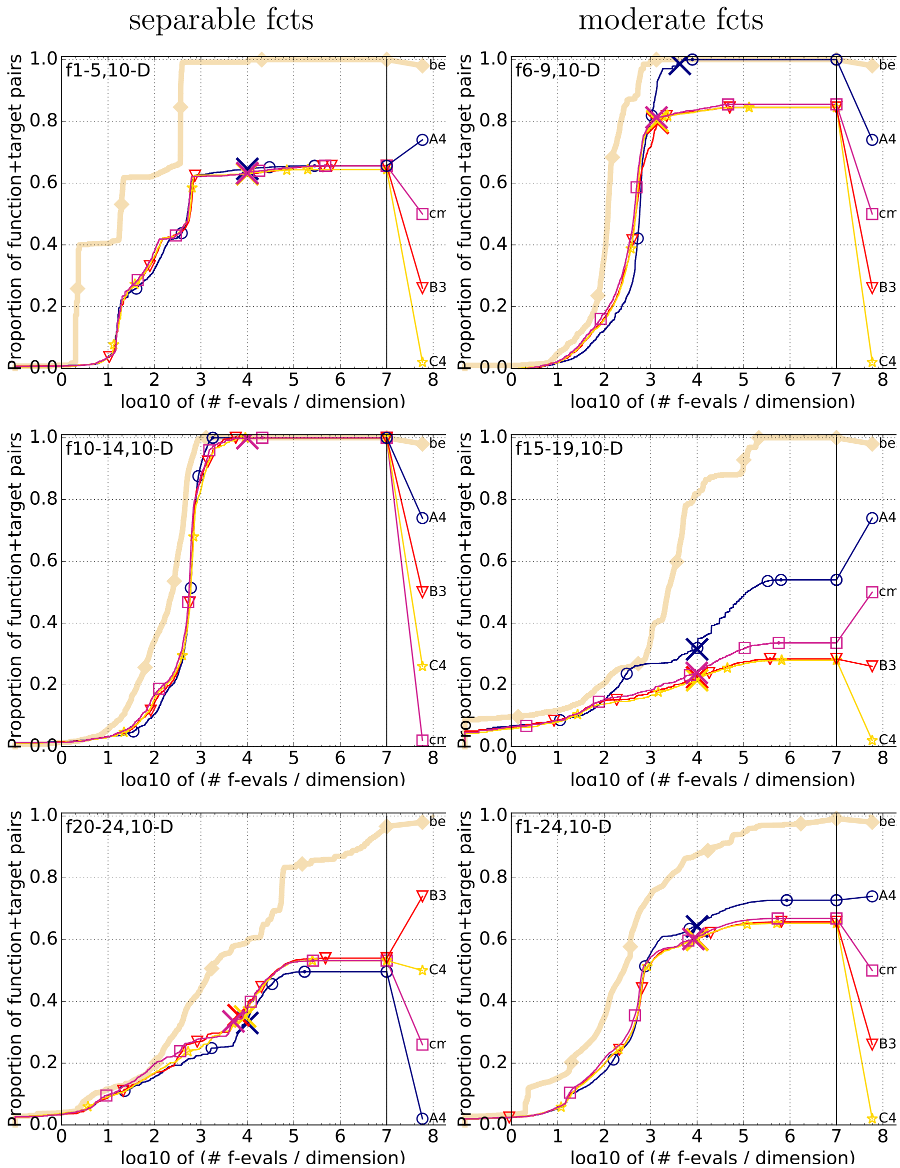

Figure A.18: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

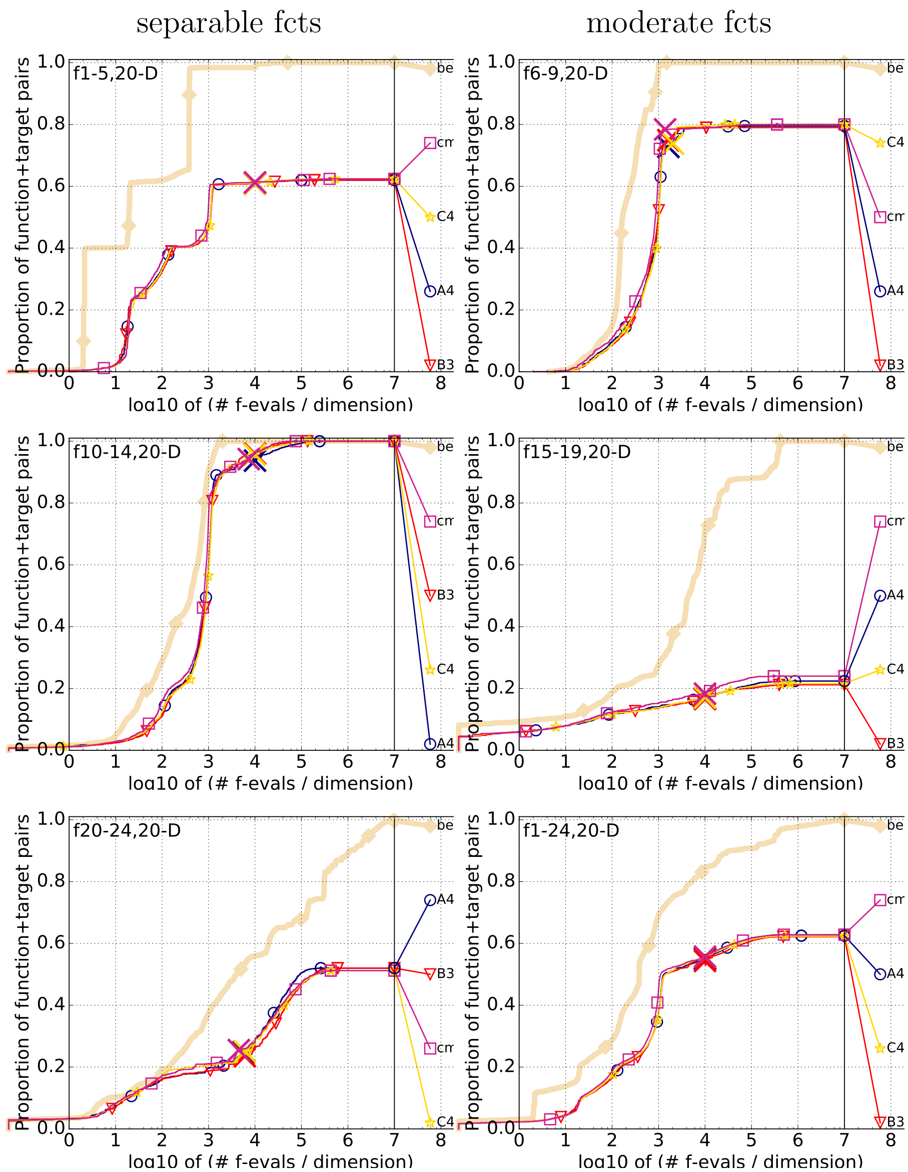

Figure A.19: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

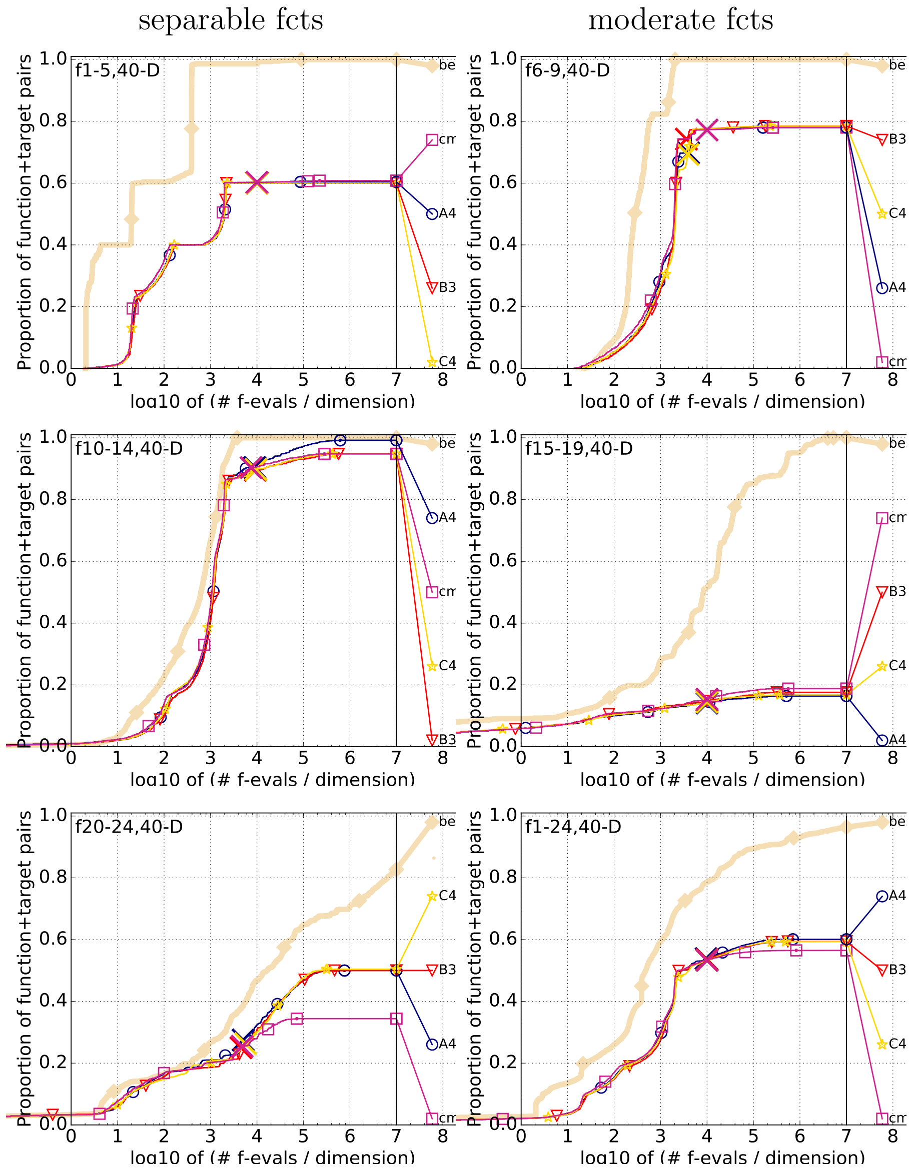

Figure A.20: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	1.8	5.7	5.7	6.2	6.2	6.2	6.2	15/15
A4	5.0$_{(4)}$	**4.5**$_{(4)}$	9.0$_{(5)}$	14$_{(5)}$	19$_{(5)}$	29$_{(4)}$	39$_{(7)}$	15/15
B3	6.0$_{(5)}$	5.3$_{(3)}$	**9.0**$_{(4)}$	**12**$_{(3)}$	**16**$_{(6)}$	**24**$_{(5)}$	32$_{(6)}$	15/15
C4	**4.9**$_{(6)}$	4.6$_{(3)}$	10$_{(3)}$	12$_{(3)}$	16$_{(4)}$	24$_{(4)}$	**32**$_{(4)}$	15/15
cmaes+	7.9$_{(8)}$	5.4$_{(3)}$	10$_{(8)}$	13$_{(6)}$	17$_{(6)}$	25$_{(5)}$	33$_{(6)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	16	19	25	25	26	28	29	15/15
A4	11$_{(10)}$	**17**$_{(9)}$	**15**$_{(6)}$	**18**$_{(4)}$	**19**$_{(4)}$	**20**$_{(3)}$	**22**$_{(3)}$	15/15
B3	**7.0**$_{(4)}$	20$_{(7)}$	20$_{(4)}$	21$_{(3)}$	22$_{(3)}$	22$_{(3)}$	23$_{(4)}$	15/15
C4	11$_{(8)}$	20$_{(12)}$	19$_{(3)}$	21$_{(3)}$	22$_{(2)}$	22$_{(2)}$	23$_{(3)}$	15/15
cmaes+	8.7$_{(11)}$	17$_{(12)}$	19$_{(7)}$	21$_{(3)}$	21$_{(3)}$	22$_{(2)}$	23$_{(3)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	15	271	445	446	450	454	464	15/15
A4	6.5$_{(5)}$	**5.9**$_{(6)}$	**17**$_{(19)}$	**19**$_{(29)}$	**19**$_{(10)}$	**19**$_{(26)}$	**19**$_{(24)}$	12/15
B3	19$_{(3)}$	17$_{(38)}$	68$_{(67)}$	67$_{(110)}$	67$_{(67)}$	67$_{(94)}$	65$_{(72)}$	4/15
C4	**5.4**$_{(3)}$	17$_{(51)}$	25$_{(36)}$	25$_{(32)}$	25$_{(66)}$	25$_{(31)}$	24$_{(20)}$	21/35
cmaes+	89$_{(22)}$	12$_{(10)}$	33$_{(31)}$	33$_{(34)}$	33$_{(63)}$	33$_{(23)}$	32$_{(53)}$	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	22	344	459	496	523	544	566	15/15
A4	60$_{(193)}$	14$_{(18)}$	**32**$_{(60)}$	**30**$_{(43)}$	**28**$_{(40)}$	**27**$_{(26)}$	**26**$_{(31)}$	9/15
B3	59$_{(2)}$	17$_{(29)}$	66$_{(71)}$	62$_{(122)}$	58$_{(38)}$	56$_{(72)}$	54$_{(35)}$	5/15
C4	**5.9**$_{(29)}$	**12**$_{(8)}$	45$_{(26)}$	42$_{(53)}$	40$_{(43)}$	39$_{(72)}$	45$_{(81)}$	5/15
cmaes+	67$_{(687)}$	15$_{(19)}$	58$_{(73)}$	54$_{(73)}$	51$_{(78)}$	49$_{(86)}$	48$_{(80)}$	6/15

Figure A.21: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	3.7	4.4	4.4	4.4	4.4	4.4	4.4	15/15
A4	8.1(1)	9.4(1)	10(3)	10(4)	10(3)	10(3)	10(2)	15/15
B3	9.3(3)	11(4)	11(4)	11(6)	11(4)	11(6)	11(6)	15/15
C4	**6.5**(2)	**8.2**(4)	**8.6**(5)	**8.6**(1)	**8.6**(7)	**8.6**(3)	**8.6**(3)	15/15
cmaes+	7.9(2)	10(4)	11(4)	11(4)	11(4)	11(5)	11(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	13	23	41	54	67	95	124	15/15
A4	**2.0**(3)	3.2(0.6)	3.2(1)	3.6(1)	3.5(0.8)	4.0(0.7)	4.1(0.7)	15/15
B3	2.0(1)	2.9(2)	2.5(1)	2.7(1.0)	2.7(0.6)	**2.6**(0.7)	2.6(0.5)	15/15
C4	2.4(3)	2.8(1)	2.4(1)	2.8(0.9)	2.7(0.5)	2.7(0.3)	2.6(0.6)	15/15
cmaes+	2.1(1)	**2.5**(1)	**2.4**(1)	**2.5**(0.4)	**2.6**(0.5)	2.7(0.4)	**2.5**(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	3.2	21	60	193	217	217	241	15/15
A4	**4.5**(6)	2.7(2)	**3.7**(5)	**1.5**(0.9)	**1.4**(0.8)	**1.4**(1.0)	**1.5**(1.0)	15/15
B3	5.7(5)	**2.5**(5)	6.0(11)	3.7(5)	4.6(5)	4.6(4)	4.2(4)	15/15
C4	4.8(4)	4.9(3)	5.3(12)	2.9(4)	4.6(6)	4.6(5)	4.2(6)	15/15
cmaes+	4.7(3)	3.8(0.6)	6.3(9)	2.7(1)	3.4(4)	3.4(5)	3.1(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	5.4	12	37	46	86	94	112	15/15
A4	**3.5**(3)	**7.1**(9)	6.3(6)	6.7(4)	4.2(3)	5.0(4)	4.8(3)	15/15
B3	4.1(3)	8.7(14)	6.2(4)	7.4(5)	4.3(4)	**4.7**(2)	4.5(1)	15/15
C4	3.6(4)	8.2(21)	6.3(4)	**6.6**(6)	**4.2**(2)	4.8(2)	4.4(2)	15/15
cmaes+	4.1(5)	9.0(5)	**6.2**(9)	6.8(4)	4.3(2)	4.7(3)	**4.4**(2)	15/15

Figure A.22: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	1	18	30	44	68	81	92	15/15
A4	3.7(2)	**0.84**(0.6)	2.9(3)	4.6(3)	3.8(2)	4.1(1)	4.4(1)	15/15
B3	6.5(6)	1.3(1)	**2.9**(1)	**4.0**(2)	**3.3**(2)	**3.8**(1)	**3.9**(0.6)	15/15
C4	**3.6**(2)	1.4(0.9)	3.6(3)	4.6(2)	4.0(0.8)	4.1(0.5)	4.3(0.7)	15/15
cmaes+	6.0(2)	1.7(1)	3.9(4)	4.7(3)	4.0(2)	4.0(1)	4.1(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	30	46	54	61	68	82	98	15/15
A4	6.4(6)	**6.7**(3)	**7.7**(1)	**7.3**(1)	**7.0**(0.9)	**6.7**(1)	**6.3**(0.8)	15/15
B3	6.4(5)	9.1(5)	10(2)	8.9(0.2)	8.4(1)	7.6(0.8)	6.9(0.9)	15/15
C4	**6.1**(7)	7.4(5)	9.3(1)	8.9(2)	8.3(0.9)	7.6(1)	6.9(1)	15/15
cmaes+	6.9(7)	8.5(4)	8.9(1)	8.3(1)	7.8(0.8)	7.1(1)	6.5(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	35	45	50	62	67	81	97	15/15
A4	4.8(4)	**7.2**(4)	**7.9**(2)	**7.1**(1)	**7.1**(0.9)	**6.8**(0.4)	**6.2**(0.8)	15/15
B3	4.4(5)	7.9(3)	8.8(3)	8.3(2)	8.1(1)	7.4(0.8)	6.7(0.9)	15/15
C4	**4.2**(6)	8.1(5)	10(2)	8.6(1)	8.6(0.6)	7.8(0.5)	7.0(0.6)	15/15
cmaes+	4.5(5)	8.2(3)	9.1(3)	8.5(2)	8.3(2)	7.5(2)	6.9(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	35	46	75	94	105	153	195	15/15
A4	12(8)	16(8)	13(11)	13(21)	12(3)	10(14)	9.0(14)	15/15
B3	10(17)	16(14)	13(4)	12(9)	12(5)	9.4(5)	8.6(13)	15/15
C4	**4.7**(3)	**7.3**(8)	**8.6**(2)	**9.0**(6)	**8.9**(11)	**7.9**(7)	**7.2**(8)	15/15
cmaes+	7.0(11)	12(16)	10(1)	10(20)	9.4(19)	8.0(15)	7.4(9)	15/15

Figure A.23: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	23	35	46	60	71	95	122	15/15
A4	5.0(4)	**6.7**(2)	**6.9**(1)	**6.9**(2)	**7.0**(0.8)	**7.1**(0.9)$^{\star 2}$	**6.9**(0.8)$^{\star 2}$	15/15
B3	5.9(12)	10(7)	10(8)	10(4)	10(3)	10(2)	9.3(1)	15/15
C4	**4.0**(2)	11(6)	10(6)	8.8(4)	10(2)	10(2)	9.1(2)	15/15
cmaes+	7.9(8)	8.5(6)	10(6)	9.5(4)	8.8(2)	9.4(2)	8.7(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	1.4	7.4	16	24	38	67	90	15/15
A4	2.5(2)	**2.2**(2)	2.7(2)	4.3(2)	4.7(2)	6.1(1)	**6.6**(1.0)	15/15
B3	**2.2**(3)	2.5(3)	**2.7**(2)	**3.6**(2)	4.4(2)	6.1(3)	6.7(3)	15/15
C4	3.1(5)	2.8(3)	2.8(2)	3.7(2)	**4.3**(2)	6.1(2)	7.0(0.9)	15/15
cmaes+	4.9(11)	3.1(3)	3.1(2)	3.6(2)	4.6(2)	**5.8**(1)	6.7(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	37	291	1033	1066	1113	1231	1412	5/5
A4	**2.8**(6)	**3.6**(13)	6.4(7)	6.3(5)	6.0(7)	5.5(4)	4.9(11)	12/15
B3	3.2(0.7)	10(9)	8.4(7)	8.2(7)	7.8(7)	7.1(6)	6.3(6)	10/15
C4	9.0(2)	8.0(2)	7.1(11)	6.9(11)	6.7(11)	6.1(7)	5.3(10)	19/28
cmaes+	5.3(1)	3.7(4)	**3.5**(3)	**3.4**(4)	**3.3**(2)	**3.0**(3)	**2.7**(1)	13/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	9.1	50	174	326	358	409	538	15/15
A4	3.2(2)	**13**(15)	**4.4**(2)	**2.8**(2)	**3.8**(2)	**3.5**(3)	**2.8**(5)	15/15
B3	**2.2**(2)	25(43)	12(22)	8.0(3)	8.1(11)	8.9(9)	8.9(15)	13/15
C4	2.4(2)	27(45)	12(7)	7.3(15)	7.4(13)	7.8(6)	6.4(7)	15/15
cmaes+	3.7(0.8)	22(41)	15(29)	9.2(9)	10(13)	11(8)	8.3(10)	14/15

Figure A.24: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	2.7	61	133	275	396	1086	1657	5/5
A4	78$_{(5)}$	10$_{(7)}$	**5.6**$_{(4)}$	**3.3**$_{(5)}$	**3.2**$_{(2)}$	**4.1**$_{(5)}$	4.0$_{(9)}$	14/15
B3	64$_{(92)}$	16$_{(33)}$	11$_{(20)}$	6.4$_{(11)}$	5.4$_{(10)}$	10$_{(11)}$	6.6$_{(7)}$	8/15
C4	**16**$_{(51)}$	**8.9**$_{(9)}$	6.5$_{(9)}$	3.6$_{(5)}$	4.8$_{(6)}$	4.7$_{(4)}$	3.1$_{(3)}$	12/15
cmaes+	29$_{(102)}$	32$_{(25)}$	18$_{(24)}$	9.1$_{(8)}$	7.1$_{(5)}$	4.5$_{(5)}$	**3.0**$_{(3)}$	12/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	19	134	666	1249	1708	2438	2858	15/15
A4	**3.6**$_{(2)}$	**12**$_{(6)}$	**3.8**$_{(7)}$	**4.3**$_{(2)}{}^{\star}$	**3.3**$_{(2)}{}^{\star 2}$	**6.4**$_{(15)}{}^{\star 2}$	**5.7**$_{(3)}{}^{\star 2}$	10/15
B3	101$_{(339)}$	58$_{(160)}$	54$_{(49)}$	191$_{(158)}$	∞	∞	∞ *2e4*	0/15
C4	230$_{(534)}$	200$_{(188)}$	85$_{(60)}$	199$_{(162)}$	145$_{(144)}$	∞	∞ *2e4*	0/15
cmaes+	202$_{(495)}$	151$_{(124)}$	44$_{(34)}$	30$_{(44)}$	62$_{(66)}$	∞	∞ *2e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	26	216	227	252	276	15/15
A4	**1.5**$_{(1)}$	**6.6**$_{(4)}$	**2.3**$_{(0.7)}$	12$_{(10)}$	15$_{(13)}$	20$_{(45)}$	19$_{(31)}$	14/15
B3	1.9$_{(0.5)}$	51$_{(159)}$	15$_{(21)}$	11$_{(14)}$	11$_{(13)}$	10$_{(13)}$	9.2$_{(8)}$	15/15
C4	2.6$_{(2)}$	11$_{(12)}$	9.3$_{(30)}$	**5.1**$_{(4)}$	**5.1**$_{(8)}$	**4.9**$_{(9)}$	**4.7**$_{(4)}$	15/15
cmaes+	2.1$_{(2)}$	9.1$_{(3)}$	4.8$_{(8)}$	11$_{(34)}$	11$_{(18)}$	10$_{(16)}$	9.3$_{(7)}$	14/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	3.7	61	365	366	366	370	375	15/15
A4	2.5$_{(1)}$	**14**$_{(19)}$	**10**$_{(10)}$	**10**$_{(12)}$	**10**$_{(9)}$	**10**$_{(12)}$	**10**$_{(11)}$	15/15
B3	**2.2**$_{(2)}$	31$_{(29)}$	15$_{(23)}$	15$_{(9)}$	15$_{(36)}$	15$_{(26)}$	15$_{(20)}$	11/15
C4	3.5$_{(4)}$	19$_{(13)}$	12$_{(13)}$	12$_{(7)}$	12$_{(6)}$	12$_{(13)}$	12$_{(12)}$	13/15
cmaes+	2.5$_{(1)}$	29$_{(36)}$	13$_{(12)}$	14$_{(28)}$	14$_{(20)}$	14$_{(8)}$	14$_{(14)}$	12/15

Figure A.25: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	1.7	51	174	276	290	324	330	15/15
A4	2.2(3)	52(124)	**35**(55)	**23**(21)	**22**(18)	**20**(25)	**19**(32)	10/15
B3	2.5(1)	**47**(106)	43(77)	27(35)	26(27)	23(48)	23(21)	9/15
C4	4.7(14)	87(110)	52(35)	33(48)	31(31)	28(43)	28(30)	8/15
cmaes+	**1.6**(0.6)	90(176)	50(80)	32(39)	30(37)	27(5)	27(39)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	5.1	27	168	218	249	289	306	15/15
A4	**15**(36)	97(99)	26(58)	20(20)	18(21)	16(24)	15(42)	13/15
B3	24(120)	**42**(80)	12(3)	9.0(14)	8.0(24)	7.1(8)	6.8(10)	13/15
C4	46(160)	60(101)	12(32)	10(2)	8.5(20)	7.4(12)	7.2(6)	14/15
cmaes+	19(31)	62(170)	**10**(16)	**8.2**(3)	**7.3**(19)	**6.5**(8)	**6.2**(15)	13/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	7.8	193	234	263	299	348	379	15/15
A4	1.5(1)	9.0(22)	61(87)	62(82)	54(54)	47(59)	44(54)	9/15
B3	**1.2**(0.7)	**3.0**(5)	**3.6**(6)	**3.5**(4)	**3.3**(4)	**3.1**(4)	**3.1**(3)	15/15
C4	2.0(0.5)	4.9(3)	10(11)	9.0(19)	8.1(23)	7.3(3)	6.9(7)	15/15
cmaes+	2.2(3)	6.6(3)	5.9(3)	5.6(6)	5.1(7)	4.7(5)	4.5(5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	18	857	8515	23399	24113	24721	24721	5/15
A4	**1.5**(4)	**5.0**(4)	**2.9**(3)	∞	∞	∞	∞ *2e4*	0/15
B3	9.0(20)	5.7(10)	3.7(4)	**2.8**(2)	**2.7**(2)	**2.7**(2)	**2.7**(2)	2/15
C4	10(17)	5.4(4)	8.0(12)	∞	∞	∞	∞ *9714*	0/15
cmaes+	5.6(17)	6.1(10)	8.3(9)	3.0(3)	3.0(4)	2.9(2)	2.9(3)	2/15

Figure A.26: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	3.6	8.0	8.0	8.0	8.0	8.0	8.0	15/15
A4	**4.3**(4)	6.0(5)	12(4)	18(5)	26(5)	37(6)	48(7)	15/15
B3	4.4(4)	5.3(3)	11(5)	16(4)	21(3)	30(4)	40(6)	15/15
C4	4.4(3)	**5.0**(3)	**9.4**(3)	**14**(4)	**20**(5)	**29**(6)	**39**(5)	15/15
cmaes+	4.6(5)	6.7(4)	11(6)	16(7)	21(3)	30(5)	39(5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	38	42	43	44	45	47	48	15/15
A4	13(7)	16(6)	17(7)	19(3)	**19**(3)	21(2)	22(2)	15/15
B3	9.4(7)	**13**(7)	**17**(4)	**19**(4)	20(3)	21(2)	22(1)	15/15
C4	**9.1**(7)	15(3)	18(5)	19(3)	20(2)	**20**(2)	**21**(2)	15/15
cmaes+	10(7)	14(6)	17(3)	20(2)	20(1)	21(2)	22(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	38	822	830	835	842	847	853	15/15
A4	63(13)	**18**(46)	**68**(117)	**67**(48)	**67**(93)	**67**(68)	**66**(81)	5/15
B3	28(59)	43(50)	149(112)	149(166)	147(327)	147(124)	146(253)	2/15
C4	**24**(7)	35(28)	108(120)	107(116)	106(98)	106(234)	105(152)	3/15
cmaes+	51(152)	28(39)	161(66)	160(78)	159(312)	158(124)	157(88)	2/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	40	808	866	921	952	1015	1044	15/15
A4	60(10)	**32**(20)	**388**(559)	**365**(302)	**353**(409)	**331**(397)	**322**(665)	1/15
B3	**11**(12)	56(77)	∞	∞	∞	∞	∞ *2e4*	0/15
C4	80(48)	66(62)	∞	∞	∞	∞	∞ *1e4*	0/15
cmaes+	60(165)	78(55)	∞	∞	∞	∞	∞ *1e4*	0/15

Figure A.27: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	6.6	6.6	6.6	6.6	6.6	6.6	6.6	15/15
A4	7.3(2)	10(2)	10(2)	10(3)	10(2)	10(2)	10(3)	15/15
B3	**6.4**(2)	**8.8**(1)	**9.0**(2)	**9.1**(3)	**9.1**(2)	**9.1**(2)	**9.1**(3)	15/15
C4	7.3(2)	10(4)	10(4)	10(4)	10(5)	10(4)	10(3)	15/15
cmaes+	8.5(4)	12(6)	13(3)	13(5)	13(6)	13(17)	13(2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	34	56	90	117	149	215	265	15/15
A4	2.1(1)	3.0(1)	2.8(0.8)	3.1(1.0)	3.2(0.6)	3.2(0.3)	3.4(0.5)	15/15
B3	**1.4**(0.9)	**2.1**(0.8)	2.1(0.7)	2.2(0.5)	2.1(0.4)	2.1(0.4)	2.1(0.3)	15/15
C4	1.8(2)	**2.1**(1)	**2.1**(0.8)	**2.1**(0.5)	**2.0**(0.6)	**2.0**(0.3)	**2.0**(0.2)	15/15
cmaes+	1.9(1)	2.6(2)	2.3(0.9)	2.4(1)	2.3(0.7)	2.1(0.4)	2.2(0.6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	11	65	342	464	482	482	535	15/15
A4	1.8(1)	**1.8**(1)	**1.2**(2)	**1.1**(0.8)	**1.4**(0.5)	**1.4**(2)	**1.3**(0.3)	15/15
B3	2.6(2)	4.0(5)	3.5(3)	3.7(3)	6.9(7)	6.9(13)	6.3(13)	10/15
C4	**1.6**(0.7)	1.8(3)	2.9(5)	2.9(8)	5.4(7)	5.4(14)	4.9(4)	12/15
cmaes+	2.1(2)	2.8(2)	2.9(3)	3.1(5)	3.9(6)	3.9(13)	3.6(4)	12/15
Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	27	45	152	179	188	198	208	15/15
A4	2.4(2)	**4.7**(3)	**2.8**(3)	**3.2**(1)	**3.5**(2)	4.0(2)	4.3(1)	15/15
B3	**1.9**(1)	6.1(6)	3.2(2)	3.6(4)	3.8(3)	4.1(2)	4.3(1)	15/15
C4	2.3(1)	5.9(15)	3.1(2)	3.4(0.9)	3.6(1)	3.9(0.9)	4.1(0.9)	15/15
cmaes+	2.1(2)	4.8(5)	2.9(3)	3.4(2)	3.5(2)	**3.8**(2)	**4.0**(2)	15/15

Figure A.28: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	21	65	127	149	159	169	178	15/15
A4	**0.66**(0.9)	1.9(3)	2.6(3)	3.4(2)	3.7(2)	4.3(2)	4.6(0.9)	15/15
B3	0.75(0.8)	2.6(6)	2.7(1)	3.2(2)	3.5(2)	3.9(3)	4.2(1)	15/15
C4	0.91(0.9)	2.3(4)	2.6(3)	3.1(2)	3.3(3)	3.8(0.9)	4.0(0.6)	15/15
cmaes+	0.78(0.8)	**1.3**(2)	**2.1**(2)	**2.9**(1)	**3.2**(0.8)	**3.6**(0.9)	**3.9**(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	114	152	168	180	194	218	242	15/15
A4	3.9(2)	**4.1**(1)	**4.3**(0.4)	**4.3**(0.6)	**4.3**(0.4)	**4.3**(0.4)	4.3(0.4)	15/15
B3	3.7(2)	4.5(2)	5.0(0.6)	4.9(0.9)	4.9(0.5)	4.7(0.5)	4.5(0.3)	15/15
C4	4.2(2)	4.3(2)	5.0(0.8)	4.9(0.5)	4.8(0.6)	4.6(0.4)	4.5(0.3)	15/15
cmaes+	**3.5**(3)	4.4(1.0)	4.5(1)	4.5(0.6)	4.5(0.4)	4.3(0.5)	**4.3**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	67	105	227	263	277	302	327	15/15
A4	7.2(2)	**6.5**(2)	**3.3**(0.7)	**3.1**(0.4)*	**3.2**(0.4)*	**3.3**(0.2)	**3.4**(0.4)	15/15
B3	6.7(7)	8.1(2)	4.3(0.9)	3.9(0.6)	4.0(0.4)	3.9(0.3)	3.8(0.4)	15/15
C4	5.8(5)	7.3(2)	4.2(0.9)	4.0(0.7)	4.0(0.4)	4.0(0.4)	3.9(0.2)	15/15
cmaes+	**5.6**(6)	7.0(2)	3.9(0.6)	3.7(0.3)	3.7(0.4)	3.6(0.3)	3.6(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	65	168	338	401	445	696	790	15/15
A4	9.0(5)	**6.5**(5)	**5.4**(5)	5.4(2)	5.5(4)	4.4(3)	4.4(2)	15/15
B3	**7.6**(12)	7.8(6)	6.2(4)	6.2(5)	6.0(4)	4.7(3)	4.7(2)	15/15
C4	11(20)	8.8(10)	6.3(6)	6.1(6)	6.0(5)	4.5(4)	4.4(3)	15/15
cmaes+	10(10)	8.2(10)	5.4(7)	**5.4**(5)	**5.4**(4)	**4.2**(3)	**4.3**(3)	15/15

Figure A.29: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	49	85	108	136	215	281	365	15/15
A4	**4.3**(3)	**4.9**(2)	**5.0**(2)	**5.5**(2)	**4.4**(1)	**4.4**(0.8)	**4.5**(0.6)	15/15
B3	5.0(5)	7.4(3)	6.9(2)	6.2(2)	5.1(1)	5.1(1)	4.9(0.8)	15/15
C4	6.3(7)	5.5(3)	6.4(3)	6.7(2)	5.8(5)	5.5(3)	5.4(1)	15/15
cmaes+	5.9(4)	7.1(3)	7.4(3)	6.7(3)	5.1(0.9)	5.6(0.7)	5.3(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	2.2	17	28	43	71	110	194	15/15
A4	2.6(7)	**1.9**(0.8)	3.1(0.7)	4.1(2)	4.5(1)	6.3(1)	**5.2**(0.8)	15/15
B3	3.4(2)	2.3(1)	**2.9**(1.0)	**3.0**(0.8)	3.7(0.9)	6.9(1)	5.6(0.7)	15/15
C4	2.6(6)	2.2(2)	3.3(0.2)	3.6(1)	3.9(1)	6.6(2)	5.6(0.8)	15/15
cmaes+	**2.4**(3)	2.1(2)	3.1(1)	3.4(2)	**3.6**(1)	**5.5**(1)	5.6(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	121	1372	6285	8282	8429	8787	9041	15/15
A4	**1.7**(4)	**2.8**(2)	5.6(7)	4.3(7)	4.2(4)	4.0(3)	3.9(2)	7/15
B3	5.9(4)	11(9)	41(19)	31(39)	30(49)	29(71)	28(25)	1/15
C4	20(72)	10(19)	9.1(13)	6.9(12)	6.8(9)	6.5(8)	6.4(5)	4/15
cmaes+	2.6(5)	6.3(9)	**4.9**(7)	**3.7**(5)	**3.7**(3)	**3.5**(1)	**3.4**(6)	6/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	41	319	582	789	1864	3204	3361	15/15
A4	**1.6**(1)	**4.8**(5)	**4.2**(3)	**6.9**(21)	**3.1**(2)	**2.6**(3)	**2.5**(5)	13/15
B3	2.7(14)	7.4(15)	11(8)	11(9)	5.8(6)	4.5(3)	6.2(4)	8/15
C4	2.8(7)	5.6(5)	5.6(20)	8.5(11)	4.5(4)	4.1(3)	5.0(5)	10/15
cmaes+	2.4(0.6)	8.0(17)	10(28)	11(16)	5.1(4)	3.8(2)	4.5(6)	11/15

Figure A.30: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	3.6	78	282	491	1134	2347	3469	15/15
A4	**5.4**(8)	50(99)	24(81)	**15**(31)	**8.0**(5)	**12**(7)	**13**(16)	7/15
B3	16(4)	**31**(25)	**17**(38)	19(20)	8.7(6)	32(45)	73(35)	1/15
C4	19(62)	42(140)	20(27)	26(38)	19(42)	51(42)	∞ *2e4*	0/15
cmaes+	17(1.0)	48(142)	41(43)	43(43)	28(43)	52(47)	∞ *2e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	40	145	1289	3084	3523	4738	5527	15/15
A4	**15**(85)	**11**(14)	**11**(25)	**10**(9)	**11**(7)	**18**(18)	**23**(22)	3/15
B3	111(163)	51(108)	23(13)	26(37)	102(116)	76(99)	65(104)	1/15
C4	245(325)	101(221)	51(49)	127(78)	∞	∞	∞ *3e4*	0/15
cmaes+	176(250)	85(177)	39(40)	38(24)	112(68)	∞	∞ *3e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	109	6764	7367	7399	7441	15/15
A4	2.5(2)	**46**(44)	65(87)	**3.6**(4)	**4.8**(6)	**4.8**(5)	**4.8**(4)	9/15
B3	2.5(1)	67(38)	**24**(56)	7.8(20)	11(7)	11(19)	11(11)	4/15
C4	**1.7**(1)	60(56)	59(119)	5.6(6)	10(13)	10(16)	10(13)	5/15
cmaes+	2.5(4)	92(243)	48(6)	5.5(7)	17(27)	17(22)	17(25)	3/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	8.3	385	2291	2398	2481	2573	2776	15/15
A4	1.3(1)	**7.1**(7)	12(19)	14(19)	14(19)	13(17)	12(13)	6/15
B3	1.4(1.0)	11(21)	38(44)	37(25)	36(35)	34(42)	32(45)	2/15
C4	1.4(0.7)	10(8)	**10**(5)	**10**(15)	**9.3**(16)	**9.0**(19)	**8.4**(12)	6/15
cmaes+	**1.2**(1)	10(10)	19(17)	18(13)	18(15)	17(21)	16(8)	4/15

Figure A.31: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	5.9	184	425	439	458	469	482	15/15
A4	**2.8**(4)	**28**(19)	33(25)	33(33)	31(19)	31(38)	30(33)	9/15
B3	10(2)	58(68)	60(101)	58(131)	55(98)	54(32)	53(83)	6/15
C4	11(32)	34(38)	**32**(38)	**31**(20)	**30**(23)	**29**(86)	**28**(52)	8/15
cmaes+	13(37)	71(112)	79(98)	77(67)	74(49)	72(51)	70(83)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	18	170	354	362	384	401	414	15/15
A4	60(408)	34(46)	25(62)	24(47)	23(44)	22(47)	22(59)	11/15
B3	55(23)	35(24)	**22**(33)	**22**(8)	**21**(45)	**20**(10)	**20**(20)	11/15
C4	**53**(175)	**34**(6)	23(31)	23(57)	22(22)	21(71)	21(65)	11/15
cmaes+	221(602)	49(139)	34(43)	33(47)	31(12)	30(12)	29(39)	10/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	2.6	407	906	1215	2214	2293	2393	15/15
A4	**2.4**(4)	34(23)	465(389)	∞	∞	∞	∞ *3e4*	0/15
B3	3.7(3)	2.4(2)	12(11)	11(20)	5.9(7)	5.8(10)	5.7(8)	13/15
C4	5.0(6)	**1.4**(3)	**5.7**(10)	5.1(8)	3.2(5)	3.2(3)	3.1(4)	15/15
cmaes+	6.1(4)	1.8(1)	5.8(9)	**4.5**(9)	**2.6**(4)	**2.6**(5)	**2.5**(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	97	10391	1.0e5	3.6e5	3.6e5	3.6e5	3.6e5	2/15
A4	**2.4**(3)	3.2(2)	∞	∞	∞	∞	∞ *3e4*	0/15
B3	14(5)	14(12)	∞	∞	∞	∞	∞ *2e4*	0/15
C4	12(33)	3.5(3)	∞	∞	∞	∞	∞ *1e4*	0/15
cmaes+	8.4(8)	**3.0**(5)	∞	∞	∞	∞	∞ *2e4*	0/15

Figure A.32: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	12	15/15
A4	3.5(2)	9.4(3)	17(4)	25(4)	32(7)	48(5)	63(5)	15/15
B3	3.7(1)	8.6(3)	15(3)	21(3)	27(5)	38(4)	49(8)	15/15
C4	**3.2**(2)	8.4(2)	13(2)	20(2)	25(3)	37(3)	47(4)	15/15
cmaes+	3.3(2)	**7.6**(3)	**13**(3)	**18**(3)	**23**(3)	**34**(4)	**44**(4)	15/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	83	87	88	89	90	92	94	15/15
A4	13(3)	**15**(2)	**17**(2)	**18**(2)	**19**(2)	**21**(2)	22(2)	15/15
B3	13(4)	17(4)	19(3)	20(3)	21(1)	22(2)	23(3)	15/15
C4	12(4)	17(4)	20(2)	20(1)	21(1)	22(1)	23(0.9)	15/15
cmaes+	**12**(3)	17(3)	19(2)	20(2)	20(2)	21(2)	**22**(2)	15/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1642	1646	1650	1654	15/15
A4	**1.5**(1)	50(44)	364(319)	363(277)	362(364)	361(219)	361(463)	1/15
B3	25(30)	172(245)	∞	∞	∞	∞	∞ *5e4*	0/15
C4	14(4)	**50**(80)	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes+	13(35)	98(110)	**305**(186)	**304**(275)	**304**(195)	**303**(453)	**302**(279)	1/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1758	1817	1886	1903	15/15
A4	**3.6**(3)	$\infty^\star$	$\infty^\star$	$\infty^\star$	$\infty^\star$	$\infty^\star$	∞ *4e4**	0/15
B3	37(47)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
C4	26(31)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes+	29(35)	∞	∞	∞	∞	∞	∞ *5e4*	0/15

Figure A.33: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	10	15/15
A4	7.7(2)	10(2)	10(2)	10(2)	10(5)	10(1)	10(3)	15/15
B3	**6.9**(2)	**8.8**(3)	**9.0**(3)	**9.0**(3)	**9.2**(3)	**9.2**(2)	**9.2**(2)	15/15
C4	7.5(2)	9.2(1)	9.4(2)	9.4(1)	9.4(2)	9.4(1)	9.4(2)	15/15
cmaes+	7.9(2)	10(2)	10(2)	10(2)	10(2)	10(3)	10(3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	114	214	281	404	580	1038	1332	15/15
A4	1.9(2)	2.2(0.6)	2.5(0.8)	2.4(0.6)	2.0(0.3)	1.6(0.2)	1.6(0.2)	15/15
B3	1.4(0.4)	1.4(0.4)	1.6(0.7)	1.4(0.1)	1.2(0.3)	0.92(0.1)	0.93(0.1)	15/15
C4	1.4(1.0)	1.5(0.6)	1.7(0.3)	1.4(0.5)	1.2(0.1)	0.91(0.2)	0.89(0.1)	15/15
cmaes+	**1.2**(0.5)	**1.3**(0.3)	**1.5**(0.4)	**1.4**(0.3)	**1.2**(0.2)	**0.86**(0.1)	**0.87**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1451	1572	1572	1597	15/15
A4	2.4(2)	**1.2**(3)	**1.0**(1.0)	**1.0**(1.0)*	**1.0**(0.6)*	**1.0**(0.9)*	**1.1**(1)*	15/15
B3	2.7(2)	2.4(3)	3.4(4)	10(24)	9.1(7)	9.1(6)	9.0(5)	6/15
C4	**2.4**(2)	2.6(1)	3.7(3)	3.9(3)	5.6(10)	5.6(6)	5.5(3)	8/15
cmaes+	2.6(2)	4.0(4)	5.7(4)	9.0(11)	64(86)	64(32)	63(73)	1/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	73	273	336	372	391	410	422	15/15
A4	3.5(2)	**3.9**(2)	**4.6**(0.9)	4.8(2)	4.9(0.9)	5.2(0.7)	5.5(1)	15/15
B3	2.7(2)	4.1(1)	4.7(3)	4.8(6)	4.9(4)	5.1(4)	5.3(0.8)	15/15
C4	**2.3**(1.0)	4.1(9)	4.7(4)	4.8(2)	4.9(0.5)	5.1(6)	5.4(4)	15/15
cmaes+	2.4(0.8)	4.3(1)	4.6(8)	**4.7**(6)	**4.7**(2)	**4.9**(6)	**5.1**(5)	15/15

Figure A.34: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	35	127	214	263	300	335	369	15/15
A4	3.0(1)	**5.5**(2)	**5.5**(1)	**5.3**(0.5)	**5.1**(1.0)	**5.3**(0.8)	**5.4**(0.6)	15/15
B3	2.2(0.5)	8.6(6)	7.1(7)	6.7(5)	6.3(4)	6.2(3)	6.0(4)	15/15
C4	**2.1**(0.9)	8.9(16)	7.3(5)	6.8(8)	6.4(1)	6.2(6)	6.1(2)	15/15
cmaes+	2.4(0.7)	8.5(10)	7.0(4)	6.5(5)	6.1(3)	5.9(4)	5.7(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	349	500	574	607	626	829	880	15/15
A4	3.5(1.0)	**2.8**(0.6)	**2.7**(0.4)	**2.7**(0.4)	**2.8**(0.3)	**2.3**(0.2)	2.4(0.2)	15/15
B3	3.1(2)	3.0(0.9)	3.0(0.1)	3.0(0.3)	3.1(0.1)	2.5(0.2)	2.5(0.2)	15/15
C4	**2.6**(0.8)	2.9(0.6)	2.7(0.6)	2.9(0.7)	3.0(0.4)	2.4(0.2)	2.4(0.2)	15/15
cmaes+	3.0(1)	3.1(0.8)	2.9(0.2)	2.9(0.3)	2.9(0.2)	2.4(0.2)	**2.4**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	143	202	763	977	1177	1467	1673	15/15
A4	10(4)	8.4(1)	**2.4**(0.3)	**1.9**(0.3)	**1.7**(0.2)	**1.5**(0.2)	**1.4**(0.1)	15/15
B3	6.6(6)	8.3(4)	2.7(0.8)	2.3(0.5)	2.0(0.2)	1.7(0.2)	1.6(0.1)	15/15
C4	7.0(4)	8.8(3)	2.9(0.4)	2.5(0.2)	2.1(0.1)	1.8(0.1)	1.7(0.1)	15/15
cmaes+	**6.5**(4)	**7.7**(1)	2.6(0.7)	2.2(0.3)	1.9(0.2)	1.7(0.2)	1.5(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	108	268	371	413	461	1303	1494	15/15
A4	9.3(6)	6.4(6)	6.8(3)	7.0(6)	7.2(4)	3.3(2)	3.3(1)	15/15
B3	6.3(8)	6.7(3)	7.2(3)	7.6(3)	7.6(3)	3.4(2)	3.4(2)	15/15
C4	7.4(5)	5.8(3)	5.8(6)	6.1(3)	6.4(4)	2.9(1)	2.8(2)	15/15
cmaes+	**5.9**(4)	**4.8**(4)	**5.2**(6)	**5.7**(3)	**5.9**(2)	**2.7**(3)	**2.7**(3)	15/15

Figure A.35: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	132	195	250	319	1310	1752	2255	15/15
A4	**3.6**(2)	**4.3**(1)	**5.2**(1)	**5.0**(1.0)	**1.4**(0.2)	**1.5**(0.2)	**1.4**(0.2)	15/15
B3	5.4(5)	7.0(3)	6.0(2)	6.1(4)	1.8(1)	1.9(0.5)	2.0(0.1)	15/15
C4	5.0(3)	6.3(2)	5.9(2)	5.7(1)	1.8(0.4)	1.8(0.2)	1.8(0.2)	15/15
cmaes+	4.1(3)	6.4(4)	6.0(2)	5.4(2)	1.5(0.5)	1.5(0.2)	1.5(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	10	41	58	90	139	251	476	15/15
A4	1.5(1)	2.2(1)	3.9(0.9)	4.4(1)	4.9(1)	5.2(0.1)	4.3(0.6)	15/15
B3	1.5(1)	**1.9**(0.9)	2.9(1)	**2.9**(0.8)	3.2(0.6)	5.5(2)	4.8(0.4)	15/15
C4	**1.1**(1)	1.9(1)	2.9(1)	3.0(0.5)	3.8(0.9)	**5.0**(0.5)	4.6(0.7)	15/15
cmaes+	1.7(2)	2.1(0.9)	**2.6**(1)	3.0(0.8)	**3.0**(0.7)	5.2(0.5)	**4.2**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	19743	20073	20769	21359	14/15
A4	**4.7**(3)	**4.4**(6)$^{\star 2}$	**34**(46)	**34**(46)	**33**(56)	**32**(28)	**31**(27)	1/15
B3	42(23)	70(112)	∞	∞	∞	∞	∞ *5e4*	0/15
C4	50(61)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes+	42(31)	62(63)	∞	∞	∞	∞	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10163	10449	11644	12095	15/15
A4	2.1(5)	**4.2**(6)	**3.0**(1)	**1.1**(1)	**1.1**(1)	**1.3**(2)*	**1.2**(2)*	14/15
B3	3.4(14)	7.0(16)	13(13)	13(14)	19(15)	17(21)	∞ *3e4*	0/15
C4	2.2(0.6)	14(7)	9.4(6)	7.7(7)	42(24)	38(28)	∞ *3e4*	0/15
cmaes+	**0.70**(0.6)	6.0(4)	8.2(16)	3.2(6)	7.9(19)	35(49)	34(67)	1/15

Figure A.36: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	2861	3669	6351	7934	15/15
A4	3.5(3)	**1.2**(0.6)	**1.8**(0.1)	**0.68**(1)^{*3}	**1.8**(4)^{*3}	**24**(15)	**42**(66)	2/15
B3	**2.4**(5)	15(15)	14(18)	25(14)	∞	∞	∞ *3e4*	0/15
C4	20(7)	39(84)	13(7)	32(39)	110(148)	∞	∞ *3e4*	0/15
cmaes+	96(7)	33(23)	21(23)	24(14)	105(88)	∞	∞ *3e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	8451	9280	10905	12469	15/15
A4	**2.7**(1)	**5.3**(15)	**1.5**(2)	**3.5**(4)	**7.4**(10)	**21**(15)	**59**(60)	1/15
B3	106(174)	64(70)	51(36)	∞	∞	∞	∞ *5e4*	0/15
C4	142(317)	105(147)	43(22)	∞	∞	∞	∞ *5e4*	0/15
cmaes+	47(141)	65(133)	8.9(5)	15(27)	68(140)	∞	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.0e5	1.2e5	1.2e5	1.2e5	15/15
A4	**1.6**(2)	2110(3251)	3005(2632)	∞	∞	∞	∞ *5e4*	0/15
B3	2.3(1)	511(169)	136(236)	**6.9**(7)	**6.0**(4)	**6.0**(5)	**5.9**(10)	1/15
C4	1.9(2)	302(111)	224(413)	∞	∞	∞	∞ *5e4*	0/15
cmaes+	2.9(3)	**242**(143)	**96**(109)	∞	∞	∞	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	51362	54470	54861	55313	14/15
A4	1.5(2)	12(15)	14(13)	11(7)	10(9)	10(11)	10(15)	1/15
B3	1.7(0.7)	13(16)	∞	∞	∞	∞	∞ *2e4*	0/15
C4	**1.5**(0.9)	**9.2**(7)	**8.1**(11)	**6.0**(8)	**5.6**(6)	**5.6**(4)	**5.6**(6)	1/15
cmaes+	1.7(1)	15(13)	∞	∞	∞	∞	∞ *2e4*	0/15

Figure A.37: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1692	1705	1729	1757	14/15
A4	**6.1**(36)	24(42)	20(23)	20(22)	20(25)	20(15)	20(19)	7/15
B3	17(0.7)	82(88)	56(55)	56(106)	55(25)	55(88)	54(88)	3/15
C4	14(24)	**20**(45)	**17**(28)	**17**(42)	**17**(16)	**17**(35)	**17**(16)	7/15
cmaes+	16(28)	33(46)	27(26)	27(54)	27(30)	27(29)	26(46)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	71	386	938	980	1008	1040	1068	14/15
A4	53(177)	64(82)	51(86)	49(87)	47(114)	46(77)	45(24)	6/15
B3	55(4)	59(137)	40(29)	39(60)	38(76)	37(27)	36(49)	6/15
C4	**7.6**(0.7)	**37**(44)	**23**(25)	**22**(21)	**21**(17)	**21**(35)	20(27)	8/15
cmaes+	16(52)	38(16)	23(33)	22(13)	21(32)	21(10)	**20**(34)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	27890	31654	33030	34256	15/15
A4	**1.8**(1)	107(101)	∞	∞	∞	∞	∞ *5e4*	0/15
B3	2.0(0.5)	2.4(0.1)	4.3(4)	5.7(9)	5.2(5)	5.0(7)	4.8(4)	4/15
C4	2.7(3)	2.4(3)	1.8(2)	4.0(4)	3.6(5)	3.5(3)	3.4(2)	5/15
cmaes+	2.0(3)	**1.9**(0.5)	**1.5**(1)	**2.8**(3)	**2.5**(3)	**2.4**(3)	**2.4**(2)	6/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	9.6e6	1.3e7	1.3e7	3/15
A4	**16**(24)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
B3	28(20)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
C4	36(44)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes+	22(12)	**2.9**(3)	∞	∞	∞	∞	∞ *5e4*	0/15

Figure A.38: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	22	23	23	23	23	23	23	15/15
A4	4.7$_{(4)}$	14$_{(3)}$	24$_{(4)}$	32$_{(8)}$	41$_{(6)}$	60$_{(3)}$	79$_{(6)}$	15/15
B3	**4.0**$_{(1)}$	11$_{(2)}$	17$_{(2)}$	25$_{(2)}$	31$_{(3)}$	45$_{(3)}$	60$_{(4)}$	15/15
C4	4.0$_{(1)}$	11$_{(2)}$	17$_{(2)}$	24$_{(2)}$	30$_{(3)}$	44$_{(3)}$	58$_{(3)}$	15/15
cmaes+	4.1$_{(2)}$	**10**$_{(3)}$	**16**$_{(4)}$	**21**$_{(2)}^{\star}$	**27**$_{(3)}^{\star 2}$	**38**$_{(2)}^{\star 3}$	**49**$_{(4)}^{\star 3}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	187	190	191	191	193	194	195	15/15
A4	23$_{(3)}$	24$_{(5)}$	26$_{(2)}$	**27**$_{(3)}$	**28**$_{(3)}$	30$_{(2)}$	32$_{(3)}$	15/15
B3	22$_{(8)}$	26$_{(4)}$	30$_{(4)}$	31$_{(2)}$	32$_{(1)}$	34$_{(3)}$	35$_{(2)}$	15/15
C4	**19**$_{(3)}$	24$_{(6)}$	28$_{(5)}$	30$_{(1)}$	31$_{(1)}$	33$_{(1)}$	34$_{(1)}$	15/15
cmaes+	20$_{(3)}$	**23**$_{(3)}$	**26**$_{(4)}$	28$_{(1)}$	29$_{(0.8)}$	**30**$_{(0.9)}$	**31**$_{(1)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	1739	3600	3609	3636	3642	3646	3651	15/15
A4	**91**$_{(112)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	838$_{(540)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	802$_{(761)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	233$_{(214)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	2234	3626	3660	3695	3707	3744	28767	12/15
A4	**18**$_{(17)}^{\star 3}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	320$_{(221)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Figure A.39: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	20	20	20	20	20	20	20	15/15
A4	**6.5**(2)	**8.1**(1)	**8.2**(1)	**8.2**(2)	**8.2**(3)	**8.2**(2)	**8.2**(0.9)	15/15
B3	7.3(2)	8.6(2)	8.8(3)	8.8(2)	8.8(3)	8.8(2)	8.8(3)	15/15
C4	7.1(2)	8.6(1)	8.7(2)	8.7(2)	8.7(2)	8.7(1)	8.7(2)	15/15
cmaes+	7.2(1)	8.4(1)	8.6(1)	8.6(2)	8.6(2)	8.6(1)	8.6(1.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	412	623	826	1039	1292	1841	2370	15/15
A4	4.0(2)	4.8(0.8)	5.4(2)	5.4(1)	5.1(2)	4.7(0.8)	4.7(0.5)	15/15
B3	1.2(0.4)	1.5(0.5)	1.7(0.4)	1.7(0.3)	1.8(0.3)	1.7(0.1)	1.8(0.2)	15/15
C4	1.3(0.5)	1.5(0.5)	1.6(0.3)	1.7(0.3)	1.7(0.2)	1.6(0.2)	1.7(0.3)	15/15
cmaes+	**1.2**(0.2)	**1.3**(0.5)	**1.4**(0.2)	**1.5**(0.3)	**1.5**(0.2)	**1.4**(0.1)	**1.5**(0.2)*	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	172	1611	4195	5099	5141	5141	5389	15/15
A4	2.2(1)	**2.1**(1)	**2.3**(2)$^{\star 2}$	**2.5**(6)$^{\star 2}$	**2.8**(3)$^{\star 2}$	**2.8**(1)$^{\star 2}$	**2.7**(4)	14/15
B3	1.6(0.9)	5.4(6)	24(22)	∞	∞	∞	∞ *1e4*	0/15
C4	2.3(4)	5.8(9)	25(16)	∞	∞	∞	∞ *1e4*	0/15
cmaes+	**1.4**(0.4)	3.0(3)	14(20)	38(69)	∞	∞	∞ *1e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	326	921	1114	1217	1267	1315	1343	15/15
A4	3.2(2)	6.6(6)	6.3(4)	6.3(4)	6.3(4)	6.4(3)	6.7(4)	15/15
B3	2.8(0.8)	6.3(5)	6.2(2)	6.0(4)	6.0(2)	6.1(4)	6.3(4)	15/15
C4	3.1(3)	**4.1**(1)	**4.2**(2)	**4.2**(1)	**4.3**(0.8)	**4.5**(0.9)	**4.6**(1)	15/15
cmaes+	**2.4**(1)	4.3(3)	4.4(1)	4.4(1)	4.4(1)	4.5(1)	4.6(1)	15/15

Figure A.40: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	200	648	857	993	1065	1138	1185	15/15
A4	2.8(0.6)	5.5(0.7)	5.4(0.3)	5.3(0.4)	5.2(0.3)	5.3(0.3)	5.5(0.1)	15/15
B3	1.9(0.7)	5.9(5)	5.6(3)	5.3(4)	5.2(2)	5.3(2)	5.4(0.9)	15/15
C4	2.1(0.7)	7.2(1)	6.6(5)	6.2(4)	6.0(1)	6.0(6)	6.0(4)	15/15
cmaes+	**1.7**(0.7)	**5.3**(4)	**5.1**(3)	**4.8**(4)	**4.7**(0.8)	**4.7**(3)	**4.8**(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	1835	2172	2455	2728	2802	4543	4739	15/15
A4	2.3(0.4)	**2.2**(0.3)	**2.1**(0.3)	**2.0**(0.2)	**2.0**(0.2)	1.3(0.1)	1.4(0.1)	15/15
B3	**2.0**(0.4)	2.3(0.5)	2.2(0.3)	2.1(0.2)	2.2(0.1)	1.4(0.1)	1.4(0.1)	15/15
C4	2.2(0.5)	2.2(0.4)	2.2(0.3)	2.1(0.3)	2.1(0.2)	1.4(0.1)	1.4(0.1)	15/15
cmaes+	2.1(0.5)	2.2(0.3)	2.1(0.3)	2.0(0.2)	2.0(0.1)	**1.3**(0.1)	**1.3**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	266	1041	2602	2954	3338	4092	4843	15/15
A4	24(2)	6.3(0.4)	2.6(0.2)	2.4(0.1)	2.1(0.1)	1.9(0.1)	1.7(0.1)	15/15
B3	12(2)	4.6(0.6)	2.1(0.2)	2.0(0.3)	1.9(0.1)	1.7(0.1)	1.5(0.1)	15/15
C4	12(2)	4.4(0.8)	2.1(0.2)	2.0(0.2)	1.9(0.1)	1.7(0.1)	1.5(0.1)	15/15
cmaes+	**11**(2)	**4.1**(0.6)	**2.0**(0.2)	**1.9**(0.1)	**1.8**(0.1)	**1.6**(0.1)$^{\star 2}$	**1.4**(0.1)$^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	515	896	1240	1390	1569	3660	5154	15/15
A4	4.0(3)	4.2(3)	4.8(3)	5.1(3)	5.3(3)	3.0(1)	2.5(0.6)	15/15
B3	3.0(0.1)	4.1(3)	4.6(3)	5.0(2)	5.0(4)	2.8(0.7)	2.3(1.0)	15/15
C4	**2.8**(2)	5.2(3)	5.4(4)	5.6(3)	5.5(3)	3.0(0.8)	2.4(0.9)	15/15
cmaes+	3.3(4)	**4.0**(4)	**4.5**(3)	**4.8**(2)	**4.8**(2)	**2.6**(1.0)	**2.2**(1)	15/15

Figure A.41: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	387	596	797	1014	4587	6208	7779	15/15
A4	2.8(0.3)	4.8(2)	**5.0**(1)	**4.7**(0.8)	**1.2**(0.3)	**1.3**(0.2)	**1.3**(0.2)	15/15
B3	4.0(4)	7.5(6)	8.0(7)	6.8(6)	1.8(1)	2.4(2)	3.3(3)	15/15
C4	**2.4**(4)	6.7(8)	10(6)	8.7(4)	2.2(1)	3.2(1)	3.6(4)	15/15
cmaes+	2.9(4)	**4.6**(3)	8.9(4)	7.8(6)	2.3(1)	2.4(3)	2.5(2)	15/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	37	98	133	205	392	687	4305	15/15
A4	1.5(1)	3.0(0.3)	4.2(0.9)	5.1(0.9)	4.8(0.7)	5.5(0.6)	1.4(0.1)	15/15
B3	**1.1**(0.6)	2.3(0.6)	3.2(0.4)	3.6(0.4)	3.4(0.5)	5.3(0.5)	1.5(0.2)	15/15
C4	1.2(0.9)	2.3(0.8)	3.1(0.6)	3.4(0.7)	3.4(0.5)	5.1(0.7)	1.5(0.2)	15/15
cmaes+	1.2(0.6)	**2.1**(0.5)	**2.7**(0.2)	**2.9**(0.2)	**3.0**(0.3)	**4.8**(0.6)	**1.4**(0.1)	15/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	4774	39246	73643	74669	75790	77814	79834	12/15
A4	**36**(53)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	131(118)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	141(227)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	46(59)	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_opt	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	425	7029	15779	45669	51151	65798	71570	15/15
A4	100(84)	22(25)	18(30)	**8.8**(11)	**13**(9)	**10**(9)	**20**(19)	1/15
B3	0.55(0.5)$\downarrow$2	3.9(7)	62(38)	∞	∞	∞	∞ *7e4*	0/15
C4	2.0(0.3)	3.2(2)	∞	∞	∞	∞	∞ *7e4*	0/15
cmaes+	**0.50**(0.3)$\downarrow$3	**2.0**(3)	**18**(23)	10(11)	∞	∞	∞ *6e4*	0/15

Figure A.42: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	26	429	2203	6329	9851	20190	26503	15/15
A4	1.7(2)	**1.5**(0.5)	**0.64**(0.2)$^{\star 3}$	**1.9**(0.2)$^{\star 3}$	**5.6**(9)$^{\star 3}$	**33**(52)	∞ *1e5*	0/15
B3	2.3(4)	90(170)	123(207)	∞	∞	∞	∞ *8e4*	0/15
C4	2.5(2)	92(95)	268(415)	∞	∞	∞	∞ *8e4*	0/15
cmaes+	**1.5**(1)	62(48)	50(64)	185(118)	∞	∞	∞ *7e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	238	836	7012	15928	27536	37234	42708	15/15
A4	**1.4**(0.8)	**4.5**(0.2)$^{\star 2}$	**2.1**(5)$^{\star 4}$	**7.6**(5)$^{\star 4}$	**26**(24)	∞	∞ *1e5*	0/15
B3	98(97)	208(227)	∞	∞	∞	∞	∞ *1e5*	0/15
C4	124(30)	284(152)	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	54(210)	339(427)	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	10609	9.8e5	1.4e6	1.4e6	1.4e6	15/15
A4	2.7(3)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	2.5(2)	8626(1331)	132(181)	∞	∞	∞	∞ *1e5*	0/15
C4	2.5(2)	**1532**(950)	62(52)	∞	∞	∞	∞ *1e5*	0/15
cmaes+	**2.3**(2)	2485(4651)	**29**(40)	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	32	15426	5.5e5	5.7e5	5.7e5	5.8e5	5.9e5	15/15
A4	2.4(2)	17(15)	∞	∞	∞	∞	∞ *1e5*	0/15
B3	1.8(1)	14(25)	∞	∞	∞	∞	∞ *5e4*	0/15
C4	**1.4**(0.8)	**10**(12)	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes+	1.6(0.8)	38(54)	∞	∞	∞	∞	∞ *4e4*	0/15

Figure A.43: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	130	2236	4392	4487	4618	5074	11329	8/15
A4	108(345)	**27**(67)	**47**(25)	**46**(52)	**45**(50)	**41**(85)	**18**(11)	4/15
B3	**14**(54)	28(31)	54(179)	53(57)	51(76)	47(66)	21(40)	3/15
C4	144(588)	40(76)	58(60)	57(57)	56(91)	51(57)	23(16)	3/15
cmaes+	132(210)	42(65)	54(46)	53(42)	52(79)	47(78)	21(27)	3/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	98	2839	6353	6620	6798	8296	10351	6/15
A4	138(239)	19(9)	13(27)	13(14)	13(19)	10(18)	8.3(9)	7/15
B3	218(269)	13(16)	8.7(10)	8.4(7)	8.2(25)	6.8(11)	5.5(8)	8/15
C4	137(279)	14(13)	**7.8**(11)	**7.5**(17)	**7.3**(16)	**6.1**(9)	**4.9**(9)	9/15
cmaes+	**114**(358)	**12**(18)	8.1(9)	7.8(15)	7.7(16)	6.3(12)	5.1(9)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	2.8	915	16425	1.8e5	2.0e5	2.1e5	2.1e5	15/15
A4	2.8(2)	∞	∞	$\infty^{\star4}$	$\infty^{\star4}$	$\infty^{\star4}$	∞ *1e5*^{*4}	0/15
B3	**2.0**(2)	2.3(9)	**10**(9)	∞	∞	∞	∞ *1e5*	0/15
C4	2.9(2)	**1.0**(2)	25(34)	∞	∞	∞	∞ *1e5*	0/15
cmaes+	2.7(2)	1.4(2)	20(27)	∞	∞	∞	∞ *7e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	98761	1.0e6	7.5e7	7.5e7	7.5e7	7.5e7	7.5e7	1/15
A4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	**14**(17)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Figure A.44: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	43	15/15
A4	$6.8_{(2)}$	$14_{(1)}$	$21_{(2)}$	$29_{(2)}$	$36_{(3)}$	$51_{(4)}$	$66_{(3)}$	15/15
B3	$7.3_{(1)}$	$15_{(3)}$	$23_{(3)}$	$31_{(3)}$	$39_{(2)}$	$55_{(3)}$	$71_{(4)}$	15/15
C4	$7.7_{(2)}$	$15_{(1)}$	$23_{(1)}$	$30_{(2)}$	$38_{(2)}$	$53_{(3)}$	$68_{(3)}$	15/15
cmaes+	$\mathbf{6.0}_{(2)}$	$\mathbf{12}_{(0.8)}^{\star}$	$\mathbf{18}_{(2)}^{\star 3}$	$\mathbf{24}_{(2)}^{\star 3}$	$\mathbf{30}_{(1)}^{\star 3}$	$\mathbf{42}_{(2)}^{\star 4}$	$\mathbf{54}_{(3)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	385	386	387	388	390	391	393	15/15
A4	$38_{(5)}$	$46_{(5)}$	$52_{(4)}$	$55_{(2)}$	$57_{(4)}$	$59_{(3)}$	$60_{(2)}$	15/15
B3	$39_{(9)}$	$48_{(8)}$	$53_{(4)}$	$56_{(7)}$	$57_{(5)}$	$60_{(4)}$	$61_{(3)}$	15/15
C4	$38_{(4)}$	$48_{(6)}$	$54_{(4)}$	$56_{(3)}$	$57_{(2)}$	$59_{(2)}$	$60_{(2)}$	15/15
cmaes+	$\mathbf{32}_{(3)}$	$\mathbf{39}_{(5)}^{\star 2}$	$\mathbf{46}_{(4)}^{\star}$	$\mathbf{48}_{(4)}^{\star 2}$	$\mathbf{50}_{(3)}^{\star 3}$	$\mathbf{52}_{(1)}^{\star 4}$	$\mathbf{53}_{(0.7)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	5066	7626	7635	7637	7643	7646	7651	15/15
A4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	4722	7628	7666	7686	7700	7758	1.4e5	9/15
A4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Figure A.45: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	41	41	41	41	41	41	41	15/15
A4	7.8(1)	8.8(1)	8.8(1.0)	8.8(2)	8.8(1)	8.8(1)	8.8(2)	15/15
B3	**6.8**(1)	**8.1**(2)	**8.4**(2)	**8.6**(2)	**8.6**(2)	**8.6**(1.0)	**8.6**(1)	15/15
C4	8.1(2)	9.5(2)	10(2)	10(1)	10(1)	10(1)	10(1)	15/15
cmaes+	8.2(1)	9.3(0.7)	10(0.8)	10(0.7)	10(0.8)	10(0.6)	10(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	1296	2343	3413	4255	5220	6728	8409	15/15
A4	1.5(0.4)	1.3(0.3)	1.3(0.2)	1.4(0.3)	1.4(0.3)	1.5(0.2)	1.6(0.5)	15/15
B3	1.6(0.1)	1.4(0.2)	1.4(0.2)	1.4(0.4)	1.5(0.2)	1.8(0.3)	1.9(0.4)	15/15
C4	1.6(0.4)	1.4(0.3)	1.4(0.2)	1.4(0.1)	1.4(0.2)	1.6(0.1)	1.8(0.3)	15/15
cmaes+	**1.1**(0.1)$^{\star 2}$	**0.98**(0.1)$^{\star 3}$	**0.99**(0.1)$^{\star 2}$	**1.0**(0.2)$^{\star 2}$	**1.1**(0.3)*	**1.3**(0.3)	**1.4**(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	1351	4274	9503	16523	16524	16524	16969	15/15
A4	11(48)	∞	∞	∞	∞	∞	∞ *3e4*	0/15
B3	7.6(21)	∞	∞	∞	∞	∞	∞ *3e4*	0/15
C4	**2.7**(3)	∞	∞	∞	∞	∞	∞ *4e4*	0/15
cmaes+	56(75)	∞	∞	∞	∞	∞	∞ *3e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	2039	3871	4040	4148	4219	4371	4484	15/15
A4	4.2(0.3)	5.8(0.9)	6.2(0.8)	6.3(5)	6.3(4)	6.3(6)	6.3(5)	15/15
B3	4.1(2)	4.2(2)	4.6(0.9)	4.7(2)	4.8(1)	4.9(2)	4.9(1)	15/15
C4	4.2(1)	6.1(3)	6.5(3)	6.5(0.3)	6.6(3)	6.6(3)	6.6(3)	15/15
cmaes+	**3.7**(1)	**3.9**(0.9)	**4.3**(1)	**4.4**(1)	**4.5**(1)	**4.5**(1)	**4.6**(0.6)	15/15

Figure A.46: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	1716	3102	3277	3379	3455	3594	3727	15/15
A4	4.0(2)	5.6(0.6)	6.0(2)	6.1(2)	6.2(1)	6.2(0.4)	6.3(0.5)	15/15
B3	4.5(1)	6.2(4)	6.5(3)	6.6(4)	6.7(2)	6.7(3)	6.7(4)	15/15
C4	4.7(1)	5.6(0.8)	6.0(0.6)	6.1(0.7)	6.2(0.8)	6.2(0.6)	6.2(0.8)	15/15
cmaes+	**3.8**(0.7)	**4.8**(0.5)	**5.1**(0.8)	**5.2**(0.4)	**5.3**(0.7)	**5.3**(0.5)	**5.3**(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	7413	8661	10735	13641	14920	17073	17476	15/15
A4	2.0(0.3)	2.1(0.4)	1.9(0.1)	1.6(0.1)	1.5(0.1)	1.3(0.0)	1.4(0.1)	15/15
B3	1.9(0.2)	2.1(0.3)	1.9(0.2)	1.6(0.1)	1.5(0.1)	1.4(0.1)	1.4(0.1)	15/15
C4	2.0(0.2)	2.2(0.3)	1.9(0.2)	1.6(0.1)	1.5(0.1)	1.4(0.0)	1.4(0.1)	15/15
cmaes+	**1.8**(0.4)	**1.9**(0.1)	**1.7**(0.1)	**1.4**(0.1)$^{\star 2}$	**1.3**(0.0)$^{\star 3}$	**1.2**(0.0)$^{\star 4}$	**1.2**(0.0)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	1002	2228	6278	8586	9762	12285	14831	15/15
A4	8.4(0.9)	5.4(0.7)	2.3(0.2)	1.8(0.1)	1.8(0.1)	1.6(0.1)	1.4(0.1)	15/15
B3	8.7(0.7)	5.5(0.8)	2.3(0.3)	1.9(0.2)	1.8(0.1)	1.6(0.1)	1.4(0.1)	15/15
C4	8.8(1)	5.9(0.7)	2.4(0.2)	1.9(0.2)	1.8(0.1)	1.6(0.0)	1.4(0.0)	15/15
cmaes+	**8.3**(1)	**5.1**(0.6)	**2.1**(0.2)	**1.7**(0.1)$^{\star 2}$	**1.6**(0.1)$^{\star 2}$	**1.4**(0.1)$^{\star 3}$	**1.2**(0.0)$^{\star 3}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	1042	1938	2740	3156	4140	12407	13827	15/15
A4	**2.1**(0.1)	**1.9**(2)	**3.2**(1)	4.0(1)	3.7(0.8)	1.7(0.4)	1.8(0.3)	15/15
B3	3.6(4)	3.9(4)	3.9(3)	4.4(3)	4.1(2)	1.7(0.4)	1.8(0.6)	15/15
C4	3.6(4)	4.7(4)	5.1(4)	5.3(2)	4.7(2)	1.9(0.7)	2.0(0.9)	15/15
cmaes+	2.2(0.2)	2.8(2)	3.4(2)	**3.9**(1)	**3.5**(2)	**1.6**(0.7)	**1.7**(0.6)	15/15

Figure A.47: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	3507	18749	24455	30201	15/15
A4	$6.0_{(6)}$	$\mathbf{6.5}_{(5)}$	$\mathbf{8.4}_{(6)}$	$\mathbf{9.0}_{(3)}$	$5.7_{(12)}$	$11_{(14)}$	$29_{(35)}$	3/15
B3	$5.1_{(3)}$	$10_{(9)}$	$10_{(18)}$	$12_{(9)}$	$\mathbf{4.0}_{(2)}$	$\mathbf{6.1}_{(5)}$	$16_{(12)}$	5/15
C4	$5.9_{(6)}$	$7.2_{(6)}$	$11_{(7)}$	$16_{(11)}$	$4.6_{(5)}$	$8.1_{(13)}$	$17_{(17)}$	5/15
cmaes+	$\mathbf{2.2}_{(0.5)}^{\star 2}$	$7.2_{(15)}$	$9.3_{(8)}$	$15_{(24)}$	$4.7_{(7)}$	$7.2_{(5)}$	$\mathbf{10}_{(12)}$	7/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	75	239	304	451	932	1648	15661	15/15
A4	$1.6_{(1)}$	$2.3_{(0.5)}$	$3.2_{(0.2)}$	$3.8_{(0.5)}$	$3.6_{(0.4)}$	$6.4_{(0.8)}$	$1.4_{(0.1)}$	15/15
B3	$1.9_{(1)}$	$2.5_{(0.5)}$	$3.5_{(0.5)}$	$4.0_{(0.7)}$	$3.7_{(0.2)}$	$6.8_{(1)}$	$1.4_{(0.2)}$	15/15
C4	$\mathbf{1.5}_{(0.8)}$	$2.1_{(0.5)}$	$3.1_{(0.5)}$	$3.7_{(0.6)}$	$3.7_{(0.5)}$	$6.4_{(0.5)}$	$1.4_{(0.1)}$	15/15
cmaes+	$1.6_{(1)}$	$\mathbf{2.0}_{(0.3)}$	$\mathbf{2.7}_{(0.4)}$	$\mathbf{3.3}_{(0.4)}$	$\mathbf{3.2}_{(0.4)}$	$\mathbf{6.0}_{(0.6)}$	$\mathbf{1.2}_{(0.1)}^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
A4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
A4	$0.95_{(0.2)}$	$10_{(14)}$	∞	∞	∞	∞	∞ *2e5*	0/15
B3	$\mathbf{0.43}_{(0.2)}$	$20_{(16)}$	∞	∞	∞	∞	∞ *2e5*	0/15
C4	$0.45_{(0.2)}$	$16_{(9)}$	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	$0.81_{(0.1)}$	$\mathbf{2.2}_{(3)}$	∞	∞	∞	∞	∞ *2e5*	0/15

Figure A.48: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	63	1030	4005	12242	30677	56288	80472	15/15
A4	1.2(0.8)	326(409)	∞	∞	∞	∞	∞ *2e5*	0/15
B3	1.3(1.0)	463(486)	∞	∞	∞	∞	∞ *2e5*	0/15
C4	**1.2**(0.9)	**152**(92)	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	1.4(0.6)	204(486)	**221**(292)	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
A4	206(540)	723(629)	∞	∞	∞	∞	∞ *2e5*	0/15
B3	148(153)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
C4	157(232)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	**108**(260)	**156**(171)	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	3.4e5	4.7e6	6.2e6	6.7e6	6.7e6	15/15
A4	2.1(3)	2.1e4(8101)	∞	∞	∞	∞	∞ *2e5*	0/15
B3	**1.7**(1)	1.7e4(3e4)	**8.0**(9)	∞	∞	∞	∞ *2e5*	0/15
C4	2.5(3)	8319(5436)	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	2.8(1)	**6721**(6716)	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
A4	2.3(1)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
B3	2.3(1)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	2.3(1)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes+	**2.0**(0.5)	∞	∞	∞	∞	∞	∞ *9e4*	0/15

Figure A.49: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	561	6541	14103	14318	14643	15567	17589	15/15
A4	109(369)	55(89)	36(29)	35(49)	35(43)	33(12)	29(29)	3/15
B3	**80**(118)	60(65)	39(32)	39(44)	38(26)	36(46)	32(61)	3/15
C4	95(215)	37(36)	23(14)	23(28)	22(29)	21(21)	19(27)	4/15
cmaes+	90(153)	**35**(77)	**21**(51)	**21**(27)	**21**(26)	**20**(11)	**17**(12)	4/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	467	5580	23491	24163	24948	26847	1.3e5	12/15
A4	52(47)	**29**(55)	**30**(59)	**30**(27)	**29**(30)	**27**(27)	**5.3**(5)	2/15
B3	43(41)	41(31)	62(50)	61(73)	59(63)	55(81)	11(6)	1/15
C4	**42**(128)	55(87)	62(45)	61(71)	59(84)	55(53)	11(18)	1/15
cmaes+	49(184)	46(65)	65(130)	63(66)	61(102)	57(50)	11(14)	1/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	3.2	1614	67457	3.7e5	4.9e5	8.1e5	8.4e5	15/15
A4	1.9(1)	**0.99**(0.4)	12(22)	∞	∞	∞	∞ *1e5*	0/15
B3	1.5(1)	2.6(0.7)	29(25)	∞	∞	∞	∞ *1e5*	0/15
C4	1.2(0.5)	1.5(3)	**9.2**(14)	∞	∞	∞	∞ *2e5*	0/15
cmaes+	**1.2**(2)	1.6(0.2)	15(12)	∞	∞	∞	∞ *8e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	5.2e7	3/15
A4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Figure A.50: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	83	83	83	83	83	83	83	30/30
A4	$9.3_{(1)}$	$17_{(2)}$	$25_{(0.7)}$	$32_{(1)}$	$39_{(1)}$	$54_{(2)}$	$70_{(3)}$	15/15
B3	$10_{(1)}$	$18_{(2)}$	$26_{(1.0)}$	$34_{(2)}$	$42_{(1)}$	$58_{(2)}$	$74_{(2)}$	15/15
C4	$10_{(0.8)}$	$17_{(2)}$	$25_{(2)}$	$32_{(2)}$	$40_{(1)}$	$56_{(3)}$	$71_{(3)}$	15/15
cmaes+	$\mathbf{8.1}_{(1)}$	$\mathbf{14}_{(1)}^{\star 3}$	$\mathbf{21}_{(2)}^{\star 3}$	$\mathbf{27}_{(2)}^{\star 4}$	$\mathbf{33}_{(2)}^{\star 4}$	$\mathbf{46}_{(2)}^{\star 4}$	$\mathbf{59}_{(2)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	796	797	799	799	800	802	804	15/15
A4	$63_{(9)}$	$79_{(5)}$	$89_{(7)}$	$96_{(10)}$	$100_{(7)}$	$105_{(3)}$	$107_{(3)}$	15/15
B3	$63_{(8)}$	$79_{(7)}$	$88_{(7)}$	$94_{(2)}$	$100_{(4)}$	$104_{(1)}$	$107_{(2)}$	15/15
C4	$63_{(12)}$	$79_{(10)}$	$88_{(10)}$	$96_{(7)}$	$100_{(5)}$	$105_{(2)}$	$107_{(3)}$	15/15
cmaes+	$\mathbf{57}_{(5)}$	$\mathbf{70}_{(6)}^{\star 2}$	$\mathbf{81}_{(5)}$	$\mathbf{87}_{(6)}^{\star}$	$\mathbf{92}_{(5)}^{\star 2}$	$\mathbf{97}_{(3)}^{\star 3}$	$\mathbf{99}_{(4)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	15526	15602	15612	15641	15646	15651	15656	15/15
A4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	15536	15601	15659	15678	15703	15733	2.8e5	9/15
A4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.51: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	98	116	120	121	121	121	121	15/15
A4	7.4(1)	7.1(0.9)	7.1(1)	7.1(1)	7.1(0.6)	7.1(1.0)	7.1(2)	15/15
B3	7.3(0.9)	7.5(1)	7.3(1)	7.3(0.8)	7.3(1)	7.3(0.9)	7.3(0.9)	15/15
C4	7.1(1)	6.9(0.5)	6.7(0.6)	6.7(0.6)	6.7(0.7)	6.7(0.3)	6.7(0.6)	15/15
cmaes+	**6.7**(0.6)	**6.6**(0.3)	**6.7**(0.7)	**6.7**(0.7)	**6.7**(0.6)	**6.7**(0.6)	**6.7**(0.8)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	3507	5523	7168	9470	11538	15007	19222	15/15
A4	1.6(0.3)	1.7(0.2)	1.8(0.1)	1.8(0.2)	1.8(0.2)	2.2(0.2)	2.6(0.3)	15/15
B3	1.8(0.3)	1.8(0.2)	2.0(0.3)	2.0(0.4)	2.1(0.2)	2.4(0.4)	2.7(0.4)	15/15
C4	1.7(0.3)	1.7(0.2)	1.8(0.3)	1.8(0.3)	2.0(0.3)	2.3(0.3)	2.6(0.5)	15/15
cmaes+	**1.3**(0.2)*	**1.4**(0.3)*	**1.5**(0.2)*	**1.5**(0.2)$^{\star 2}$	**1.6**(0.1)$^{\star 2}$	**1.9**(0.3)	**2.2**(0.4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	10698	17839	41037	66294	66294	66294	68145	15/15
A4	116(114)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
B3	**66**(92)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
C4	68(40)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes+	410(368)	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	7080	10655	11012	11265	11430	11701	11969	15/15
A4	7.6(0.9)	8.5(2)	8.6(7)	8.6(5)	8.6(4)	8.6(0.5)	8.6(2)	15/15
B3	7.5(0.7)	7.2(2)	7.5(4)	7.5(3)	7.6(1)	7.6(2)	7.6(3)	15/15
C4	7.9(1)	9.3(5)	9.5(3)	9.5(5)	9.4(5)	9.4(4)	9.4(5)	15/15
cmaes+	**7.2**(0.6)	**6.5**(1)	**6.8**(0.5)	**6.8**(2)	**6.8**(0.5)	**6.8**(1)	**6.8**(1)	15/15

Figure A.52: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	6122	12982	13300	13496	13651	13909	14142	15/15
A4	8.7(4)	7.0(4)	7.2(5)	7.2(5)	7.3(4)	7.3(4)	7.3(1)	15/15
B3	9.1(0.4)	**6.3**(2)	**6.5**(2)	**6.6**(0.1)	**6.6**(2)	**6.7**(0.1)	**6.7**(0.1)	15/15
C4	8.6(1)	7.7(2)	7.9(3)	8.0(0.3)	8.0(4)	8.0(3)	8.0(3)	15/15
cmaes+	**8.2**(2)	6.5(3)	6.6(5)	6.7(6)	6.7(1)	6.8(5)	6.8(0.9)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	25890	30368	36796	51579	56007	65128	70824	15/15
A4	1.9(0.2)	2.0(0.1)	1.9(0.1)	1.5(0.1)	1.4(0.1)	1.3(0.0)	1.2(0.0)	15/15
B3	1.9(0.3)	2.1(0.2)	1.9(0.2)	1.5(0.1)	1.4(0.1)	1.3(0.0)	1.2(0.0)	15/15
C4	1.9(0.3)	2.0(0.2)	1.9(0.1)	1.5(0.1)	1.4(0.1)	1.3(0.0)	1.2(0.0)	15/15
cmaes+	**1.8**(0.2)	**1.9**(0.1)	**1.8**(0.1)	**1.4**(0.0)*	**1.3**(0.1)*	**1.2**(0.0)$^{\star 3}$	**1.1**(0.0)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	2368	4855	11681	25315	29749	38949	48211	15/15
A4	9.3(0.8)	6.1(0.4)	3.0(0.4)	1.6(0.1)	1.4(0.1)	1.2(0.0)	1.1(0.0)	15/15
B3	**9.0**(0.7)	6.2(0.4)	3.0(0.2)	1.6(0.1)	1.5(0.0)	1.2(0.1)	1.1(0.0)	15/15
C4	10(0.9)	6.3(0.3)	3.0(0.2)	1.5(0.1)	1.4(0.1)	1.2(0.1)	1.1(0.0)	15/15
cmaes+	9.2(0.5)	**6.0**(0.3)	**2.9**(0.1)	**1.5**(0.1)*	**1.3**(0.1)$^{\star 2}$	**1.1**(0.0)$^{\star 3}$	**0.99**(0.0)$^{\star 3}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	4169	7452	9174	10751	13146	22758	25192	15/15
A4	1.6(1)	2.3(1)	2.7(1)	3.0(1)	2.8(0.9)	2.1(0.8)	2.1(0.6)	15/15
B3	1.9(2)	2.3(1)	2.8(1)	3.0(0.6)	2.9(0.9)	2.1(0.7)	2.2(0.6)	15/15
C4	**1.4**(0.1)	**1.5**(2)	**2.1**(2)	**2.4**(1)	**2.3**(0.7)	**1.8**(0.5)	**1.9**(0.4)	15/15
cmaes+	1.4(1)	1.7(1)	2.4(0.8)	2.6(0.8)	2.5(0.7)	1.9(0.7)	2.0(0.9)	15/15

Figure A.53: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	2029	6916	8734	11861	71936	98467	1.2e5	15/15
A4	2.5(3)	5.1(3)	15(9)	30(34)	8.6(9)	**17**(8)	**45**(22)	1/15
B3	4.3(3)	6.7(7)	11(12)	25(21)	11(10)	56(65)	∞ *4e5*	0/15
C4	2.5(0.2)	**4.3**(5)	**11**(5)	25(10)	11(8)	56(75)	∞ *4e5*	0/15
cmaes+	**1.7**(0.2)	4.3(11)	17(8)	**18**(10)	**7.3**(8)	46(31)	∞ *3e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	304	616	777	1105	2207	4825	57711	15/15
A4	1.4(0.4)	2.1(0.2)	2.9(0.2)	3.7(0.2)	4.0(0.4)	6.7(0.5)	1.2(0.1)	15/15
B3	1.6(0.6)	2.3(0.3)	3.1(0.3)	3.9(0.3)	4.2(0.3)	6.8(0.8)	1.3(0.1)	15/15
C4	1.4(0.8)	2.1(0.2)	2.9(0.3)	3.7(0.2)	4.0(0.2)	6.8(0.4)	1.3(0.1)	15/15
cmaes+	**1.2**(0.6)	**1.8**(0.1)*	**2.4**(0.2)$^{\star 3}$	**3.2**(0.3)$^{\star 3}$	**3.6**(0.2)	**6.2**(0.6)	**1.2**(0.1)*	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	1.9e5	7.9e5	1.0e6	1.1e6	1.1e6	1.1e6	1.1e6	15/15
A4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	5244	72122	3.2e5	7.1e5	1.4e6	2.0e6	2.0e6	15/15
A4	0.31(0.1)$_{\downarrow 4}$	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	0.29(0.1)$_{\downarrow 4}$	78(155)	∞	∞	∞	∞	∞ *4e5*	0/15
C4	0.30(0.1)$_{\downarrow 4}$	81(66)	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	**0.26**(0.1)$_{\downarrow 4}$	**78**(41)	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.54: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	399	4220	14158	34948	51958	1.3e5	2.7e5	14/15
A4	0.59(0.5)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	0.64(0.7)	619(767)	∞	∞	∞	∞	∞ *4e5*	0/15
C4	0.90(0.6)	653(753)	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	**0.58**(0.8)	**225**(288)	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	1442	16998	47068	1.3e5	1.9e5	6.7e5	9.5e5	6/15
A4	362(336)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	249(541)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
C4	270(303)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	**131**(636)	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	1.4e6	1.7e7	2.6e7	4.5e7	4.5e7	8/15
A4	3.3(2)	4.9e4(2e4)	∞	∞	∞	∞	∞ *1e4*	0/15
B3	**2.7**(2)	7.7e4(7e4)	∞	∞	∞	∞	∞ *1e4*	0/15
C4	2.7(2)	**4.4e4**(2e4)	∞	∞	∞	∞	∞ *1e4*	0/15
cmaes+	2.9(3)	8.9e4(7e4)	∞	∞	∞	∞	∞ *1e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	222	1.3e5	1.6e8	∞	∞	∞	∞	0
A4	2.3(0.7)	∞	∞	.	.	.	.	0/15
B3	2.4(1)	∞	∞	.	.	.	.	0/15
C4	2.0(0.8)	∞	∞	.	.	.	.	0/15
cmaes+	**1.7**(0.5)	∞	∞	.	.	.	.	0/15

Figure A.55: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	1044	21144	1.0e5	1.0e5	1.0e5	1.0e5	1.0e5	26/30
A4	1.4(2)	9.1(13)	**3.3**(8)	**3.3**(2)	**3.3**(4)	**3.3**(5)	**3.3**(2)	5/15
B3	1.5(0.3)	11(35)	3.8(4)	3.8(8)	3.8(3)	3.7(7)	3.7(4)	5/15
C4	2.2(6)	12(12)	4.1(9)	4.1(2)	4.1(4)	4.1(7)	4.1(8)	5/15
cmaes+	**1.2**(0.2)	**8.8**(15)	4.1(5)	4.1(5)	4.1(8)	4.1(5)	4.1(7)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	3090	35442	6.5e5	6.5e5	6.5e5	6.5e5	6.5e5	8/30
A4	14(77)	8.5(17)	4.0(4)	4.0(4)	4.0(4)	4.0(3)	4.0(9)	1/15
B3	14(65)	8.6(14)	4.0(5)	4.0(8)	4.0(7)	4.0(4)	4.0(3)	1/15
C4	6.3(0.1)	**7.5**(7)	**3.8**(5)	**3.8**(4)	**3.8**(4)	**3.8**(5)	**3.8**(4)	1/15
cmaes+	**3.3**(43)	7.7(19)	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	7.1	11925	75453	6.6e5	1.3e6	3.2e6	3.4e6	15/15
A4	1.8(1)	0.69(0.4)	∞	∞	∞	∞	∞ *4e4*	0/15
B3	1.1(1)	1.1(0.9)	∞	∞	∞	∞	∞ *3e4*	0/15
C4	1.3(0.5)	**0.51**(0.4)	∞	∞	∞	∞	∞ *4e4*	0/15
cmaes+	**0.99**(0.8)	1.5(2)	∞	∞	∞	∞	∞ *1e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	5.8e6	9.8e7	3.0e8	3.0e8	3.0e8	3.0e8	3.0e8	1/15
A4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
B3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
C4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes+	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.56: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

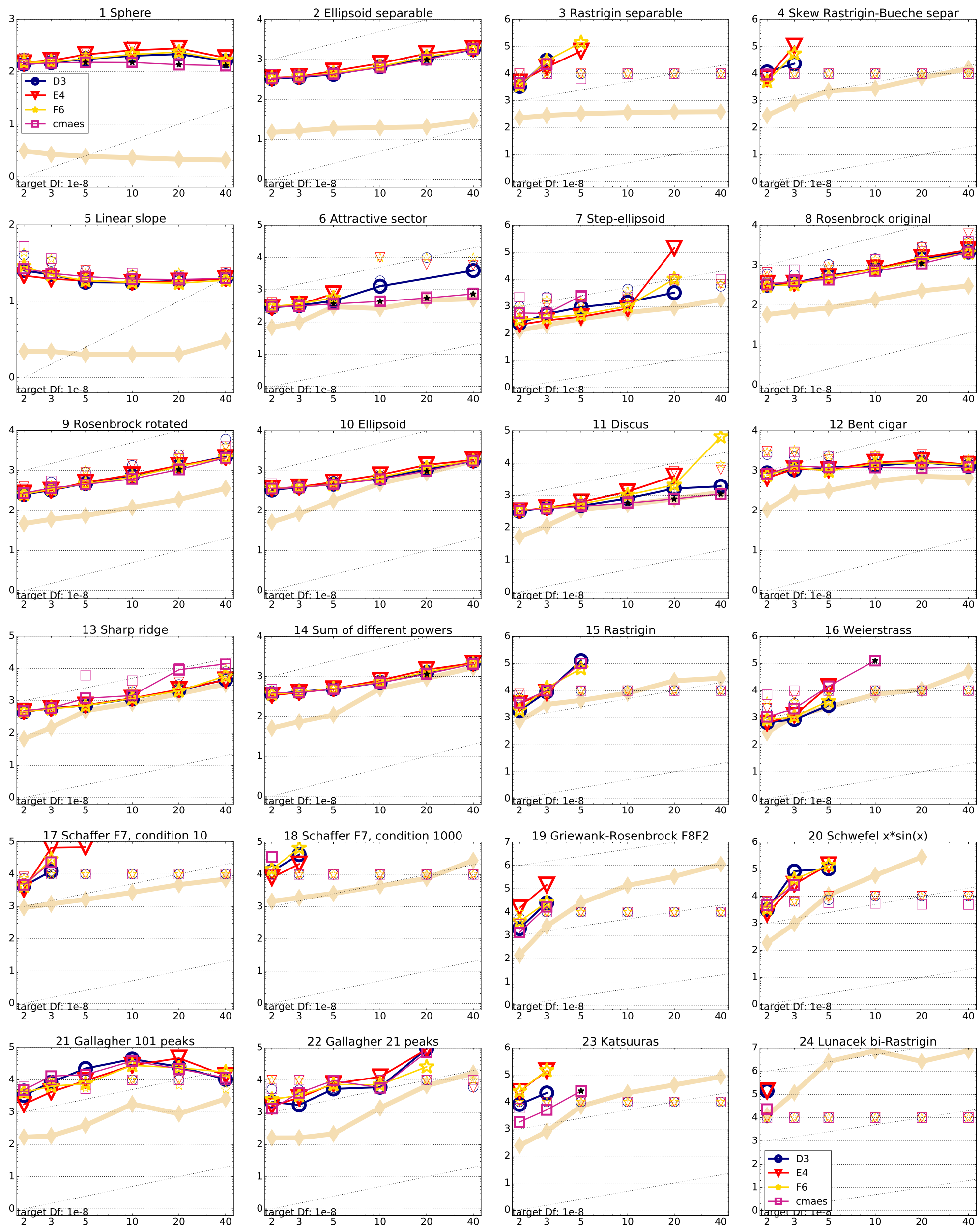

Figure A.57: Expected running time (ERT in number of f-evaluations as $\log_{10}$ value), divided by dimension for target function value 10^{-8} versus dimension. Slanted grid lines indicate quadratic scaling with the dimension. Different symbols correspond to different algorithms given in the legend of f_1 and f_{24}. Light symbols give the maximum number of function evaluations from the longest trial divided by dimension. Black stars indicate a statistically better result compared to all other algorithms with $p < 0.01$ and Bonferroni correction number of dimensions (six). Legend: ○:A1, ▽:A2, ⋆:A3, □:A4, △:A5, ◇:A6, ⬡:A7, ⬠:A8, ⬡:A9, ◁:A10

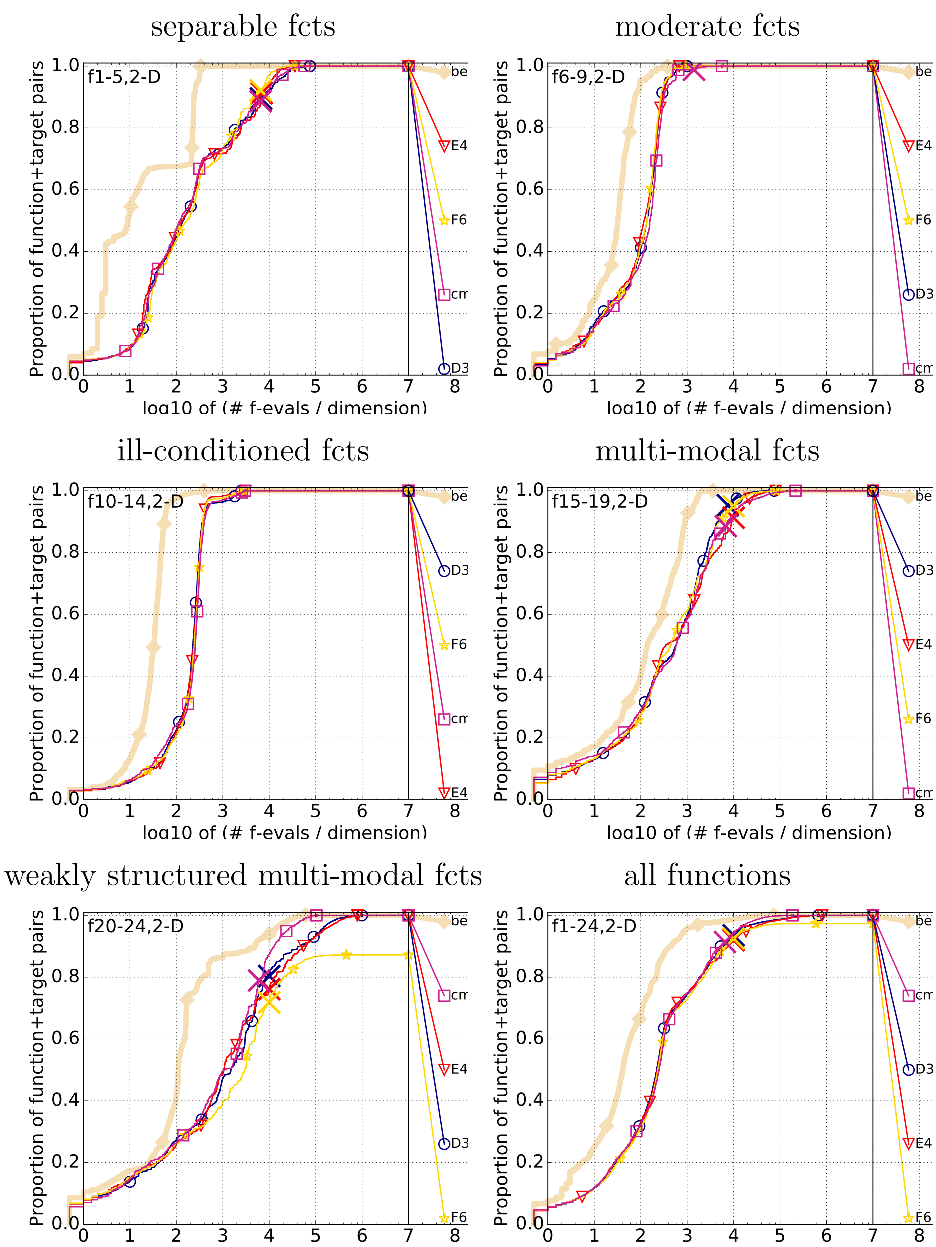

Figure A.58: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

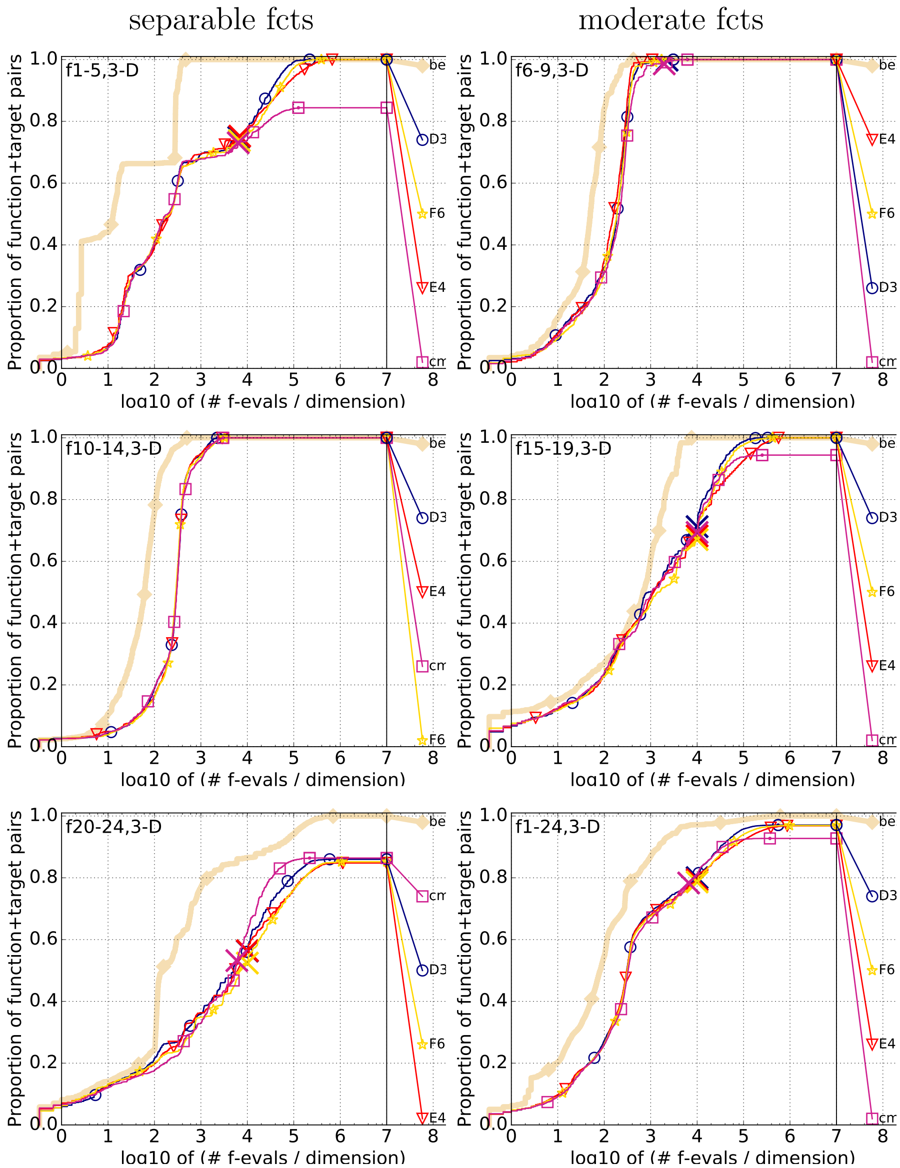

Figure A.59: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

separable fcts

moderate fcts

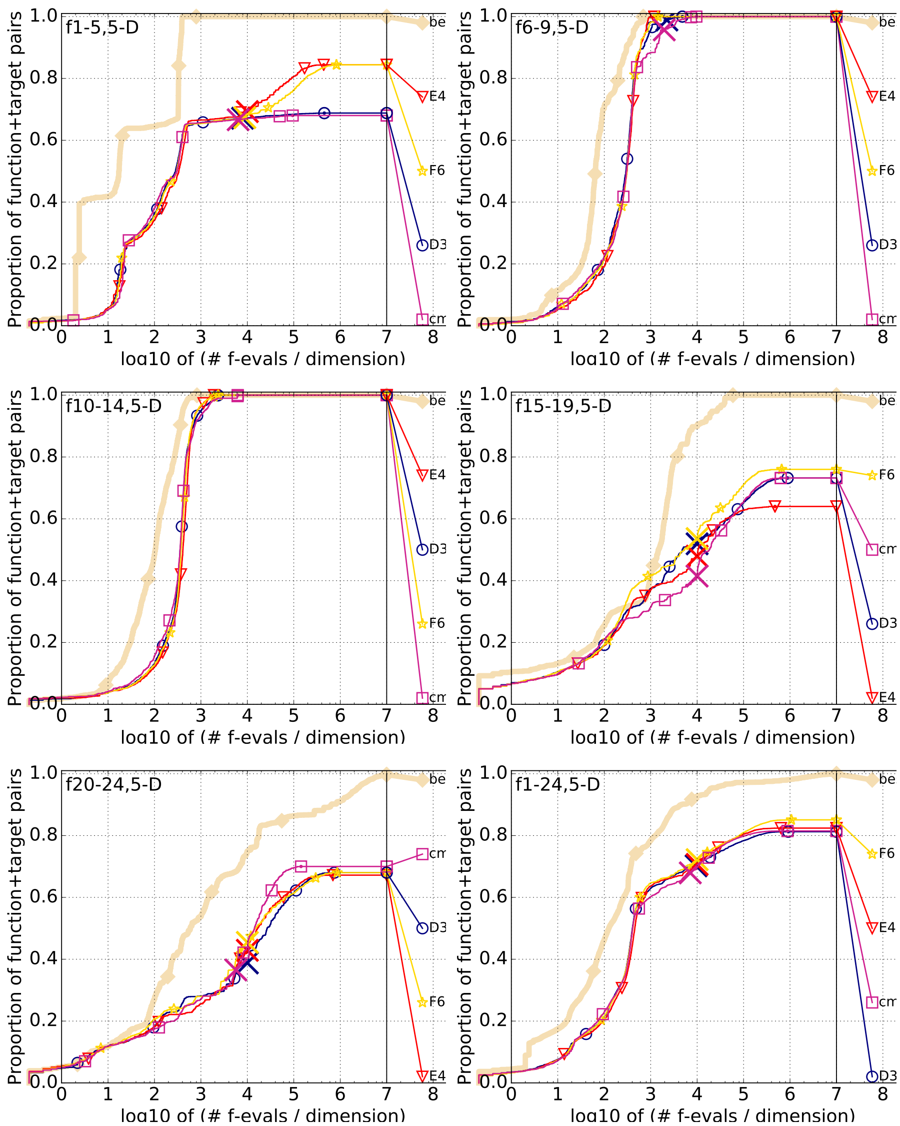

Figure A.60: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

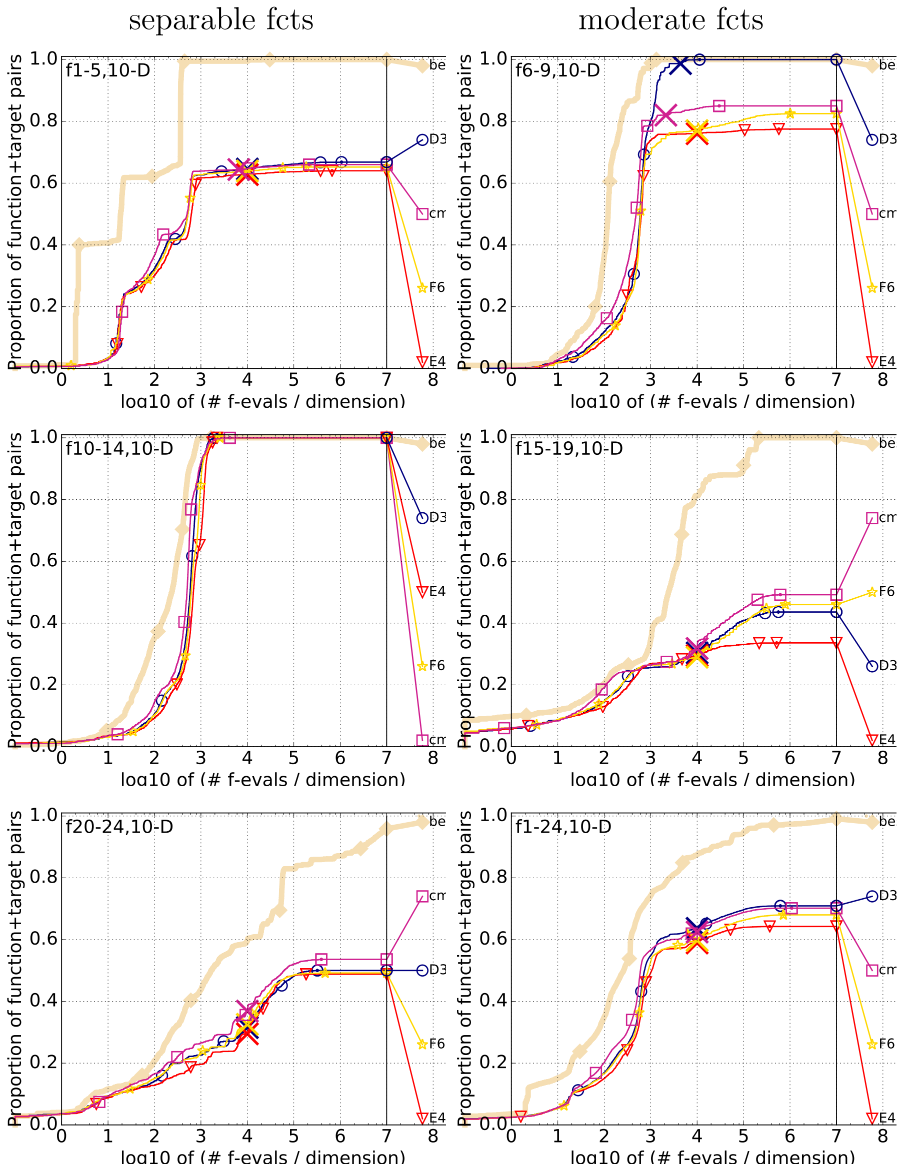

Figure A.61: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

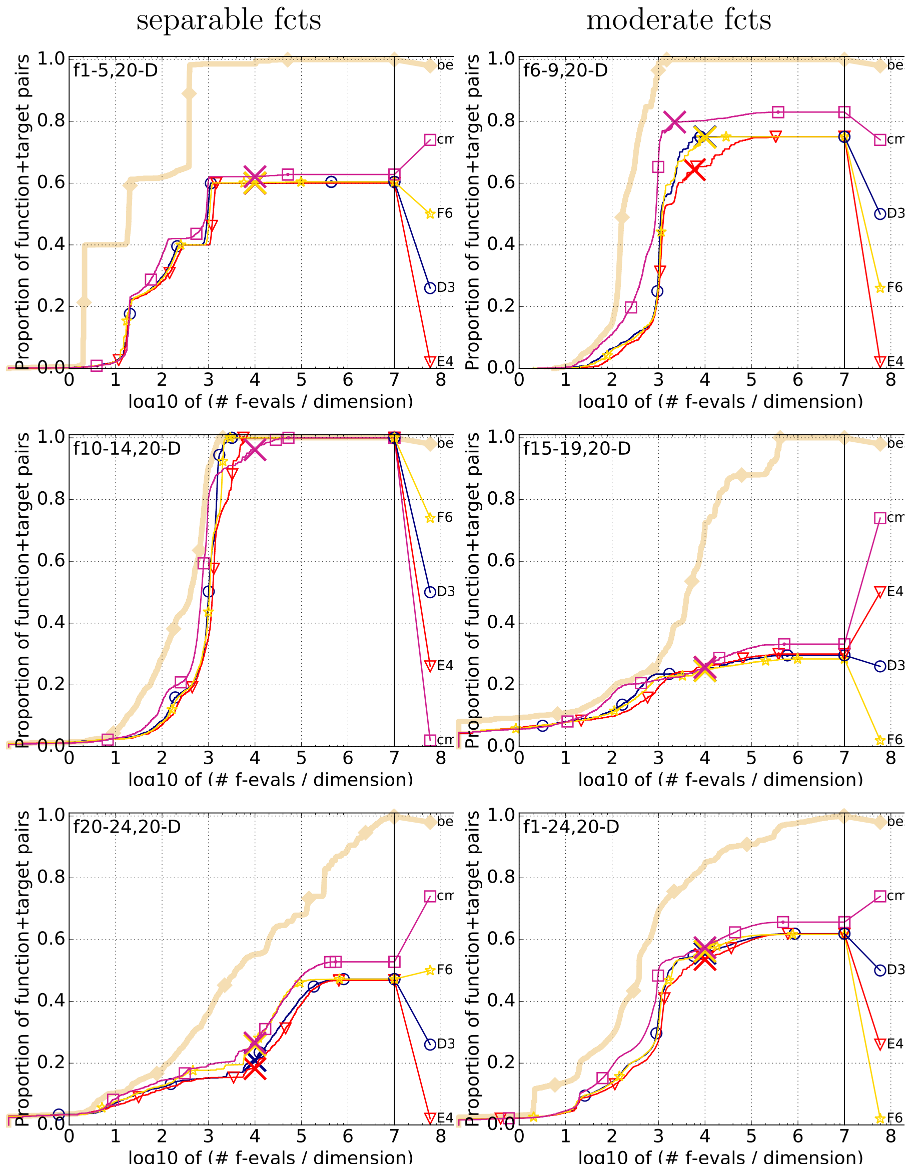

Figure A.62: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

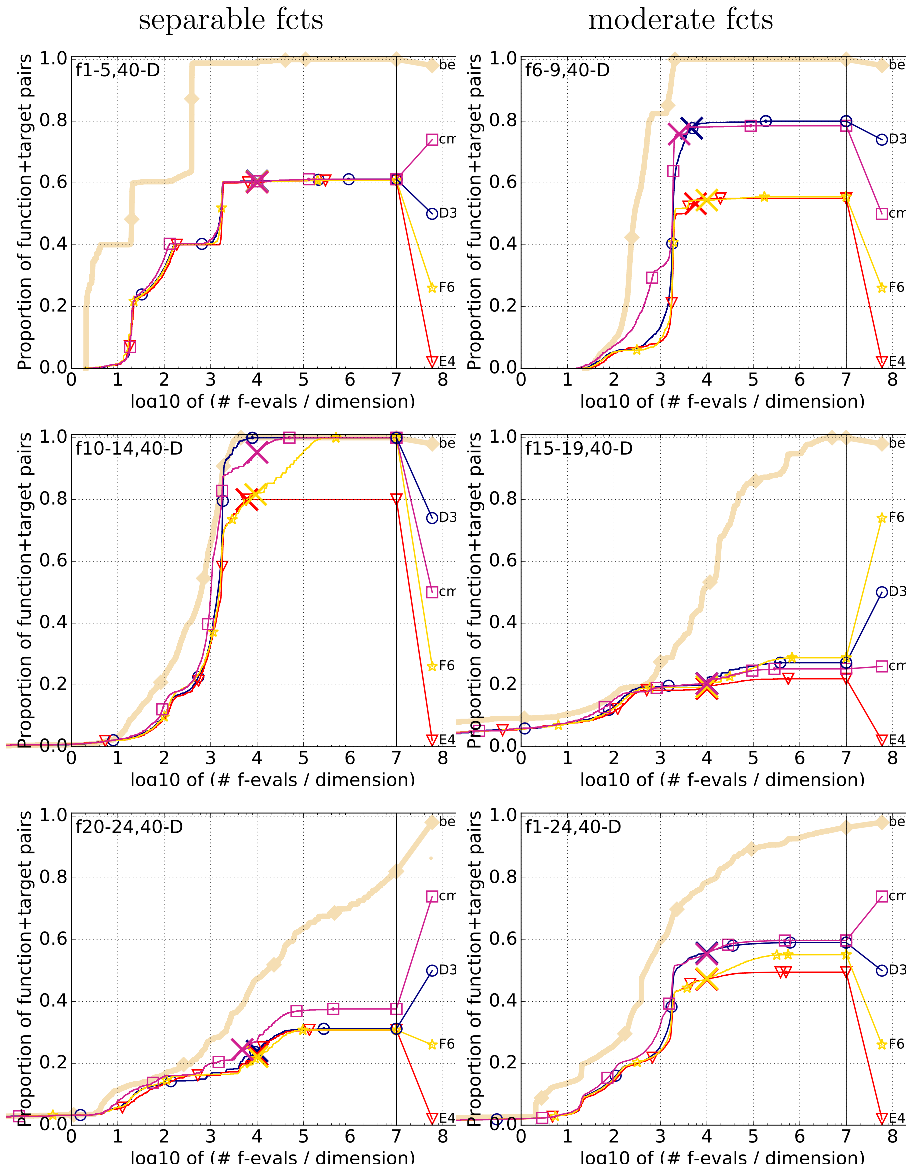

Figure A.63: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	1.8	5.7	5.7	6.2	6.2	6.2	6.2	15/15
D3	4.8$_{(7)}$	4.7$_{(2)}$	8.7$_{(4)}$	14$_{(6)}$	20$_{(8)}$	31$_{(6)}$	41$_{(6)}$	15/15
E4	**4.4**$_{(6)}$	**3.6**$_{(3)}$	**8.2**$_{(5)}$	**13**$_{(6)}$	20$_{(9)}$	31$_{(4)}$	40$_{(5)}$	15/15
F6	7.5$_{(7)}$	4.9$_{(4)}$	10$_{(7)}$	14$_{(9)}$	20$_{(10)}$	32$_{(7)}$	43$_{(9)}$	15/15
cmaes	5.5$_{(5)}$	4.5$_{(3)}$	10$_{(6)}$	14$_{(5)}$	**20**$_{(3)}$	**29**$_{(7)}$	**38**$_{(10)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	16	19	25	25	26	28	29	15/15
D3	12$_{(10)}$	**17**$_{(5)}$	17$_{(5)}$	18$_{(4)}$	**19**$_{(3)}$	21$_{(5)}$	**22**$_{(3)}$	15/15
E4	13$_{(10)}$	18$_{(9)}$	**15**$_{(6)}$	19$_{(3)}$	20$_{(5)}$	21$_{(4)}$	23$_{(4)}$	15/15
F6	12$_{(10)}$	19$_{(9)}$	17$_{(5)}$	19$_{(4)}$	19$_{(4)}$	21$_{(3)}$	23$_{(4)}$	15/15
cmaes	**11**$_{(11)}$	18$_{(10)}$	17$_{(7)}$	**18**$_{(10)}$	20$_{(5)}$	**21**$_{(4)}$	22$_{(4)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	15	271	445	446	450	454	464	15/15
D3	**2.2**$_{(2)}$	7.5$_{(4)}$	**15**$_{(15)}$	**15**$_{(8)}$	**15**$_{(14)}$	**15**$_{(7)}$	**14**$_{(14)}$	13/15
E4	6.0$_{(13)}$	15$_{(14)}$	19$_{(24)}$	22$_{(38)}$	21$_{(19)}$	21$_{(26)}$	21$_{(16)}$	12/15
F6	5.2$_{(2)}$	**5.6**$_{(3)}$	15$_{(16)}$	15$_{(33)}$	15$_{(23)}$	15$_{(22)}$	15$_{(29)}$	12/15
cmaes	10$_{(4)}$	8.4$_{(9)}$	20$_{(27)}$	20$_{(8)}$	20$_{(20)}$	20$_{(36)}$	20$_{(14)}$	11/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	22	344	459	496	523	544	566	15/15
D3	6.5$_{(1)}$	12$_{(11)}$	50$_{(52)}$	47$_{(126)}$	44$_{(46)}$	43$_{(58)}$	41$_{(36)}$	7/15
E4	**2.2**$_{(2)}$	**5.9**$_{(12)}$	23$_{(25)}$	27$_{(44)}$	26$_{(29)}$	25$_{(45)}$	24$_{(21)}$	10/15
F6	3.9$_{(9)}$	11$_{(4)}$	**22**$_{(29)}$	**20**$_{(39)}$	**19**$_{(18)}$	**19**$_{(19)}$	**18**$_{(5)}$	11/15
cmaes	2.2$_{(1)}$	10$_{(5)}$	40$_{(58)}$	37$_{(30)}$	35$_{(26)}$	34$_{(45)}$	33$_{(22)}$	8/15

Figure A.64: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	3.7	4.4	4.4	4.4	4.4	4.4	4.4	15/15
D3	8.9(3)	11(3)	11(2)	11(5)	11(5)	11(4)	11(5)	15/15
E4	**7.5**(2)	**9.1**(2)	**10**(3)	**10**(2)	**10**(3)	**10**(2)	**10**(2)	15/15
F6	9.1(3)	12(4)	13(4)	13(4)	13(5)	13(4)	13(5)	15/15
cmaes	7.7(3)	11(4)	12(6)	12(4)	12(6)	12(4)	12(4)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	13	23	41	54	67	95	124	15/15
D3	3.0(3)	3.6(5)	3.5(3)	4.2(1)	4.5(0.9)	4.3(0.9)	4.2(0.6)	15/15
E4	1.7(2)	**3.2**(2)	**3.3**(2)	3.8(1.0)	**4.0**(0.3)	4.2(1)	**4.0**(0.8)	15/15
F6	**1.7**(1)	3.8(2)	3.5(0.6)	**3.7**(2)	4.1(0.8)	4.3(0.8)	4.4(0.7)	15/15
cmaes	2.2(3)	3.4(2)	3.4(1)	3.7(1)	4.0(1)	**4.2**(1)	4.3(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	3.2	21	60	193	217	217	241	15/15
D3	**3.3**(3)	2.8(6)	3.7(2)	1.8(1)	1.7(0.7)	1.7(0.7)	1.8(0.7)	15/15
E4	4.0(5)	**2.3**(2)	**2.4**(2)	**1.2**(2)	**1.3**(0.7)	**1.3**(0.4)	**1.3**(1)	15/15
F6	5.1(6)	2.6(1)	3.4(2)	2.1(3)	2.0(3)	2.0(3)	1.9(1)	15/15
cmaes	4.4(6)	6.2(8)	10(15)	3.6(12)	3.4(2)	3.4(4)	4.5(8)	13/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	5.4	12	37	46	86	94	112	15/15
D3	3.3(6)	8.8(24)	6.9(10)	8.4(7)	5.0(3)	5.6(2)	5.4(3)	15/15
E4	4.5(8)	13(4)	7.5(9)	8.7(4)	5.4(2)	6.2(0.9)	5.9(3)	15/15
F6	3.2(2)	**7.8**(10)	**5.5**(2)	**7.1**(4)	**4.5**(1)	**5.2**(3)	**5.0**(2)	15/15
cmaes	**3.1**(2)	8.3(16)	7.9(5)	8.6(6)	5.2(1)	6.0(4)	5.5(0.8)	15/15

Figure A.65: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	1	18	30	44	68	81	92	15/15
D3	**5.2**(6)	1.1(1)	4.1(3)	5.7(2)	4.6(2)	4.8(1)	**5.0**(2)	15/15
E4	5.6(3)	1.4(3)	**3.5**(4)	5.5(3)	4.6(1)	5.2(1)	5.4(2)	15/15
F6	6.5(6)	**1.0**(0.8)	4.0(3)	**5.3**(2)	**4.6**(0.8)	**4.7**(0.9)	5.0(0.6)	15/15
cmaes	7.0(6)	1.7(1)	5.8(3)	6.8(2)	5.3(1)	5.4(1)	5.5(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	30	46	54	61	68	82	98	15/15
D3	7.7(5)	7.3(3)	**7.4**(2)	**7.2**(1)	**6.8**(0.8)	**6.6**(0.7)	**6.3**(1)	15/15
E4	7.1(2)	**7.3**(3)	7.5(2)	7.7(0.7)	7.5(1)	7.4(0.9)	6.9(1)	15/15
F6	8.2(5)	9.0(2)	8.6(2)	8.2(2)	7.9(1)	7.6(2)	7.0(1)	15/15
cmaes	**6.8**(6)	8.0(5)	9.2(3)	8.7(2)	8.3(3)	7.7(1)	7.1(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	35	45	50	62	67	81	97	15/15
D3	**4.6**(4)	**5.9**(3)	**7.4**(2)	**6.8**(2)	**7.0**(1)	**6.5**(0.7)	**6.3**(0.9)	15/15
E4	5.9(4)	6.3(3)	7.9(1)	7.3(1)	7.2(1)	6.8(0.5)	6.5(0.7)	15/15
F6	6.2(4)	8.1(1)	8.2(1)	7.6(2)	7.6(1)	7.1(1)	6.6(1.0)	15/15
cmaes	5.6(5)	7.7(3)	8.3(4)	7.8(1.0)	7.6(1)	7.2(0.8)	6.6(0.9)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	35	46	75	94	105	153	195	15/15
D3	**6.3**(4)	14(10)	13(11)	12(0.8)	12(14)	10(12)	8.8(11)	15/15
E4	9.1(15)	12(17)	**10**(25)	**8.8**(3)	**8.7**(11)	**7.1**(5)	**6.4**(10)	15/15
F6	7.5(5)	**11**(17)	10(6)	10(11)	10(5)	8.2(12)	7.5(8)	15/15
cmaes	8.9(5)	13(15)	12(17)	11(16)	11(9)	9.3(14)	8.3(5)	15/15

Figure A.66: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	23	35	46	60	71	95	122	15/15
D3	6.2(7)	7.1(3)	**6.6**(3)	6.9(1)	6.8(1)	7.0(1.0)	6.8(1)	15/15
E4	**3.4**(2)	**6.1**(2)	6.6(2)	**6.7**(1)	**6.7**(1)	6.9(0.9)	**6.8**(0.6)	15/15
F6	5.9(4)	6.4(2)	6.7(3)	6.8(2)	6.9(1)	**6.8**(0.9)	6.9(1)	15/15
cmaes	5.0(8)	8.3(5)	8.2(3)	7.3(3)	7.5(2)	7.0(2)	7.4(1.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	1.4	7.4	16	24	38	67	90	15/15
D3	3.7(8)	2.5(3)	3.0(2)	**3.8**(3)	**4.5**(1)	5.8(0.6)	6.6(1.0)	15/15
E4	**2.0**(3)	**2.3**(2)	**2.4**(0.7)	5.0(2)	4.8(2)	**5.4**(1)	**6.3**(1)	15/15
F6	2.4(2)	**2.3**(2)	2.9(3)	4.9(1)	5.4(2)	6.3(0.5)	6.9(0.9)	15/15
cmaes	4.6(11)	2.7(3)	2.8(2)	4.2(3)	5.2(3)	6.3(2)	7.4(0.9)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	37	291	1033	1066	1113	1231	1412	5/5
D3	2.8(7)	**2.4**(2)	3.1(3)	**3.2**(2)	**3.1**(2)	**2.9**(4)	**2.6**(3)	15/15
E4	1.4(0.7)	3.8(6)	4.1(7)	6.0(5)	5.8(5)	5.3(10)	4.7(6)	13/15
F6	**0.97**(1)	4.9(11)	**2.9**(1.0)	3.4(5)	3.3(4)	3.0(5)	2.7(7)	14/15
cmaes	1.1(1)	4.9(4)	5.2(5)	5.8(6)	6.3(8)	5.7(4)	5.0(6)	13/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	9.1	50	174	326	358	409	538	15/15
D3	3.9(5)	8.9(10)	**3.6**(3)	**2.4**(3)	3.1(4)	2.9(4)	2.3(4)	15/15
E4	**3.2**(5)	8.4(15)	4.1(6)	2.9(5)	**2.9**(4)	**2.8**(3)	**2.3**(4)	15/15
F6	3.6(4)	20(33)	7.5(5)	4.1(8)	4.1(11)	3.8(6)	3.0(3)	15/15
cmaes	3.4(4)	**7.5**(17)	5.7(24)	5.9(3)	5.5(30)	5.0(12)	3.9(5)	14/15

Figure A.67: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	2.7	61	133	275	396	1086	1657	5/5
D3	105(317)	10(7)	6.5(10)	5.3(3)	3.9(6)	4.4(6)	5.1(5)	12/15
E4	24(137)	**3.3**(4)	**2.2**(0.7)	**2.2**(4)	**2.3**(1)	3.9(5)	**4.1**(3)	13/15
F6	**5.1**(5)	11(19)	5.6(12)	3.3(3)	3.2(4)	4.7(5)	5.0(7)	14/15
cmaes	100(358)	15(17)	8.1(11)	5.0(4)	4.0(3)	**3.7**(1.0)	5.7(9)	10/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	19	134	666	1249	1708	2438	2858	15/15
D3	39(272)	13(13)	4.8(7)	3.1(4)	3.9(4)	8.3(11)	8.7(12)	8/15
E4	20(67)	13(31)	3.2(6)	2.5(2)	3.4(2)	**6.2**(6)	**5.5**(6)	12/15
F6	**2.4**(2)	**7.0**(10)	**2.8**(4)	**1.9**(2)	**1.5**(2)	8.3(5)	10(14)	7/15
cmaes	77(0.6)	22(5)	8.1(7)	4.6(5)	3.9(5)	16(18)	25(35)	3/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	26	216	227	252	276	15/15
D3	1.9(2)	**6.0**(6)	**4.7**(7)	**6.4**(12)	12(16)	13(17)	14(18)	15/15
E4	1.9(1)	8.2(8)	7.7(14)	37(28)	61(118)	118(166)	108(111)	7/15
F6	**1.7**(1)	12(42)	15(47)	17(14)	27(22)	31(33)	28(31)	13/15
cmaes	1.8(2)	10(27)	22(44)	7.6(13)	**7.6**(12)	**10**(10)	**9.4**(7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	3.7	61	365	366	366	370	375	15/15
D3	3.4(4)	14(13)	16(14)	17(8)	17(16)	17(14)	17(13)	14/15
E4	**2.6**(2)	**10**(22)	**10**(13)	**11**(5)	**12**(16)	**12**(20)	**12**(6)	14/15
F6	2.8(2)	12(12)	17(16)	18(16)	18(15)	18(17)	18(14)	14/15
cmaes	3.8(2)	26(24)	22(23)	22(15)	22(23)	25(28)	25(24)	11/15

Figure A.68: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	1.7	51	174	276	290	324	330	15/15
D3	**1.9**(3)	60(94)	37(31)	24(23)	22(27)	20(17)	20(18)	11/15
E4	4.5(14)	**40**(15)	**17**(36)	**11**(11)	**12**(17)	**11**(9)	**11**(14)	13/15
F6	2.3(0)	65(69)	39(28)	29(56)	28(62)	25(26)	25(39)	10/15
cmaes	3.4(0.4)	98(187)	43(29)	35(36)	33(31)	30(54)	29(53)	9/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	5.1	27	168	218	249	289	306	15/15
D3	13(43)	**58**(152)	19(18)	18(18)	16(15)	14(29)	14(22)	12/15
E4	295(3)	93(53)	17(13)	13(29)	12(23)	11(36)	10(3)	14/15
F6	79(357)	130(113)	29(17)	22(58)	20(25)	17(15)	16(26)	12/15
cmaes	**11**(35)	83(201)	**14**(18)	**11**(13)	**10**(36)	**8.8**(2)	**8.4**(18)	13/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	7.8	193	234	263	299	348	379	15/15
D3	3.1(3)	13(29)	54(42)	60(45)	53(62)	46(100)	43(114)	10/15
E4	2.7(3)	14(6)	163(234)	145(208)	162(218)	140(136)	128(219)	5/15
F6	**2.2**(0.7)	46(40)	172(203)	189(240)	166(286)	144(159)	132(119)	5/15
cmaes	2.3(2)	**9.1**(12)	**13**(23)	**12**(20)	**11**(17)	**10**(4)	**9.3**(5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	18	857	8515	23399	24113	24721	24721	5/15
D3	3.9(4)	4.3(6)	6.8(17)	12(13)	12(13)	11(15)	11(12)	1/15
E4	5.8(1)	3.4(6)	6.9(13)	12(13)	12(11)	11(15)	11(8)	1/15
F6	**1.5**(0.6)	**3.0**(1)	5.8(7)	∞	∞	∞	∞ *2e4*	0/15
cmaes	2.0(2)	3.9(9)	**4.5**(9)	**2.0**(3)	**1.9**(2)	**1.9**(1)	**1.9**(1)	5/15

Figure A.69: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 2-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	3.6	8.0	8.0	8.0	8.0	8.0	8.0	15/15
D3	4.3(4)	5.4(2)	**11**(6)	19(4)	25(3)	38(6)	**51**(7)	15/15
E4	**4.1**(8)	**4.8**(3)	12(6)	**18**(6)	**25**(6)	40(6)	54(3)	15/15
F6	5.2(5)	5.9(2)	13(5)	19(3)	25(7)	39(5)	52(10)	15/15
cmaes	5.7(3)	6.5(5)	12(4)	19(6)	26(10)	**37**(12)	51(5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	38	42	43	44	45	47	48	15/15
D3	12(4)	15(2)	17(4)	18(2)	19(3)	**20**(3)	**21**(2)	15/15
E4	14(4)	15(4)	**16**(5)	**17**(2)	**18**(2)	20(2)	22(3)	15/15
F6	**11**(7)	**14**(5)	17(2)	18(3)	19(2)	21(2)	22(2)	15/15
cmaes	12(5)	16(4)	18(3)	19(3)	19(3)	21(2)	22(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	38	822	830	835	842	847	853	15/15
D3	71(39)	16(32)	113(113)	112(109)	111(137)	111(69)	110(181)	3/15
E4	5.8(12)	8.4(34)	**62**(78)	**62**(103)	**62**(118)	**61**(144)	**61**(73)	5/15
F6	**5.4**(3)	**6.8**(8)	108(201)	107(117)	106(151)	106(136)	105(182)	3/15
cmaes	67(2)	12(22)	82(35)	81(63)	81(152)	80(71)	80(114)	4/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	40	808	866	921	952	1015	1044	15/15
D3	30(14)	28(17)	**81**(71)	**76**(85)	**74**(46)	**69**(66)	**67**(73)	4/15
E4	11(12)	**26**(27)	360(339)	338(456)	327(332)	307(452)	299(191)	1/15
F6	70(19)	27(21)	188(282)	177(177)	171(113)	161(146)	156(294)	2/15
cmaes	**9.2**(31)	38(51)	∞	∞	∞	∞	∞ *2e4*	0/15

Figure A.70: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	6.6	6.6	6.6	6.6	6.6	6.6	6.6	15/15
D3	7.0$_{(1)}$	10$_{(1)}$	10$_{(3)}$	10$_{(3)}$	10$_{(3)}$	10$_{(3)}$	10$_{(3)}$	15/15
E4	7.0$_{(0.9)}$	**8.6**$_{(1)}$	**8.9**$_{(2)}$	**8.9**$_{(2)}$	**8.9**$_{(3)}$	**8.9**$_{(3)}$	**8.9**$_{(1)}$	15/15
F6	7.1$_{(2)}$	10$_{(3)}$	10$_{(4)}$	10$_{(2)}$	10$_{(4)}$	10$_{(4)}$	10$_{(2)}$	15/15
cmaes	**6.9**$_{(2)}$	10$_{(5)}$	11$_{(3)}$	11$_{(4)}$	11$_{(3)}$	11$_{(4)}$	11$_{(4)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	34	56	90	117	149	215	265	15/15
D3	1.9$_{(1)}$	2.8$_{(1)}$	2.9$_{(1.0)}$	**3.1**$_{(0.8)}$	3.2$_{(1)}$	**3.0**$_{(0.6)}$	3.2$_{(0.5)}$	15/15
E4	1.8$_{(2)}$	3.0$_{(1)}$	3.1$_{(1)}$	3.2$_{(0.6)}$	3.2$_{(0.9)}$	3.4$_{(0.5)}$	3.5$_{(0.5)}$	15/15
F6	2.2$_{(1)}$	3.3$_{(2)}$	3.1$_{(1)}$	3.7$_{(1)}$	3.7$_{(0.5)}$	3.6$_{(0.6)}$	3.7$_{(0.7)}$	15/15
cmaes	**1.8**$_{(2)}$	**2.7**$_{(1)}$	**2.8**$_{(0.8)}$	3.2$_{(0.8)}$	**3.1**$_{(0.7)}$	3.0$_{(0.3)}$	**3.1**$_{(0.2)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	11	65	342	464	482	482	535	15/15
D3	2.1$_{(1)}$	1.6$_{(3)}$	1.0$_{(1)}$	1.4$_{(0.9)}$	1.9$_{(1)}$	1.9$_{(2)}$	1.9$_{(1)}$	15/15
E4	**1.8**$_{(2)}$	**1.1**$_{(0.8)}$	**0.89**$_{(1)}$	**1.1**$_{(2)}$	**1.3**$_{(1)}$	**1.3**$_{(2)}$	**1.4**$_{(3)}$	15/15
F6	2.4$_{(2)}$	2.6$_{(4)}$	1.5$_{(0.7)}$	1.6$_{(2)}$	1.9$_{(2)}$	1.9$_{(2)}$	1.9$_{(3)}$	15/15
cmaes	2.6$_{(3)}$	4.3$_{(9)}$	1.8$_{(2)}$	1.8$_{(2)}$	3.2$_{(4)}$	3.2$_{(2)}$	2.9$_{(3)}$	14/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	27	45	152	179	188	198	208	15/15
D3	**2.1**$_{(1)}$	6.6$_{(7)}$	3.9$_{(2)}$	4.1$_{(3)}$	4.3$_{(1)}$	4.7$_{(2)}$	5.0$_{(1)}$	15/15
E4	2.5$_{(3)}$	**5.3**$_{(3)}$	3.4$_{(0.9)}$	3.6$_{(1)}$	3.9$_{(1)}$	4.4$_{(0.9)}$	4.7$_{(1.0)}$	15/15
F6	3.0$_{(2)}$	5.8$_{(4)}$	**3.2**$_{(0.6)}$	**3.5**$_{(2)}$	**3.7**$_{(2)}$	**4.1**$_{(1)}$	**4.5**$_{(1)}$	15/15
cmaes	2.2$_{(2)}$	7.7$_{(5)}$	4.4$_{(3)}$	4.8$_{(4)}$	4.9$_{(4)}$	5.3$_{(1)}$	5.5$_{(1.0)}$	15/15

Figure A.71: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	21	65	127	149	159	169	178	15/15
D3	0.84$_{(0.9)}$	**2.8**$_{(2)}$	3.4$_{(2)}$	4.0$_{(2)}$	4.3$_{(2)}$	4.8$_{(2)}$	5.2$_{(2)}$	15/15
E4	0.86$_{(1)}$	3.1$_{(3)}$	**3.4**$_{(2)}$	**3.8**$_{(1)}$	**4.1**$_{(0.9)}$	**4.6**$_{(0.8)}$	**5.1**$_{(0.7)}$	15/15
F6	0.95$_{(2)}$	3.9$_{(2)}$	4.1$_{(2)}$	4.5$_{(2)}$	4.7$_{(1)}$	5.2$_{(1)}$	5.5$_{(1)}$	15/15
cmaes	**0.76**$_{(0.5)}$	3.7$_{(3)}$	4.5$_{(4)}$	5.0$_{(2)}$	5.2$_{(1)}$	5.6$_{(2)}$	5.9$_{(1)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	114	152	168	180	194	218	242	15/15
D3	4.4$_{(2)}$	**4.4**$_{(2)}$	4.7$_{(0.6)}$	4.8$_{(0.9)}$	4.7$_{(0.8)}$	4.6$_{(0.6)}$	4.6$_{(0.7)}$	15/15
E4	5.2$_{(2)}$	4.8$_{(0.5)}$	4.7$_{(0.5)}$	4.7$_{(0.6)}$	4.7$_{(0.4)}$	4.6$_{(0.6)}$	4.6$_{(0.6)}$	15/15
F6	5.0$_{(0.9)}$	4.5$_{(0.9)}$	**4.5**$_{(0.7)}$	**4.6**$_{(0.8)}$	**4.5**$_{(0.5)}$	**4.5**$_{(0.9)}$	4.6$_{(0.3)}$	15/15
cmaes	**3.9**$_{(2)}$	4.6$_{(1)}$	4.8$_{(1)}$	4.7$_{(0.5)}$	4.7$_{(0.8)}$	4.6$_{(0.7)}$	**4.5**$_{(0.8)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	67	105	227	263	277	302	327	15/15
D3	6.6$_{(4)}$	7.2$_{(3)}$	3.7$_{(0.5)}$	3.5$_{(0.3)}$	3.5$_{(0.4)}$	3.5$_{(0.3)}$	3.6$_{(0.3)}$	15/15
E4	6.4$_{(4)}$	7.0$_{(2)}$	3.7$_{(0.5)}$	3.4$_{(0.3)}$	3.4$_{(0.3)}$	3.5$_{(0.4)}$	3.6$_{(0.3)}$	15/15
F6	7.1$_{(3)}$	6.5$_{(2)}$	3.4$_{(0.4)}$	**3.3**$_{(0.6)}$	3.3$_{(0.3)}$	3.4$_{(0.4)}$	3.5$_{(0.3)}$	15/15
cmaes	**5.5**$_{(4)}$	**6.4**$_{(2)}$	**3.4**$_{(0.6)}$	3.3$_{(0.7)}$	**3.3**$_{(0.6)}$	**3.4**$_{(0.3)}$	**3.3**$_{(0.5)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	65	168	338	401	445	696	790	15/15
D3	8.4$_{(7)}$	**6.4**$_{(5)}$	**4.6**$_{(3)}$	**4.6**$_{(4)}$	**4.7**$_{(4)}$	**3.7**$_{(3)}$	**3.8**$_{(3)}$	15/15
E4	8.9$_{(7)}$	7.6$_{(8)}$	5.4$_{(4)}$	5.1$_{(4)}$	5.2$_{(5)}$	4.1$_{(3)}$	4.1$_{(4)}$	15/15
F6	11$_{(12)}$	8.8$_{(7)}$	6.2$_{(4)}$	6.0$_{(5)}$	5.9$_{(4)}$	4.5$_{(3)}$	4.5$_{(5)}$	15/15
cmaes	**7.1**$_{(5)}$	7.7$_{(7)}$	6.1$_{(4)}$	6.2$_{(5)}$	6.1$_{(6)}$	4.8$_{(4)}$	4.8$_{(5)}$	15/15

Figure A.72: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	49	85	108	136	215	281	365	15/15
D3	3.6(1)	5.4(3)	**5.7**(3)	**5.6**(2)	**4.2**(0.7)	4.4(0.6)	4.5(0.6)	15/15
E4	4.7(2)	5.4(3)	5.8(3)	5.7(2)	4.7(1)	4.5(0.8)	4.3(0.4)	15/15
F6	**3.6**(1)	6.1(2)	6.1(3)	5.7(1)	4.3(0.9)	**4.3**(1)	**4.3**(0.3)	15/15
cmaes	4.4(4)	**5.0**(2)	6.3(2)	6.4(1)	4.6(0.9)	4.8(0.7)	4.5(0.7)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	2.2	17	28	43	71	110	194	15/15
D3	2.6(4)	2.4(0.7)	3.7(1)	4.0(0.7)	**4.1**(2)	6.2(0.7)	5.5(0.5)	15/15
E4	**2.2**(5)	**1.9**(2)	3.2(2)	4.3(1)	4.4(0.5)	**6.0**(0.9)	5.2(0.5)	15/15
F6	2.8(5)	2.1(2)	3.9(2)	4.6(2)	4.5(1)	6.1(1)	**5.2**(1.0)	15/15
cmaes	2.8(5)	2.1(1)	**3.1**(1)	**3.8**(1.0)	4.2(1)	6.3(1)	5.4(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	121	1372	6285	8282	8429	8787	9041	15/15
D3	19(124)	4.2(7)	4.2(4)	3.2(2)	3.2(5)	3.0(3)	3.0(6)	8/15
E4	**1.1**(0.6)	5.3(4)	**4.0**(4)	**3.1**(2)	**3.0**(5)	**2.9**(3)	**2.8**(1)	9/15
F6	1.4(0.6)	**2.9**(2)	6.0(6)	4.6(2)	4.5(4)	4.3(5)	4.2(3)	6/15
cmaes	21(3)	3.8(5)	4.7(3)	3.6(5)	3.5(7)	3.4(3)	3.3(6)	7/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	41	319	582	789	1864	3204	3361	15/15
D3	12(0.6)	4.0(9)	**2.7**(1)	**2.2**(1)	**1.0**(0.4)	**0.63**(2)	**0.63**(0.2)	15/15
E4	0.86(2)	5.5(28)	3.8(8)	3.1(2)	1.5(0.4)	1.1(0.6)	1.1(2)	15/15
F6	1.5(3)	3.8(6)	3.6(7)	3.6(4)	1.5(2)	0.97(2)	0.96(1)	15/15
cmaes	**0.74**(0.7)	**3.5**(8)	3.1(3)	5.3(3)	3.3(3)	2.0(2)	1.9(3)	14/15

Figure A.73: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	3.6	78	282	491	1134	2347	3469	15/15
D3	61(6)	33(106)	**10**(4)	11(17)	**5.3**(13)	**7.9**(14)	**11**(13)	7/15
E4	23(81)	36(106)	20(30)	25(63)	11(13)	27(35)	57(26)	2/15
F6	37(132)	**14**(68)	12(3)	13(11)	8.3(9)	10(13)	25(17)	4/15
cmaes	**4.8**(16)	36(192)	15(41)	**8.8**(24)	5.6(0.9)	15(7)	20(11)	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	40	145	1289	3084	3523	4738	5527	15/15
D3	21(2)	10(8)	29(47)	28(98)	25(36)	26(31)	22(27)	3/15
E4	14(3)	17(1)	**9.0**(41)	**6.8**(15)	**6.0**(13)	**10**(11)	**11**(21)	5/15
F6	**1.9**(2)	**4.3**(1)	21(30)	15(30)	18(23)	41(60)	36(98)	2/15
cmaes	20(68)	25(104)	17(21)	8.1(8)	16(19)	45(47)	∞ *3e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	109	6764	7367	7399	7441	15/15
D3	2.2(2)	89(47)	112(129)	6.4(6)	8.1(8)	10(12)	10(15)	5/15
E4	2.1(2)	78(128)	128(89)	65(63)	59(79)	59(50)	59(51)	1/15
F6	**1.7**(0.8)	57(44)	**57**(29)	8.6(9)	10(8)	11(10)	10(11)	5/15
cmaes	1.9(1)	**30**(12)	84(163)	**4.9**(5)	**6.8**(8)	**6.8**(5)	**6.8**(10)	6/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	8.3	385	2291	2398	2481	2573	2776	15/15
D3	1.7(1)	5.6(10)	111(94)	106(90)	103(84)	99(102)	92(117)	1/15
E4	1.4(1)	**4.4**(6)	36(44)	35(29)	34(32)	32(44)	30(40)	3/15
F6	**1.3**(1)	5.2(5)	56(96)	53(31)	51(63)	50(67)	46(78)	2/15
cmaes	1.8(1)	11(14)	**34**(80)	**33**(37)	**32**(59)	**31**(28)	**28**(30)	3/15

Figure A.74: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	5.9	184	425	439	458	469	482	15/15
D3	364(2)	51(56)	59(39)	57(35)	55(49)	53(112)	52(59)	7/15
E4	13(81)	**36**(65)	**29**(69)	**28**(18)	**27**(23)	**27**(41)	**26**(34)	10/15
F6	**3.0**(5)	44(77)	44(34)	43(56)	41(41)	40(87)	39(37)	8/15
cmaes	13(3)	99(102)	91(177)	88(83)	85(132)	83(117)	81(64)	5/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	18	170	354	362	384	401	414	15/15
D3	**77**(205)	**24**(48)	**14**(1)	**14**(13)	**13**(29)	**12**(17)	**12**(19)	13/15
E4	190(809)	36(69)	23(53)	23(53)	22(49)	21(47)	21(19)	12/15
F6	122(389)	38(48)	27(35)	27(53)	26(41)	25(32)	24(30)	11/15
cmaes	213(56)	47(69)	34(27)	33(38)	31(80)	30(59)	29(41)	10/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	2.6	407	906	1215	2214	2293	2393	15/15
D3	4.8(3)	46(40)	70(25)	53(74)	29(34)	28(33)	27(19)	5/15
E4	6.0(5)	37(43)	469(604)	350(309)	192(237)	185(92)	178(169)	1/15
F6	3.6(2)	50(63)	485(397)	362(488)	199(149)	192(235)	184(226)	1/15
cmaes	**3.2**(2)	**8.0**(9)	**16**(16)	**12**(7)	**6.6**(5)	**6.5**(5)	**6.3**(3)	14/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	97	10391	1.0e5	3.6e5	3.6e5	3.6e5	3.6e5	2/15
D3	6.0(8)	1.5(2)	∞	∞	∞	∞	∞ *3e4*	0/15
E4	1.9(1)	3.5(3)	∞	∞	∞	∞	∞ *3e4*	0/15
F6	**1.4**(0.6)	1.7(3)	∞	∞	∞	∞	∞ *3e4*	0/15
cmaes	3.2(7)	**1.2**(1)	**3.8**(4)	∞	∞	∞	∞ *3e4*	0/15

Figure A.75: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 3-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	11	12	12	12	12	12	12	15/15
D3	3.3$_{(3)}$	8.9$_{(5)}$	17$_{(4)}$	25$_{(5)}$	33$_{(6)}$	48$_{(5)}$	64$_{(8)}$	15/15
E4	3.3$_{(1)}$	8.8$_{(4)}$	21$_{(5)}$	29$_{(5)}$	40$_{(3)}$	58$_{(7)}$	78$_{(11)}$	15/15
F6	3.5$_{(2)}$	10$_{(5)}$	17$_{(2)}$	25$_{(5)}$	34$_{(4)}$	50$_{(5)}$	65$_{(8)}$	15/15
cmaes	**3.1**$_{(2)}$	**8.3**$_{(3)}$	**16**$_{(4)}$	**22**$_{(3)}$	**28**$_{(3)}$	**42**$_{(5)}$	**56**$_{(5)}^{\star}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	83	87	88	89	90	92	94	15/15
D3	**12**$_{(3)}$	**15**$_{(2)}$	**16**$_{(3)}$	**17**$_{(1)}$	**19**$_{(2)}$	**20**$_{(2)}$	**22**$_{(1)}$	15/15
E4	17$_{(3)}$	19$_{(2)}$	20$_{(2)}$	21$_{(1)}$	22$_{(2)}$	24$_{(3)}$	26$_{(2)}$	15/15
F6	14$_{(5)}$	16$_{(0.5)}$	17$_{(3)}$	18$_{(2)}$	19$_{(2)}$	21$_{(3)}$	23$_{(3)}$	15/15
cmaes	12$_{(5)}$	15$_{(3)}$	18$_{(1)}$	19$_{(1)}$	20$_{(1)}$	21$_{(2)}$	22$_{(1)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	716	1622	1637	1642	1646	1650	1654	15/15
D3	3.0$_{(2)}$	**36**$_{(26)}$	∞	∞	∞	∞	∞ *4e4*	0/15
E4	2.3$_{(2)}$	59$_{(100)}$	**210**$_{(265)}$	**209**$_{(259)}$	**209**$_{(99)}$	**208**$_{(159)}$	**208**$_{(199)}$	2/15
F6	2.9$_{(2)}$	96$_{(80)}$	430$_{(736)}$	429$_{(386)}$	428$_{(381)}$	427$_{(304)}$	426$_{(372)}$	1/15
cmaes	**2.0**$_{(3)}$	48$_{(42)}$	∞	∞	∞	∞	∞ *3e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	809	1633	1688	1758	1817	1886	1903	15/15
D3	2.8$_{(5)}$	362$_{(820)}$	∞	∞	∞	∞	∞ *4e4*	0/15
E4	**2.5**$_{(2)}$	203$_{(317)}$	∞	∞	∞	∞	∞ *5e4*	0/15
F6	3.4$_{(3)}$	**113**$_{(77)}$	∞	∞	∞	∞	∞ *4e4*	0/15
cmaes	2.6$_{(1)}$	∞	∞	∞	∞	∞	∞ *3e4*	0/15

Figure A.76: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	10	10	10	10	10	10	10	15/15
D3	**6.9**(2)	**8.6**(2)	**8.9**(2)	**8.9**(2)	**8.9**(1)	**8.9**(2)	**8.9**(2)	15/15
E4	7.1(2)	9.1(1)	9.3(2)	9.3(2)	9.3(3)	9.3(2)	9.3(2)	15/15
F6	**6.9**(2)	9.0(1)	9.3(1)	9.3(2)	9.3(2)	9.3(1)	9.3(2)	15/15
cmaes	8.1(2)	10(1)	10(1)	11(1)	11(0.6)	11(0.5)	11(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	114	214	281	404	580	1038	1332	15/15
D3	**1.6**(0.7)	2.0(0.8)	2.4(0.6)	2.2(0.5)	1.9(0.3)	1.5(0.3)	1.5(0.3)	15/15
E4	3.3(2)	3.5(2)	3.8(2)	3.7(1)	3.0(0.5)	2.4(0.5)	2.5(0.4)	15/15
F6	2.1(1)	2.5(0.8)	2.9(0.8)	2.7(0.5)	2.4(0.5)	1.9(0.1)	1.8(0.3)	15/15
cmaes	1.6(0.8)	**1.7**(0.4)	**1.9**(0.5)	**1.8**(0.3)	**1.6**(0.2)	**1.2**(0.2)*	**1.2**(0.2)$^{\star 2}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	24	324	1171	1451	1572	1572	1597	15/15
D3	**2.4**(3)	1.7(2)	2.5(3)	2.2(3)	2.0(3)	2.0(4)	2.4(1)	14/15
E4	2.7(2)	1.9(1)	**0.90**(0.4)	**0.98**(1.0)	**1.0**(0.9)	**1.0**(1)	**1.2**(0.9)	15/15
F6	2.6(2)	**1.1**(0.8)	1.4(0.6)	1.3(2)	1.4(1)	1.4(2)	1.5(0.8)	15/15
cmaes	2.9(2)	2.2(0.2)	3.3(2)	4.8(5)	5.9(5)	5.9(8)	5.8(3)	11/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	73	273	336	372	391	410	422	15/15
D3	3.6(2)	4.6(6)	5.2(5)	5.5(4)	5.6(2)	5.8(1)	6.1(2)	15/15
E4	3.9(1)	3.9(3)	4.6(0.6)	4.8(2)	5.0(0.8)	5.4(0.5)	5.8(0.7)	15/15
F6	3.1(1)	3.8(1)	4.6(3)	4.8(3)	4.9(3)	5.3(2)	5.7(2)	15/15
cmaes	**2.4**(0.3)	**3.0**(1)	**4.1**(1)	**4.3**(0.9)	**4.5**(0.8)	**4.8**(0.8)	**5.0**(0.8)	15/15

Figure A.77: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	35	127	214	263	300	335	369	15/15
D3	2.9$_{(2)}$	7.7$_{(2)}$	6.8$_{(6)}$	6.5$_{(3)}$	6.2$_{(5)}$	6.2$_{(0.7)}$	6.1$_{(3)}$	15/15
E4	3.4$_{(1)}$	8.1$_{(6)}$	7.0$_{(6)}$	6.7$_{(1)}$	6.4$_{(1)}$	6.5$_{(4)}$	6.6$_{(3)}$	15/15
F6	2.7$_{(1)}$	**6.8**$_{(6)}$	**6.4**$_{(4)}$	**6.1**$_{(0.9)}$	**5.9**$_{(3)}$	**6.0**$_{(5)}$	6.0$_{(4)}$	15/15
cmaes	**2.6**$_{(0.8)}$	7.3$_{(2)}$	6.6$_{(3)}$	6.3$_{(2)}$	6.0$_{(2)}$	6.0$_{(2)}$	**5.9**$_{(0.7)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	349	500	574	607	626	829	880	15/15
D3	3.3$_{(0.9)}$	2.8$_{(0.9)}$	2.8$_{(0.2)}$	2.8$_{(0.2)}$	2.9$_{(0.3)}$	2.4$_{(0.3)}$	2.5$_{(0.2)}$	15/15
E4	4.3$_{(1)}$	3.3$_{(0.4)}$	3.1$_{(0.4)}$	3.1$_{(0.5)}$	3.2$_{(0.6)}$	2.7$_{(0.3)}$	2.8$_{(0.2)}$	15/15
F6	3.3$_{(0.7)}$	**2.8**$_{(0.4)}$	**2.7**$_{(0.3)}$	**2.7**$_{(0.2)}$	**2.8**$_{(0.2)}$	**2.4**$_{(0.2)}$	**2.4**$_{(0.3)}$	15/15
cmaes	**3.2**$_{(1)}$	2.9$_{(0.6)}$	2.9$_{(0.3)}$	2.9$_{(0.2)}$	3.0$_{(0.2)}$	2.4$_{(0.2)}$	2.5$_{(0.1)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	143	202	763	977	1177	1467	1673	15/15
D3	10$_{(3)}$	7.9$_{(1)}$	2.3$_{(0.2)}$	1.8$_{(0.2)}$	1.6$_{(0.1)}$	1.4$_{(0.1)}$	1.4$_{(0.1)}$	15/15
E4	14$_{(2)}$	11$_{(2)}$	3.0$_{(0.4)}$	2.5$_{(0.3)}$	2.1$_{(0.1)}$	1.9$_{(0.1)}$	1.8$_{(0.1)}$	15/15
F6	12$_{(3)}$	9.4$_{(1)}$	2.6$_{(0.5)}$	2.2$_{(0.3)}$	1.9$_{(0.3)}$	1.7$_{(0.2)}$	1.6$_{(0.2)}$	15/15
cmaes	**6.5**$_{(4)}^{\star}$	**6.8**$_{(4)}$	**2.1**$_{(0.5)}$	**1.8**$_{(0.2)}$	**1.6**$_{(0.2)}$	**1.4**$_{(0.1)}$	**1.3**$_{(0.1)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	108	268	371	413	461	1303	1494	15/15
D3	8.2$_{(4)}$	7.5$_{(7)}$	8.1$_{(7)}$	8.6$_{(5)}$	8.7$_{(4)}$	3.9$_{(3)}$	3.9$_{(2)}$	15/15
E4	8.4$_{(3)}$	**6.4**$_{(4)}$	**6.8**$_{(4)}$	7.3$_{(4)}$	7.5$_{(2)}$	3.4$_{(2)}$	3.4$_{(2)}$	15/15
F6	10$_{(8)}$	7.4$_{(3)}$	6.9$_{(5)}$	**7.2**$_{(8)}$	**7.2**$_{(5)}$	**3.2**$_{(2)}$	**3.1**$_{(2)}$	15/15
cmaes	**8.0**$_{(3)}$	6.8$_{(6)}$	7.6$_{(6)}$	8.1$_{(7)}$	8.4$_{(6)}$	3.8$_{(2)}$	3.9$_{(2)}$	15/15

Figure A.78: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	132	195	250	319	1310	1752	2255	15/15
D3	3.6(1.0)	4.8(2)	**5.0**(1)	**4.7**(1)	**1.4**(0.3)	**1.5**(0.1)	**1.4**(0.2)	15/15
E4	3.6(1)	**4.5**(0.9)	5.0(1)	5.0(1)	1.5(0.3)	1.5(0.4)	1.5(0.2)	15/15
F6	3.4(1.0)	4.7(2)	5.6(1)	5.3(1)	1.6(0.3)	1.5(0.2)	1.5(0.2)	15/15
cmaes	**3.4**(2)	4.9(2)	5.3(2)	5.5(2)	1.6(0.5)	2.7(0.3)	2.3(6)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	10	41	58	90	139	251	476	15/15
D3	1.4(2)	2.3(1)	3.8(1)	4.0(0.9)	4.5(0.5)	5.1(0.4)	4.4(0.3)	15/15
E4	1.4(2)	**1.8**(2)	3.9(1)	4.9(1)	5.0(1)	5.5(0.7)	4.4(0.4)	15/15
F6	1.5(2)	1.9(1)	3.7(1)	4.3(1)	4.8(0.7)	5.6(0.5)	4.3(0.6)	15/15
cmaes	**1.3**(1)	2.1(2)	**3.5**(1)	**3.6**(0.6)	**4.4**(0.7)	**5.0**(0.9)	**4.2**(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	511	9310	19369	19743	20073	20769	21359	14/15
D3	4.6(7)	15(14)	33(27)	33(31)	32(21)	31(20)	30(20)	1/15
E4	4.8(3)	14(20)	∞	∞	∞	∞	∞ *5e4*	0/15
F6	**2.8**(3)	8.2(7)	**17**(6)	**16**(9)	**16**(14)	**16**(26)	**15**(15)	2/15
cmaes	3.0(5)	**5.0**(3)	26(28)	26(35)	25(26)	24(23)	24(39)	1/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	120	612	2662	10163	10449	11644	12095	15/15
D3	1.5(2)	**2.2**(3)	**1.5**(2)	**0.79**(0.6)	**0.78**(1)	**0.78**(0.5)	**1.0**(0.9)	14/15
E4	2.5(7)	33(31)	14(35)	6.8(19)	6.6(6)	6.0(9)	5.8(10)	7/15
F6	1.4(2)	4.7(5)	4.2(13)	1.7(2)	1.8(2)	1.7(3)	1.7(2)	13/15
cmaes	**1.0**(1)	16(18)	12(10)	3.9(3)	4.4(4)	6.2(16)	6.0(11)	7/15

Figure A.79: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	5.2	215	899	2861	3669	6351	7934	15/15
D3	**2.5**(5)	8.3(29)	7.3(15)	3.5(6)	5.8(8)	111(148)	∞ *5e4*	0/15
E4	3.3(6)	**1.2**(0.8)	0.71(0.1)	**0.66**(0.1)	**1.6**(3)	**9.3**(7)	**43**(92)	2/15
F6	3.2(4)	1.2(0.5)	**0.67**(0.2)	1.2(3)	2.6(3)	35(42)	∞ *5e4*	0/15
cmaes	3.1(5)	16(54)	25(27)	8.0(9)	10(14)	110(93)	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	103	378	3968	8451	9280	10905	12469	15/15
D3	1.2(0.5)	5.8(22)	1.8(2)	2.8(4)	11(11)	∞	∞ *5e4*	0/15
E4	1.3(0.9)	4.2(21)	**1.0**(4)	**1.1**(0.9)	**2.2**(2)	**32**(43)	∞ *5e4*	0/15
F6	**1.1**(0.6)	**1.6**(0.8)	3.2(0.7)	1.8(6)	3.3(3)	64(48)	∞ *5e4*	0/15
cmaes	1.3(0.5)	21(4)	17(23)	18(21)	36(44)	∞	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	242	1.0e5	1.2e5	1.2e5	1.2e5	15/15
D3	2.3(0.5)	2966(3775)	∞	∞	∞	∞	∞ *5e4*	0/15
E4	**2.2**(2)	4568(1e4)	∞	∞	∞	∞	∞ *5e4*	0/15
F6	2.4(1)	3006(6692)	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes	2.8(4)	**1280**(1879)	**260**(442)$^{\star 2}$	∞	∞	∞	∞ *5e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	16	851	38111	51362	54470	54861	55313	14/15
D3	**1.2**(0.6)	11(7)	14(13)	10(11)	10(15)	9.5(8)	**9.4**(11)	1/15
E4	1.3(0.7)	10(8)	**9.5**(10)	**7.0**(7)	**6.6**(5)	**6.6**(6)	13(17)	1/15
F6	1.4(0.8)	**5.6**(4)	18(34)	14(13)	13(12)	13(17)	13(10)	1/15
cmaes	1.3(0.6)	21(25)	∞	∞	∞	∞	∞ *3e4*	0/15

Figure A.80: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	41	1157	1674	1692	1705	1729	1757	14/15
D3	17(114)	69(103)	67(88)	66(38)	66(144)	65(161)	64(75)	3/15
E4	8.4(1)	31(35)	28(72)	28(49)	27(22)	27(19)	27(28)	6/15
F6	7.0(20)	**26**(40)	**22**(18)	**22**(36)	**22**(20)	**22**(19)	**21**(36)	7/15
cmaes	**3.2**(9)	62(54)	43(62)	43(47)	42(80)	42(42)	41(36)	4/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	71	386	938	980	1008	1040	1068	14/15
D3	**1.2**(1)	**32**(104)	**28**(44)	**27**(38)	**26**(20)	**25**(73)	**25**(42)	8/15
E4	22(6)	57(71)	41(45)	39(35)	38(27)	37(60)	36(19)	7/15
F6	17(17)	55(92)	38(40)	37(87)	36(48)	35(49)	34(49)	7/15
cmaes	109(177)	95(154)	51(47)	49(95)	48(86)	46(63)	45(54)	6/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	3.0	518	14249	27890	31654	33030	34256	15/15
D3	2.0(2)	109(190)	∞	∞	∞	∞	∞ *5e4*	0/15
E4	2.1(4)	702(1330)	∞	∞	∞	∞	∞ *5e4*	0/15
F6	2.0(0.6)	100(112)	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes	**1.9**(1)	**7.0**(13)^{*3}	**3.6**(3)^{*3}	**4.5**(10)^{*3}	**4.0**(5)^{*3}	**3.8**(5)^{*3}	**3.7**(5)^{*3}	5/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	1622	2.2e5	6.4e6	9.6e6	9.6e6	1.3e7	1.3e7	3/15
D3	14(11)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
E4	13(11)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
F6	10(2)	∞	∞	∞	∞	∞	∞ *5e4*	0/15
cmaes	**4.0**(1)	**1.1**(1.0)	∞	∞	∞	∞	∞ *5e4*	0/15

Figure A.81: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	22	23	23	23	23	23	23	15/15
D3	$4.8_{(2)}$	$14_{(3)}$	$23_{(3)}$	$31_{(3)}$	$41_{(4)}$	$59_{(3)}$	$77_{(5)}$	15/15
E4	$4.5_{(3)}$	$17_{(5)}$	$28_{(7)}$	$40_{(2)}$	$52_{(8)}$	$76_{(12)}$	$100_{(16)}$	15/15
F6	$4.2_{(1)}$	$16_{(4)}$	$24_{(3)}$	$35_{(4)}$	$43_{(3)}$	$65_{(6)}$	$85_{(6)}$	15/15
cmaes	$\mathbf{4.0}_{(1)}$	$\mathbf{11}_{(3)}$	$\mathbf{17}_{(2)}^{\star 3}$	$\mathbf{25}_{(4)}^{\star 3}$	$\mathbf{32}_{(4)}^{\star 4}$	$\mathbf{44}_{(4)}^{\star 4}$	$\mathbf{58}_{(3)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	187	190	191	191	193	194	195	15/15
D3	$23_{(4)}$	$25_{(3)}$	$27_{(3)}$	$28_{(2)}$	$29_{(3)}$	$31_{(3)}$	$33_{(3)}$	15/15
E4	$29_{(3)}$	$30_{(3)}$	$31_{(3)}$	$33_{(3)}$	$34_{(2)}$	$37_{(2)}$	$39_{(3)}$	15/15
F6	$24_{(4)}$	$25_{(3)}$	$26_{(3)}$	$\mathbf{28}_{(3)}$	$29_{(3)}$	$31_{(2)}$	$33_{(1)}$	15/15
cmaes	$\mathbf{19}_{(4)}^{\star}$	$\mathbf{24}_{(5)}$	$\mathbf{26}_{(4)}$	$28_{(2)}$	$\mathbf{29}_{(2)}$	$\mathbf{30}_{(2)}$	$\mathbf{31}_{(2)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	1739	3600	3609	3636	3642	3646	3651	15/15
D3	$50_{(62)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	$412_{(446)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	$\mathbf{14}_{(34)}$	∞	∞	∞	∞	∞	∞ *6e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	2234	3626	3660	3695	3707	3744	28767	12/15
D3	$\mathbf{30}_{(36)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
E4	$199_{(116)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	$42_{(73)}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	$42_{(50)}$	∞	∞	∞	∞	∞	∞ *7e4*	0/15

Figure A.82: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	20	20	20	20	20	20	20	15/15
D3	**7.1**(2)	**8.3**(2)	8.7(1)	**8.7**(2)	**8.7**(2)	**8.7**(2)	**8.7**(2)	15/15
E4	7.2(0.8)	8.4(1)	**8.7**(1)	8.7(0.7)	8.7(0.8)	8.7(1)	8.7(1)	15/15
F6	7.2(0.7)	8.5(2)	8.8(1)	8.8(2)	8.8(2)	8.8(1)	8.8(2)	15/15
cmaes	7.9(0.9)	9.3(1)	10(1)	10(0.9)	10(1)	10(1)	10(1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	412	623	826	1039	1292	1841	2370	15/15
D3	4.5(3)	4.6(2)	5.1(2)	5.2(3)	5.1(1)	4.7(1)	4.8(1.0)	15/15
E4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	1030(1577)	2256(4176)	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	**1.6**(0.5)$^{\star 3}$	**1.7**(0.3)$^{\star 3}$	**1.7**(0.4)$^{\star 4}$	**1.8**(0.3)$^{\star 4}$	**1.8**(0.3)$^{\star 4}$	**1.7**(0.2)$^{\star 4}$	**1.6**(0.2)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	172	1611	4195	5099	5141	5141	5389	15/15
D3	2.2(1)	**0.89**(0.1)	2.9(4)	2.7(1)	2.7(3)	2.7(2)	2.6(3)	14/15
E4	3.8(2)	0.96(0.5)	**0.61**(0.2)	**0.67**(0.4)	**0.71**(0.4)	**0.71**(0.3)	**0.81**(0.5)	15/15
F6	2.5(0.6)	0.99(0.6)	1.4(1)	1.4(2)	1.6(2)	1.6(2)	1.8(2)	15/15
cmaes	**1.5**(0.9)	2.9(2)	12(13)	∞	∞	∞	∞ *2e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	326	921	1114	1217	1267	1315	1343	15/15
D3	3.2(1)	5.4(3)	5.4(3)	5.4(3)	5.5(2)	5.7(0.5)	5.9(3)	15/15
E4	4.5(2)	5.0(0.5)	5.1(0.5)	5.2(1.0)	5.3(0.6)	5.6(0.6)	5.9(0.7)	15/15
F6	3.4(0.6)	5.0(2)	5.1(0.6)	5.1(3)	5.2(2)	5.4(3)	5.7(0.4)	15/15
cmaes	**2.6**(2)	**4.9**(2)	**5.0**(4)	**4.9**(1)	**4.9**(3)	**5.0**(3)	**5.2**(2)	15/15

Figure A.83: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	200	648	857	993	1065	1138	1185	15/15
D3	2.6(0.6)	6.5(5)	6.2(2)	5.8(4)	5.8(2)	5.8(2)	6.0(1)	15/15
E4	3.8(1)	6.5(6)	6.2(2)	6.0(0.7)	5.9(0.6)	6.1(0.5)	6.4(0.3)	15/15
F6	3.1(0.7)	5.6(1)	5.4(0.6)	5.3(0.4)	5.3(0.4)	5.3(0.5)	5.6(0.4)	15/15
cmaes	**1.9**(0.4)*	**5.4**(1)	**5.3**(0.8)	**5.0**(0.7)	**4.9**(0.7)	**5.0**(0.6)	**5.1**(0.5)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	1835	2172	2455	2728	2802	4543	4739	15/15
D3	2.2(0.5)	2.1(0.4)	**2.0**(0.3)	**1.9**(0.2)	1.9(0.3)	1.3(0.1)	1.3(0.2)	15/15
E4	2.9(0.3)	2.6(0.3)	2.4(0.2)	2.2(0.1)	2.3(0.2)	1.5(0.2)	1.6(0.2)	15/15
F6	2.6(0.2)	2.3(0.2)	2.2(0.2)	2.0(0.2)	2.1(0.2)	1.4(0.1)	1.4(0.1)	15/15
cmaes	**1.9**(0.4)	**2.1**(0.3)	2.0(0.1)	1.9(0.0)	**1.9**(0.1)	**1.3**(0.0)	**1.3**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	266	1041	2602	2954	3338	4092	4843	15/15
D3	23(1)	6.1(0.6)	2.5(0.2)	2.3(0.1)	2.1(0.2)	1.8(0.2)	1.6(0.1)	15/15
E4	40(3)	10(0.9)	4.3(0.3)	3.9(0.3)	3.5(0.2)	3.0(0.3)	2.6(0.2)	15/15
F6	31(2)	8.1(0.8)	3.3(0.1)	3.0(0.1)	2.8(0.2)	2.4(0.1)	2.1(0.0)	15/15
cmaes	**12**(4)$^{\star 4}$	**3.8**(0.3)$^{\star 4}$	**1.7**(0.1)$^{\star 4}$	**1.6**(0.1)$^{\star 4}$	**1.5**(0.1)$^{\star 4}$	**1.3**(0.1)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	515	896	1240	1390	1569	3660	5154	15/15
D3	3.2(0.3)	4.6(3)	5.1(2)	5.4(3)	5.5(1)	3.0(0.6)	2.5(1.0)	15/15
E4	5.4(4)	6.3(4)	6.5(4)	6.7(2)	6.8(3)	3.7(1)	3.0(0.7)	15/15
F6	3.7(2)	4.9(4)	5.6(4)	6.0(5)	6.1(2)	3.4(0.8)	2.8(2)	15/15
cmaes	**3.0**(2)	**3.6**(2)	**4.3**(2)	**4.6**(4)	**4.8**(2)	**2.7**(2)	**2.2**(0.6)	15/15

Figure A.84: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	387	596	797	1014	4587	6208	7779	15/15
D3	**2.6**(0.3)	**4.6**(3)	**5.2**(2)	**4.8**(2)	**1.2**(0.4)	1.3(0.2)	1.4(0.4)	15/15
E4	3.3(0.9)	4.8(1)	5.2(2)	4.9(1)	1.3(0.3)	1.3(0.3)	1.5(0.3)	15/15
F6	2.8(0.7)	4.7(2)	5.6(2)	5.1(2)	1.3(0.3)	**1.3**(0.4)	**1.3**(0.3)	15/15
cmaes	3.2(2)	5.3(2)	5.3(2)	5.6(1)	1.4(0.3)	1.6(0.3)	1.4(0.8)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	37	98	133	205	392	687	4305	15/15
D3	**1.2**(0.7)	2.8(1)	4.4(0.7)	5.4(0.9)	4.6(0.8)	5.6(0.4)	1.4(0.1)	15/15
E4	1.3(0.9)	3.8(1)	5.6(1)	7.0(2)	5.9(0.8)	6.5(0.4)	1.6(0.1)	15/15
F6	1.2(0.7)	3.0(0.5)	4.8(1)	5.5(1)	5.2(0.4)	5.9(0.5)	1.4(0.1)	15/15
cmaes	1.3(0.8)	**2.3**(0.6)	**3.3**(0.9)*	**3.7**(0.7)$^{\star 2}$	**3.7**(0.6)$^{\star 2}$	**5.0**(0.5)*	**1.3**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	4774	39246	73643	74669	75790	77814	79834	12/15
D3	68(175)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	**3.9**(2)$^{\star 3}$	∞	∞	∞	∞	∞	∞ *7e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	425	7029	15779	45669	51151	65798	71570	15/15
D3	146(118)	62(32)	27(27)	33(42)	∞	∞	∞ *1e5*	0/15
E4	1606(1155)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	374(487)	57(57)	89(97)	∞	∞	∞	∞ *1e5*	0/15
cmaes	**0.68**(0.3)	**0.12**(0.0)$^{\star 3}$	**2.1**(0.6)$^{\star 2}$	**2.5**(5)$^{\star 3}$	**2.8**(3)$^{\star 3}$	**19**(40)$^{\star 3}$	**18**(15)$^{\star 3}$	1/15

Figure A.85: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	26	429	2203	6329	9851	20190	26503	15/15
D3	**1.3**(1)	1.5(0.5)	**0.74**(0.2)	4.4(4)	5.6(5)	**34**(35)	∞ *1e5*	0/15
E4	1.6(1)	2.0(1)	0.97(0.3)	1.3(2)	**2.2**(3)	∞	∞ *1e5*	0/15
F6	1.8(2)	**1.5**(0.8)	0.76(0.3)	**1.2**(5)	3.1(6)	70(48)	∞ *1e5*	0/15
cmaes	1.4(0.9)	24(0.4)	15(28)	16(20)	66(96)	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	238	836	7012	15928	27536	37234	42708	15/15
D3	1.3(0.3)	**1.5**(0.3)	**0.34**(0.1)	2.9(0.4)	4.9(9)	∞	∞ *1e5*	0/15
E4	1.7(2)	2.2(0.5)	1.0(2)	**1.5**(2)	**2.6**(6)	∞	∞ *1e5*	0/15
F6	**1.2**(0.2)	1.7(0.7)	1.4(4)	4.4(3)	6.5(5)	**38**(72)	∞ *1e5*	0/15
cmaes	1.3(1)	38(60)	17(12)	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	10609	9.8e5	1.4e6	1.4e6	1.4e6	15/15
D3	2.3(1)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
E4	**2.2**(0.8)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	2.3(2)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	2.9(4)	**2.4e4**(9921)	∞⋆3	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	32	15426	5.5e5	5.7e5	5.7e5	5.8e5	5.9e5	15/15
D3	**1.6**(1)	**9.4**(9)	∞	∞	∞	∞	∞ *1e5*	0/15
E4	2.0(2)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	1.6(0.7)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	1.8(1)	∞	∞	∞	∞	∞	∞ *5e4*	0/15

Figure A.86: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	130	2236	4392	4487	4618	5074	11329	8/15
D3	207(610)	50(37)	99(111)	97(150)	94(94)	86(72)	39(62)	2/15
E4	**63**(248)	53(98)	**64**(43)	**62**(95)	61(73)	55(39)	25(16)	3/15
F6	133(385)	**46**(23)	64(106)	62(63)	**61**(46)	**55**(68)	**25**(50)	3/15
cmaes	66(199)	57(70)	87(132)	85(116)	82(100)	75(21)	34(55)	2/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	98	2839	6353	6620	6798	8296	10351	6/15
D3	174(0.6)	20(21)	9.3(8)	8.9(18)	8.7(21)	7.2(18)	5.8(10)	9/15
E4	446(1571)	29(41)	20(36)	19(41)	18(31)	15(13)	12(28)	6/15
F6	**149**(422)	20(12)	11(11)	11(10)	10(19)	8.5(11)	6.9(8)	8/15
cmaes	230(373)	**14**(10)	**8.9**(16)	**8.6**(12)	**8.4**(5)	**6.9**(12)	**5.6**(3)	8/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	2.8	915	16425	1.8e5	2.0e5	2.1e5	2.1e5	15/15
D3	**1.3**(0.9)	1582(1421)	∞	∞	∞	∞	∞ *1e5*	0/15
E4	1.9(1)	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	1.7(0.7)	1560(3142)	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	2.1(3)	**12**(6)$^{\star 4}$	**40**(62)$^{\star 4}$	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	98761	1.0e6	7.5e7	7.5e7	7.5e7	7.5e7	7.5e7	1/15
D3	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
cmaes	**0.70**(1)$^{\star 4}$	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Figure A.87: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 10-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	43	43	43	43	43	43	43	15/15
D3	10$_{(1.0)}$	19$_{(3)}$	29$_{(2)}$	39$_{(2)}$	51$_{(3)}$	71$_{(6)}$	91$_{(7)}$	15/15
E4	10$_{(3)}$	22$_{(5)}$	35$_{(5)}$	49$_{(5)}$	62$_{(3)}$	88$_{(6)}$	116$_{(8)}$	15/15
F6	8.0$_{(3)}$	20$_{(3)}$	31$_{(4)}$	41$_{(4)}$	53$_{(5)}$	76$_{(4)}$	99$_{(6)}$	15/15
cmaes	**6.4**$_{(1)}$	**13**$_{(0.9)}^{\star 4}$	**19**$_{(1)}^{\star 4}$	**25**$_{(1)}^{\star 4}$	**32**$_{(2)}^{\star 4}$	**44**$_{(3)}^{\star 4}$	**57**$_{(2)}^{\star 4}$	15/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	385	386	387	388	390	391	393	15/15
D3	44$_{(2)}$	45$_{(3)}$	47$_{(2)}$	48$_{(1)}$	49$_{(2)}$	52$_{(2)}$	54$_{(2)}$	15/15
E4	59$_{(3)}$	60$_{(2)}$	62$_{(1)}$	63$_{(3)}$	65$_{(2)}$	68$_{(2)}$	70$_{(1)}$	15/15
F6	49$_{(1)}$	50$_{(2)}$	52$_{(2)}$	53$_{(2)}$	54$_{(1)}$	56$_{(2)}$	59$_{(3)}$	15/15
cmaes	**32**$_{(7)}^{\star 3}$	**39**$_{(6)}^{\star 2}$	**44**$_{(3)}$	**46**$_{(3)}^{\star}$	**47**$_{(2)}^{\star 2}$	**48**$_{(1)}^{\star 3}$	**50**$_{(0.7)}^{\star 3}$	15/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	5066	7626	7635	7637	7643	7646	7651	15/15
D3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

$\Delta f_{\rm opt}$	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	4722	7628	7666	7686	7700	7758	1.4e5	9/15
D3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Figure A.88: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	41	41	41	41	41	41	41	15/15
D3	7.4$_{(1)}$	9.1$_{(1)}$	9.1$_{(1.0)}$	9.1$_{(0.8)}$	9.1$_{(1.0)}$	9.1$_{(1)}$	9.1$_{(1)}$	15/15
E4	7.6$_{(1.0)}$	8.9$_{(1)}$	9.0$_{(1)}$	9.0$_{(1)}$	9.0$_{(1)}$	9.0$_{(0.9)}$	9.0$_{(1)}$	15/15
F6	**7.1**$_{(1)}$	**8.4**$_{(2)}$	**8.5**$_{(1)}$	**8.5**$_{(2)}$	**8.5**$_{(2)}$	**8.5**$_{(2)}$	**8.5**$_{(1)}$	15/15
cmaes	8.1$_{(1)}$	9.1$_{(0.9)}$	9.4$_{(0.9)}$	9.5$_{(0.5)}$	9.5$_{(0.4)}$	9.5$_{(0.7)}$	9.5$_{(0.6)}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	1296	2343	3413	4255	5220	6728	8409	15/15
D3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *1e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	**1.4**$_{(0.3)}^{\star 4}$	**1.2**$_{(0.1)}^{\star 4}$	**1.1**$_{(0.1)}^{\star 4}$	**1.1**$_{(0.1)}^{\star 4}$	**1.1**$_{(0.1)}^{\star 4}$	**1.2**$_{(0.1)}^{\star 4}$	**1.2**$_{(0.1)}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	1351	4274	9503	16523	16524	16524	16969	15/15
D3	1.5$_{(0.5)}$	**1.7**$_{(1)}$	3.4$_{(5)}$	**3.0**$_{(2)}$	**3.0**$_{(2)}$	**3.0**$_{(2)}$	**3.1**$_{(1.0)}$	15/15
E4	2.5$_{(0.5)}$	3.1$_{(2)}$	**2.1**$_{(2)}$	15$_{(18)}$	23$_{(16)}$	23$_{(37)}$	38$_{(100)}$	4/15
F6	1.6$_{(0.5)}$	2.7$_{(4)}$	3.9$_{(4)}$	3.7$_{(3)}$	3.8$_{(3)}$	3.8$_{(0.8)}$	5.1$_{(4)}$	15/15
cmaes	**0.96**$_{(1)}^{\star 2}$	139$_{(124)}$	129$_{(113)}$	∞	∞	∞	∞ *4e4*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	2039	3871	4040	4148	4219	4371	4484	15/15
D3	4.5$_{(1)}$	5.8$_{(2)}$	6.1$_{(4)}$	6.2$_{(4)}$	6.3$_{(4)}$	6.4$_{(4)}$	6.5$_{(0.4)}$	15/15
E4	5.1$_{(1)}$	5.8$_{(3)}$	6.2$_{(3)}$	6.4$_{(4)}$	6.5$_{(2)}$	6.6$_{(2)}$	6.8$_{(3)}$	15/15
F6	4.9$_{(1)}$	5.0$_{(0.7)}$	5.4$_{(0.7)}$	5.5$_{(2)}$	5.6$_{(2)}$	5.7$_{(2)}$	5.8$_{(0.5)}$	15/15
cmaes	**3.6**$_{(1)}$	**4.3**$_{(0.7)}$	**4.6**$_{(0.3)}$	**4.7**$_{(2)}$	**4.7**$_{(2)}$	**4.8**$_{(0.6)}$	**4.8**$_{(3)}^{\star}$	15/15

Figure A.89: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	1716	3102	3277	3379	3455	3594	3727	15/15
D3	$4.7_{(2)}$	$6.0_{(4)}$	$6.4_{(2)}$	$6.6_{(2)}$	$6.7_{(2)}$	$6.7_{(0.4)}$	$6.8_{(4)}$	15/15
E4	$5.0_{(0.8)}$	$5.9_{(2)}$	$6.3_{(2)}$	$6.5_{(0.4)}$	$6.6_{(0.4)}$	$6.8_{(2)}$	$7.0_{(2)}$	15/15
F6	$5.0_{(1)}$	$5.8_{(2)}$	$6.2_{(2)}$	$6.3_{(2)}$	$6.4_{(2)}$	$6.5_{(2)}$	$6.6_{(0.4)}$	15/15
cmaes	$\mathbf{3.9}_{(0.4)}$	$\mathbf{5.1}_{(2)}$	$\mathbf{5.5}_{(0.2)}$	$\mathbf{5.6}_{(0.5)}$	$\mathbf{5.6}_{(3)}$	$\mathbf{5.6}_{(3)}$	$\mathbf{5.6}_{(2)}{}^{\star}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	7413	8661	10735	13641	14920	17073	17476	15/15
D3	$2.3_{(0.2)}$	$2.0_{(0.1)}$	$1.7_{(0.1)}$	$1.4_{(0.1)}$	$1.3_{(0.0)}$	$1.2_{(0.1)}$	$1.2_{(0.1)}$	15/15
E4	$3.1_{(0.2)}$	$2.7_{(0.1)}$	$2.2_{(0.1)}$	$1.8_{(0.1)}$	$1.7_{(0.0)}$	$1.5_{(0.1)}$	$1.6_{(0.1)}$	15/15
F6	$2.6_{(0.2)}$	$2.3_{(0.1)}$	$1.9_{(0.1)}$	$1.5_{(0.1)}$	$1.4_{(0.1)}$	$1.3_{(0.1)}$	$1.3_{(0.1)}$	15/15
cmaes	$\mathbf{1.6}_{(0.1)}{}^{\star 4}$	$\mathbf{1.8}_{(0.2)}{}^{\star 2}$	$\mathbf{1.6}_{(0.1)}{}^{\star}$	$\mathbf{1.3}_{(0.1)}{}^{\star}$	$\mathbf{1.2}_{(0.0)}{}^{\star 2}$	$\mathbf{1.1}_{(0.0)}{}^{\star 3}$	$\mathbf{1.1}_{(0.0)}{}^{\star 3}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	1002	2228	6278	8586	9762	12285	14831	15/15
D3	$28_{(2)}$	$13_{(0.3)}$	$4.7_{(0.3)}$	$3.5_{(0.2)}$	$3.1_{(0.1)}$	$2.5_{(0.1)}$	$2.2_{(0.1)}$	15/15
E4	$73_{(27)}$	$33_{(10)}$	$12_{(3)}$	$8.8_{(3)}$	$7.8_{(2)}$	$6.3_{(1)}$	$5.3_{(2)}$	15/15
F6	$37_{(2)}$	$17_{(0.9)}$	$6.1_{(0.3)}$	$4.5_{(0.2)}$	$4.1_{(0.2)}$	$3.3_{(0.1)}$	$2.8_{(0.1)}$	15/15
cmaes	$\mathbf{10}_{(0.6)}{}^{\star 4}$	$\mathbf{4.9}_{(0.3)}{}^{\star 4}$	$\mathbf{1.9}_{(0.1)}{}^{\star 4}$	$\mathbf{1.5}_{(0.0)}{}^{\star 4}$	$\mathbf{1.3}_{(0.1)}{}^{\star 4}$	$\mathbf{1.2}_{(0.0)}{}^{\star 4}$	$\mathbf{1.0}_{(0.0)}{}^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	1042	1938	2740	3156	4140	12407	13827	15/15
D3	$4.5_{(2)}$	$4.9_{(3)}$	$5.2_{(3)}$	$5.6_{(3)}$	$5.1_{(2)}$	$2.2_{(0.2)}$	$2.2_{(0.5)}$	15/15
E4	$6.5_{(3)}$	$6.1_{(4)}$	$5.9_{(4)}$	$6.1_{(1)}$	$5.5_{(2)}$	$2.4_{(0.8)}$	$2.4_{(0.5)}$	15/15
F6	$4.0_{(2)}$	$4.2_{(4)}$	$4.7_{(2)}$	$5.1_{(2)}$	$4.6_{(2)}$	$2.1_{(0.5)}$	$2.2_{(0.4)}$	15/15
cmaes	$\mathbf{2.2}_{(0.3)}{}^{\star 2}$	$\mathbf{3.0}_{(2)}$	$\mathbf{3.5}_{(2)}$	$\mathbf{3.8}_{(2)}$	$\mathbf{3.5}_{(2)}$	$\mathbf{1.5}_{(0.4)}{}^{\star}$	$\mathbf{1.6}_{(0.3)}{}^{\star}$	15/15

Figure A.90: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	652	2021	2751	3507	18749	24455	30201	15/15
D3	4.3(0.5)	**4.1**(2)	5.8(3)	4.9(2)	1.2(0.6)	1.3(0.4)	1.3(0.4)	15/15
E4	4.8(0.4)	4.4(2)	**4.2**(2)	5.0(2)	**1.00**(0.3)	**1.1**(0.2)	1.2(0.3)	15/15
F6	4.2(0.8)	4.7(2)	4.4(1)	**4.7**(3)	1.00(0.7)	1.2(0.4)	**1.2**(0.3)	15/15
cmaes	**4.1**(3)*	4.2(8)	21(36)	36(76)	8.8(10)	7.2(7)	6.0(6)	10/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	75	239	304	451	932	1648	15661	15/15
D3	1.9(2)	3.2(0.8)	5.0(0.7)	7.5(2)	6.9(1)	8.2(1.0)	1.3(0.1)	15/15
E4	2.0(2)	4.2(0.9)	6.7(1)	11(2)	9.2(1)	10(0.9)	1.6(0.1)	15/15
F6	2.0(3)	3.4(0.9)	5.5(0.9)	8.4(2)	7.6(0.8)	8.8(0.7)	1.4(0.0)	15/15
cmaes	**1.7**(1)	**2.1**(0.7)$^{\star 2}$	**3.0**(0.6)$^{\star 4}$	**3.7**(0.4)$^{\star 4}$	**3.8**(0.4)$^{\star 4}$	**5.9**(0.6)$^{\star 4}$	**1.2**(0.1)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	30378	1.5e5	3.1e5	3.2e5	3.2e5	4.5e5	4.6e5	15/15
D3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	1384	27265	77015	1.4e5	1.9e5	2.0e5	2.2e5	15/15
D3	2025(2674)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	**0.44**(0.1)$^{\star 4}$	**0.86**(0.1)$^{\star 4}$	**2.4**(1)$^{\star 4}$	**19**(35)$^{\star 4}$	∞	∞	∞ *2e5*	0/15

Figure A.91: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	63	1030	4005	12242	30677	56288	80472	15/15
D3	1.3(1.0)	**1.7**(0.5)	**1.1**(0.2)	**0.59**(0.1)⋆	3.8(4)	∞	∞ *2e5*	0/15
E4	**1.2**(0.9)	13(11)	4.6(0.6)	2.2(3)	**2.1**(3)	∞	∞ *2e5*	0/15
F6	1.3(2)	1.8(0.6)	1.2(0.4)	0.77(0.2)	4.4(5)	∞	∞ *2e5*	0/15
cmaes	1.4(1)	15(97)	20(25)	51(76)	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	621	3972	19561	28555	67569	1.3e5	1.5e5	15/15
D3	1.9(0.8)	**1.1**(0.3)	**0.44**(0.1)⋆	3.6(4)	12(13)	∞	∞ *2e5*	0/15
E4	2.8(3)	2.0(1)	0.82(0.2)	**1.3**(1.0)	**3.1**(2)	∞	∞ *2e5*	0/15
F6	1.8(0.4)	1.4(0.2)	0.91(1)	3.5(4)	42(36)	∞	∞ *2e5*	0/15
cmaes	**1.0**(0.5)⋆	26(38)	31(26)	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	3.4e5	4.7e6	6.2e6	6.7e6	6.7e6	15/15
D3	1.9(0.5)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	2.5(2)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	1.9(3)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	**1.7**(2)	**7.6e4**(7e4)⋆4	∞	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	82	46150	3.1e6	5.5e6	5.5e6	5.6e6	5.6e6	14/15
D3	2.9(3)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	11(7)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	3.8(2)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	**1.7**(0.8)	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Figure A.92: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	561	6541	14103	14318	14643	15567	17589	15/15
D3	61(68)	**37**(49)	36(38)	35(42)	35(40)	33(16)	29(33)	3/15
E4	155(138)	59(68)	66(55)	65(46)	64(145)	60(86)	53(76)	2/15
F6	**37**(80)	49(69)	33(53)	33(30)	32(50)	30(74)	27(27)	3/15
cmaes	91(227)	47(80)	**31**(36)	**31**(50)	**30**(32)	**28**(50)	**25**(15)	3/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	467	5580	23491	24163	24948	26847	1.3e5	12/15
D3	88(63)	40(41)	79(71)	77(58)	75(45)	70(159)	14(18)	1/15
E4	42(9)	35(19)	77(63)	75(100)	73(84)	67(57)	13(16)	1/15
F6	**15**(49)	**26**(53)	**22**(37)	**21**(34)	**21**(20)	**19**(17)	**3.8**(7)	3/15
cmaes	52(33)	39(61)	62(44)	61(103)	59(101)	55(63)	11(14)	1/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	3.2	1614	67457	3.7e5	4.9e5	8.1e5	8.4e5	15/15
D3	1.6(1)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	2.7(3)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	**0.90**(0.6)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	1.9(2)	**33**(48)$^{\star 4}$	**8.7**(10)$^{\star 4}$	∞	∞	∞	∞ *2e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	1.3e6	7.5e6	5.2e7	5.2e7	5.2e7	5.2e7	5.2e7	3/15
D3	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15

Figure A.93: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f1	83	83	83	83	83	83	83	30/30
D3	9.5(1)	17(1)	24(1)	32(2)	39(1)	55(3)	70(2)	15/15
E4	11(2)	19(2)	28(2)	36(4)	45(2)	63(2)	80(3)	15/15
F6	10(2)	18(2)	25(2)	33(2)	41(2)	57(1)	72(2)	15/15
cmaes	**8.4**(1.0)	**14**(0.8)$^{\star 3}$	**21**(1)$^{\star 3}$	**26**(2)$^{\star 4}$	**33**(2)$^{\star 4}$	**45**(1)$^{\star 4}$	**56**(2)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f2	796	797	799	799	800	802	804	15/15
D3	63(4)	72(4)	77(2)	**80**(4)	**83**(5)	**86**(2)	**88**(3)	15/15
E4	84(3)	85(2)	86(2)	87(1)	88(2)	90(2)	92(2)	15/15
F6	73(4)	79(4)	82(2)	84(2)	85(2)	87(1)	89(1)	15/15
cmaes	**58**(4)*	**69**(8)	**77**(4)	81(6)	85(4)	89(3)	90(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f3	15526	15602	15612	15641	15646	15651	15656	15/15
D3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f4	15536	15601	15659	15678	15703	15733	2.8e5	9/15
D3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.94: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f5	98	116	120	121	121	121	121	15/15
D3	6.8(0.6)	6.7(0.4)	6.6(0.6)	6.6(0.6)	6.6(0.7)	6.6(0.7)	6.6(0.6)	15/15
E4	6.7(1)	6.6(1.0)	6.6(0.5)	6.6(1)	6.6(0.6)	6.6(0.8)	6.6(1)	15/15
F6	**6.6**(1)	**6.3**(1)	**6.3**(0.7)	**6.3**(1)	**6.3**(1)	**6.3**(0.7)	**6.3**(0.8)	15/15
cmaes	7.1(0.5)	6.5(0.4)	6.5(0.5)	6.6(0.6)	6.6(0.5)	6.6(0.4)	6.6(0.3)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f6	3507	5523	7168	9470	11538	15007	19222	15/15
D3	9.1(7)	7.7(4)	7.5(3)	7.0(3)	6.7(3)	6.6(2)	6.3(2)	15/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	**1.4**(0.3)$^{\star 4}$	**1.4**(0.3)$^{\star 4}$	**1.4**(0.2)$^{\star 4}$	**1.3**(0.2)$^{\star 4}$	**1.4**(0.1)$^{\star 4}$	**1.4**(0.1)$^{\star 4}$	**1.4**(0.1)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f7	10698	17839	41037	66294	66294	66294	68145	15/15
D3	**0.47**(0.1)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
E4	0.64(0.1)	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	0.55(0.6)	**177**(353)	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	21(29)	∞	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f8	7080	10655	11012	11265	11430	11701	11969	15/15
D3	**6.7**(0.6)	6.8(2)	7.0(3)	7.0(3)	7.0(1)	7.1(2)	7.1(3)	15/15
E4	7.0(0.9)	7.7(8)	7.9(2)	7.9(6)	7.9(4)	7.9(5)	7.9(5)	15/15
F6	6.8(1)	**6.4**(2)	**6.5**(1)	**6.6**(3)	**6.6**(3)	**6.7**(3)	**6.7**(0.8)	15/15
cmaes	6.9(0.8)	6.8(2)	6.9(4)	7.0(4)	7.0(0.3)	7.0(2)	6.9(3)	15/15

Figure A.95: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f9	6122	12982	13300	13496	13651	13909	14142	15/15
D3	7.9(0.7)	5.8(4)	6.0(0.4)	6.1(6)	6.1(0.3)	6.2(0.4)	6.2(3)	15/15
E4	7.9(0.9)	5.4(0.2)	5.6(2)	5.7(2)	5.7(0.3)	5.8(0.3)	5.9(0.2)	15/15
F6	**7.8**(0.8)	**5.4**(0.3)	**5.6**(2)	**5.7**(2)	**5.7**(0.4)	**5.8**(0.3)	**5.8**(0.4)	15/15
cmaes	7.9(0.4)	5.5(3)	5.7(0.2)	5.8(0.2)	5.8(0.2)	5.8(2)	5.8(2)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f10	25890	30368	36796	51579	56007	65128	70824	15/15
D3	1.9(0.2)	1.8(0.1)	**1.6**(0.1)	**1.2**(0.1)	**1.2**(0.0)	**1.1**(0.0)	**0.99**(0.0)	15/15
E4	2.6(0.1)	2.2(0.1)	1.9(0.1)	1.4(0.0)	1.3(0.0)	1.1(0.0)	1.0(0.0)	15/15
F6	2.2(0.2)	2.0(0.2)	1.7(0.1)	1.3(0.1)	1.2(0.0)	1.1(0.0)	1.00(0.0)	15/15
cmaes	**1.6**(0.1)*	**1.7**(0.1)	1.6(0.1)	1.2(0.1)	1.2(0.1)	1.1(0.0)	1.0(0.0)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f11	2368	4855	11681	25315	29749	38949	48211	15/15
D3	30(0.6)	15(0.3)	6.2(0.1)	2.9(0.0)	2.5(0.1)	1.9(0.0)	1.6(0.0)	15/15
E4	∞	∞	∞	∞	∞	∞	∞ *2e5*	0/15
F6	1116(1367)	544(780)	226(248)	104(96)	89(86)	68(76)	55(101)	2/15
cmaes	**12**(0.7)$^{\star 4}$	**6.7**(0.3)$^{\star 4}$	**3.0**(0.1)$^{\star 4}$	**1.5**(0.0)$^{\star 4}$	**1.3**(0.0)$^{\star 4}$	**1.1**(0.0)$^{\star 4}$	**0.90**(0.0)$^{\star 4}$	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f12	4169	7452	9174	10751	13146	22758	25192	15/15
D3	1.2(0.1)	1.7(1)	2.1(1)	2.3(0.8)	2.3(0.9)	1.8(0.4)	1.9(0.4)	15/15
E4	1.4(1.0)	1.8(1)	2.4(0.8)	2.5(1)	2.5(0.9)	2.0(0.5)	2.1(0.3)	15/15
F6	1.5(1)	2.0(1)	2.5(2)	2.6(2)	2.6(1)	2.0(0.7)	2.1(0.5)	15/15
cmaes	**1.0**(0.1)$^{\star 2}$	**1.5**(0.7)	**2.0**(0.8)	**2.2**(1)	**2.2**(0.6)	**1.7**(0.4)	**1.9**(0.5)	15/15

Figure A.96: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f13	2029	6916	8734	11861	71936	98467	1.2e5	15/15
D3	3.2(6)	4.8(5)	6.4(4)	**5.5**(3)	0.99(0.7)	**1.1**(0.5)	1.3(0.4)	15/15
E4	2.6(0.2)	**4.4**(3)	**6.3**(4)	5.5(2)	0.96(0.8)	1.5(0.7)	**1.3**(0.5)	15/15
F6	**2.0**(0.2)	4.7(4)	6.4(7)	5.6(4)	**0.95**(0.7)	1.4(1.0)	1.8(1)	14/15
cmaes	4.2(7)	17(22)	26(23)	27(25)	4.8(9)	3.8(5)	3.3(10)	10/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f14	304	616	777	1105	2207	4825	57711	15/15
D3	1.3(0.5)	2.2(0.3)	3.3(0.3)	5.3(0.4)	6.3(0.3)	7.7(0.4)	1.1(0.1)	15/15
E4	1.7(1.0)	2.6(0.4)	3.8(0.5)	7.4(1.0)	8.5(1)	9.0(0.6)	1.2(0.0)	15/15
F6	1.6(0.8)	2.4(0.4)	3.4(0.2)	5.6(1)	6.9(0.6)	8.0(0.4)	1.2(0.1)	15/15
cmaes	**1.2**(0.5)	**1.8**(0.3)*	**2.5**(0.3)$^{\star 4}$	**3.4**(0.4)$^{\star 4}$	**4.1**(0.2)$^{\star 4}$	**6.4**(0.3)$^{\star 4}$	**1.1**(0.1)	15/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f15	1.9e5	7.9e5	1.0e6	1.1e6	1.1e6	1.1e6	1.1e6	15/15
D3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f16	5244	72122	3.2e5	7.1e5	1.4e6	2.0e6	2.0e6	15/15
D3	67(95)	6.4(11)	18(10)	∞	∞	∞	∞ *4e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	306(744)	22(35)	**17**(9)	∞	∞	∞	∞ *4e5*	0/15
cmaes	**0.25**(0.1)$^{\star 4}_{\downarrow 4}$	**0.22**(0.3)*	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.97: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f17	399	4220	14158	34948	51958	1.3e5	2.7e5	14/15
D3	$\mathbf{0.36}_{(0.3)\downarrow 4}$	$\mathbf{0.60}_{(0.2)}$	$2.5_{(0.1)}$	$10_{(20)}$	$\mathbf{53}_{(62)}$	∞	∞ *4e5*	0/15
E4	$0.42_{(0.3)\downarrow 4}$	$0.68_{(0.1)}$	$\mathbf{0.47}_{(0.1)}$	$11_{(5)}$	$112_{(162)}$	∞	∞ *4e5*	0/15
F6	$0.41_{(0.2)\downarrow 4}$	$0.65_{(0.2)}$	$2.4_{(7)}$	$\mathbf{7.1}_{(6)}$	$113_{(90)}$	∞	∞ *4e5*	0/15
cmaes	$0.47_{(0.4)\downarrow 2}$	$15_{(71)}$	$43_{(57)}$	$75_{(89)}$	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f18	1442	16998	47068	1.3e5	1.9e5	6.7e5	9.5e5	6/15
D3	$1.1_{(0.3)}$	$\mathbf{0.33}_{(0.1)}$	$20_{(40)}$	∞	∞	∞	∞ *4e5*	0/15
E4	$1.2_{(0.4)}$	$0.43_{(0.1)}$	$\mathbf{8.5}_{(11)}$	∞	∞	∞	∞ *4e5*	0/15
F6	$1.2_{(0.3)}$	$0.35_{(0.1)}$	$11_{(16)}$	∞	∞	∞	∞ *4e5*	0/15
cmaes	$\mathbf{0.86}_{(0.2)}$	$21_{(29)}$	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f19	1	1	1.4e6	1.7e7	2.6e7	4.5e7	4.5e7	8/15
D3	$3.5_{(2)}$	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	$2.2_{(1)}$	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	$\mathbf{2.0}_{(2)}$	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	$2.5_{(2)}$	$\mathbf{1.6e5}_{(3e5)}^{\star 4}$	∞	∞	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f20	222	1.3e5	1.6e8	∞	∞	∞	∞	0
D3	$3.3_{(0.8)}$	∞	∞	.	.	.	.	0/15
E4	$11_{(12)}$	∞	∞	.	.	.	.	0/15
F6	$3.7_{(2)}$	∞	∞	.	.	.	.	0/15
cmaes	$\mathbf{1.7}_{(0.6)}^{\star 2}$	∞	∞	.	.	.	.	0/15

Figure A.98: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f21	1044	21144	1.0e5	1.0e5	1.0e5	1.0e5	1.0e5	26/30
D3	2.2(5)	**12**(17)	**4.1**(12)	**4.1**(5)	**4.0**(2)	**4.0**(8)	**4.0**(8)	5/15
E4	1.5(0.2)	14(14)	5.3(14)	5.3(4)	5.3(9)	5.3(6)	5.3(5)	4/15
F6	2.2(2)	13(16)	6.8(16)	6.8(19)	6.8(8)	6.8(14)	6.8(7)	3/15
cmaes	**0.70**(0.3)	12(26)	4.5(6)	4.5(8)	4.5(13)	4.5(3)	4.5(4)	4/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f22	3090	35442	6.5e5	6.5e5	6.5e5	6.5e5	6.5e5	8/30
D3	4.2(1)	**7.7**(19)	∞	∞	∞	∞	∞ *2e5*	0/15
E4	4.9(25)	8.1(12)	∞	∞	∞	∞	∞ *2e5*	0/15
F6	**3.9**(0.1)	8.6(17)	∞	∞	∞	∞	∞ *2e5*	0/15
cmaes	14(12)	8.8(11)	∞	∞	∞	∞	∞ *1e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f23	7.1	11925	75453	6.6e5	1.3e6	3.2e6	3.4e6	15/15
D3	1.5(2)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	2.1(4)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	**1.2**(0.7)	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	1.3(1)	**3.9**(3)$^{\star 4}$	**5.6**(4)$^{\star 4}$	**8.9**(7)$^{\star 4}$	∞	∞	∞ *4e5*	0/15

Δf_{opt}	1e1	1e0	1e-1	1e-2	1e-3	1e-5	1e-7	#succ
f24	5.8e6	9.8e7	3.0e8	3.0e8	3.0e8	3.0e8	3.0e8	1/15
D3	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
E4	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
F6	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15
cmaes	∞	∞	∞	∞	∞	∞	∞ *4e5*	0/15

Figure A.99: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/DIM) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 40-D. The "best 2009" line corresponds to the best ERT observed during BBOB 2009 for each single target.

Appendix B

Parent-Offspring Sampling Variance Results

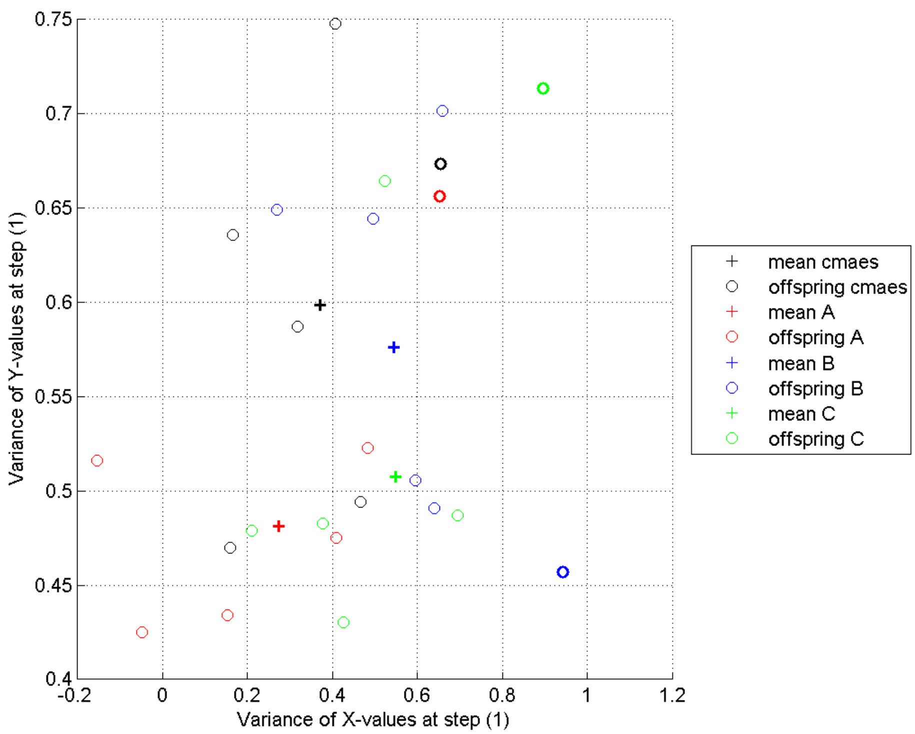

Figure B.1: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 1.

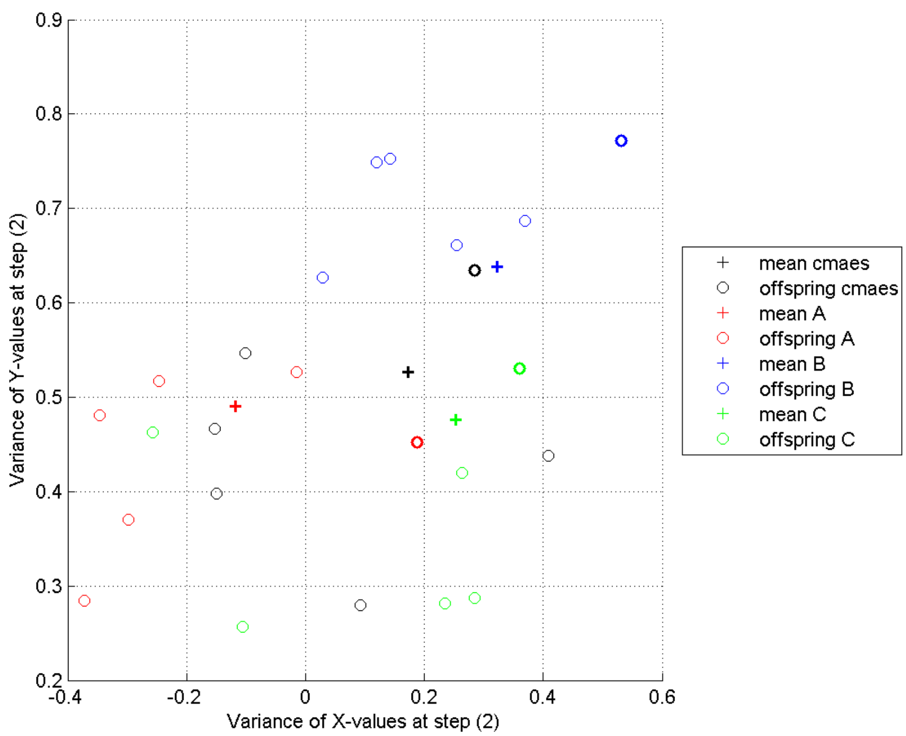

Figure B.2: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 2.

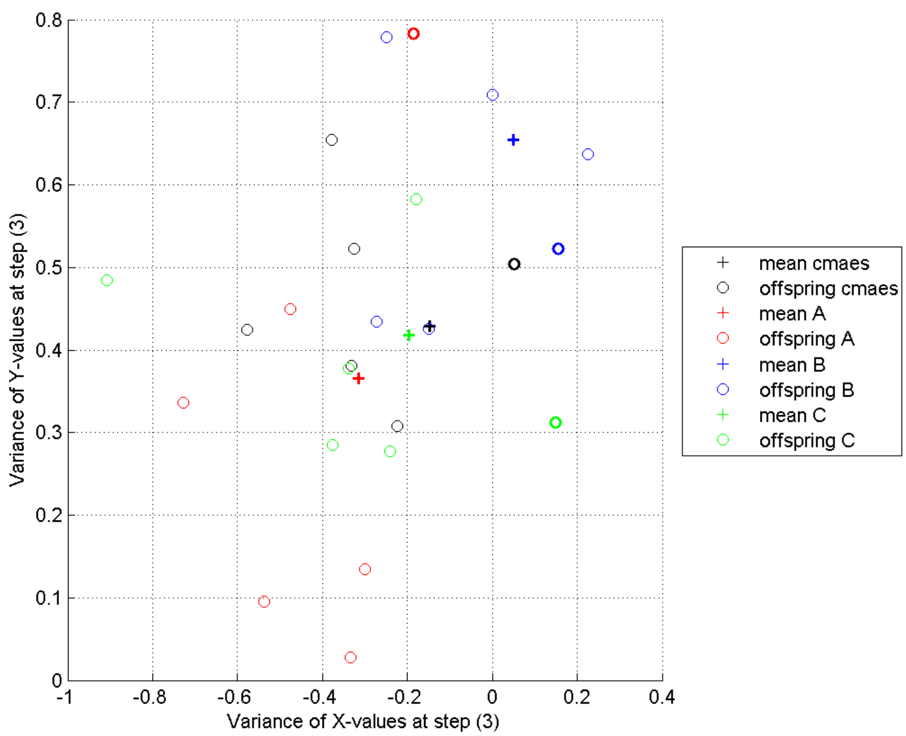

Figure B.3: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 3.

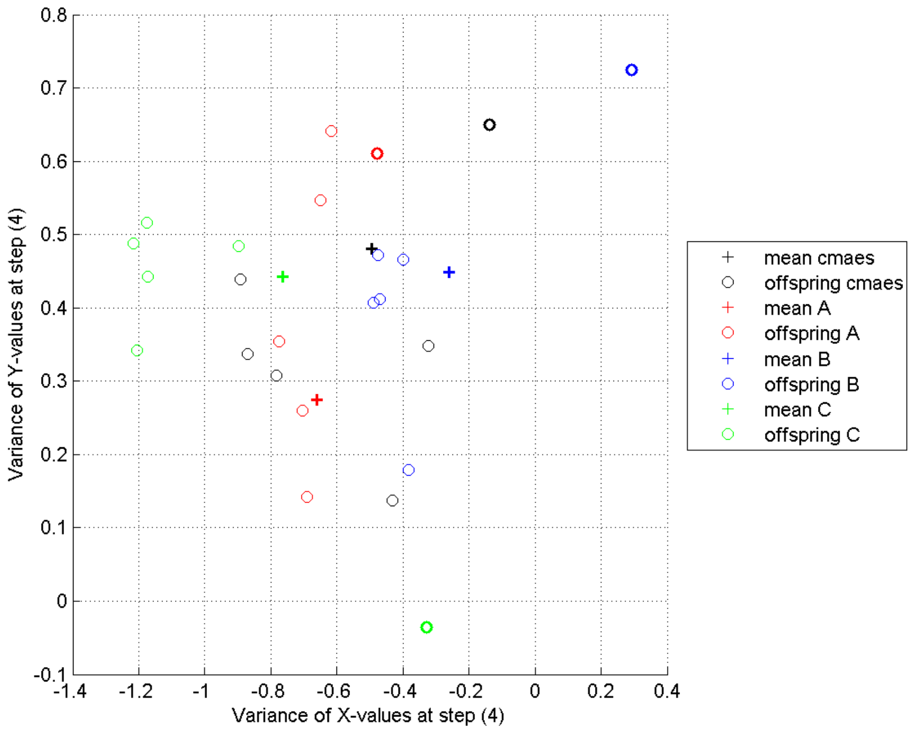

Figure B.4: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 4.

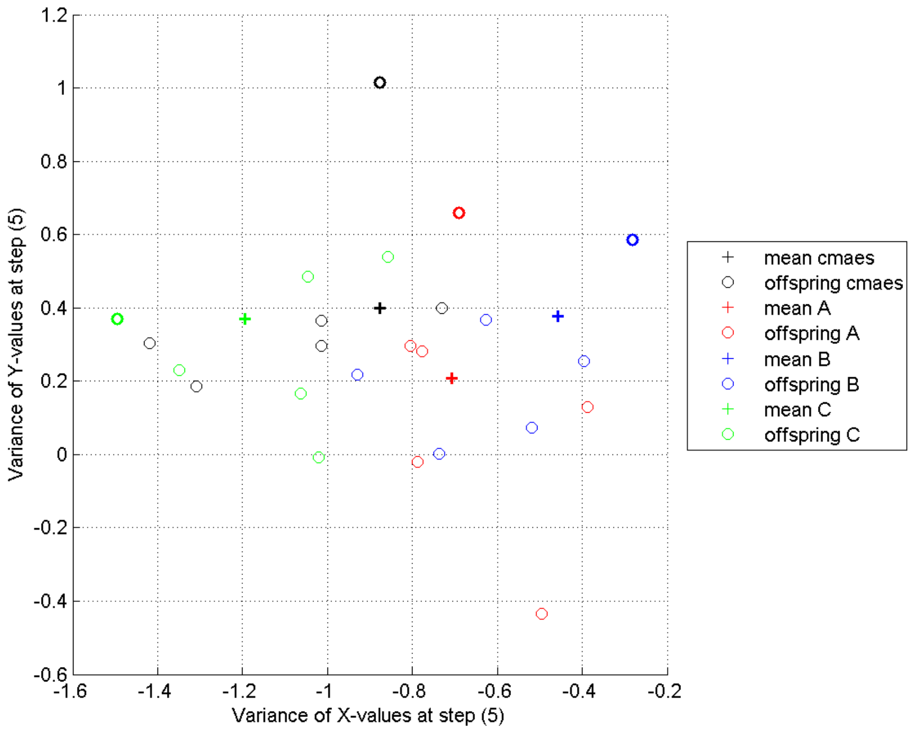

Figure B.5: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 5.

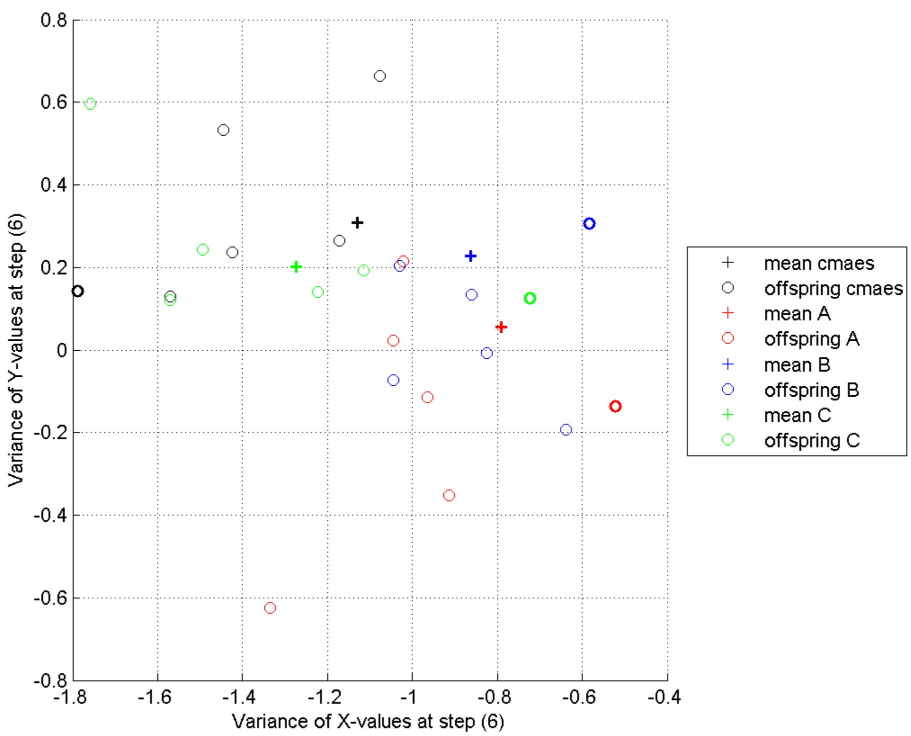

Figure B.6: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 6.

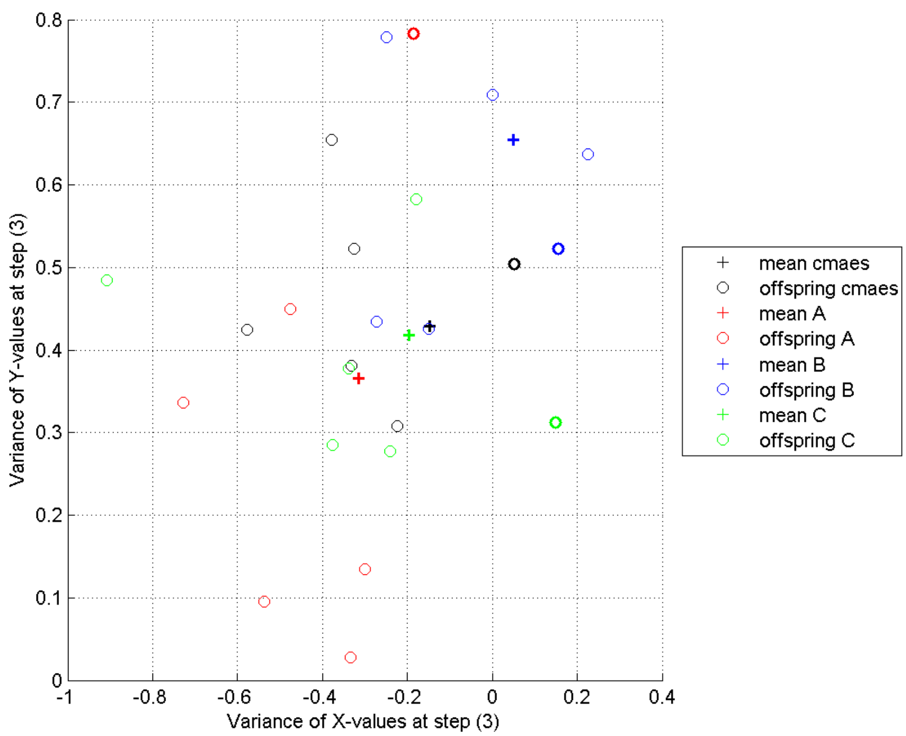

Figure B.7: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 7.

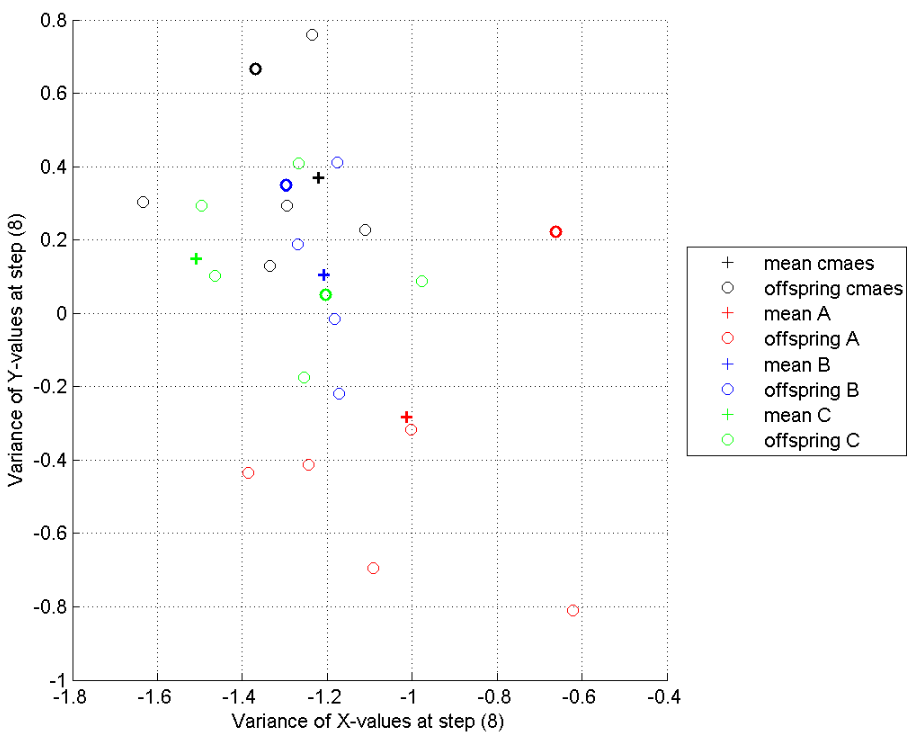

Figure B.8: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 8.

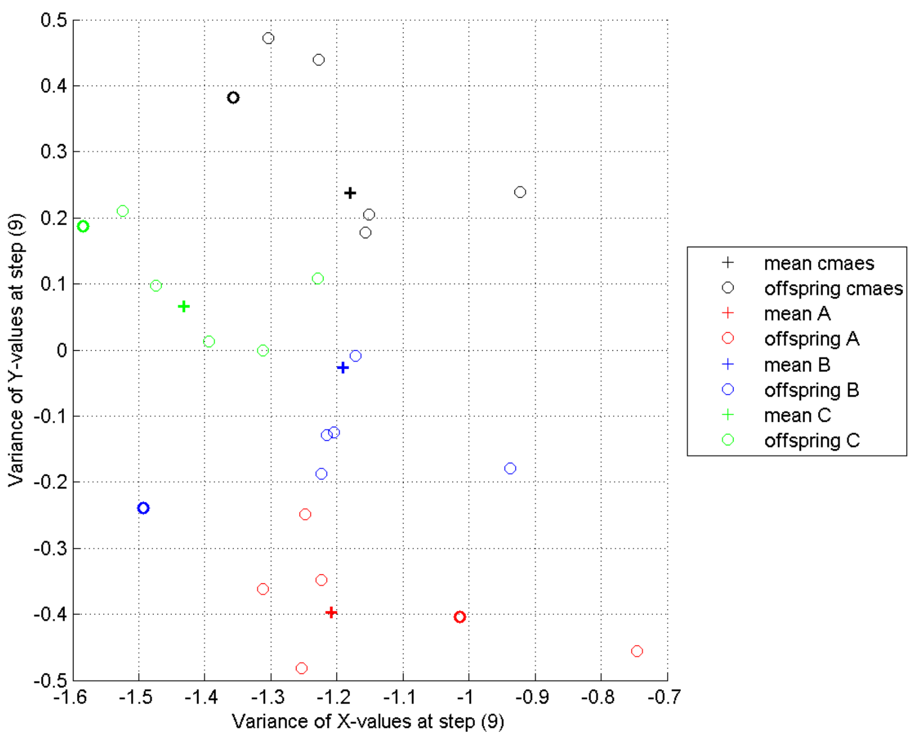

Figure B.9: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 9.

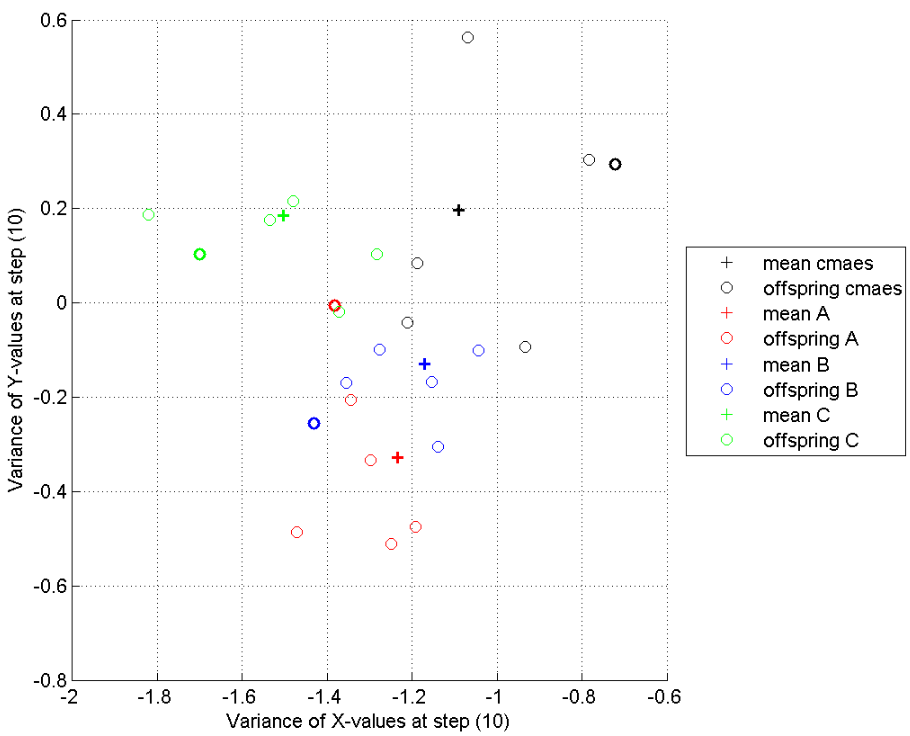

Figure B.10: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 10.

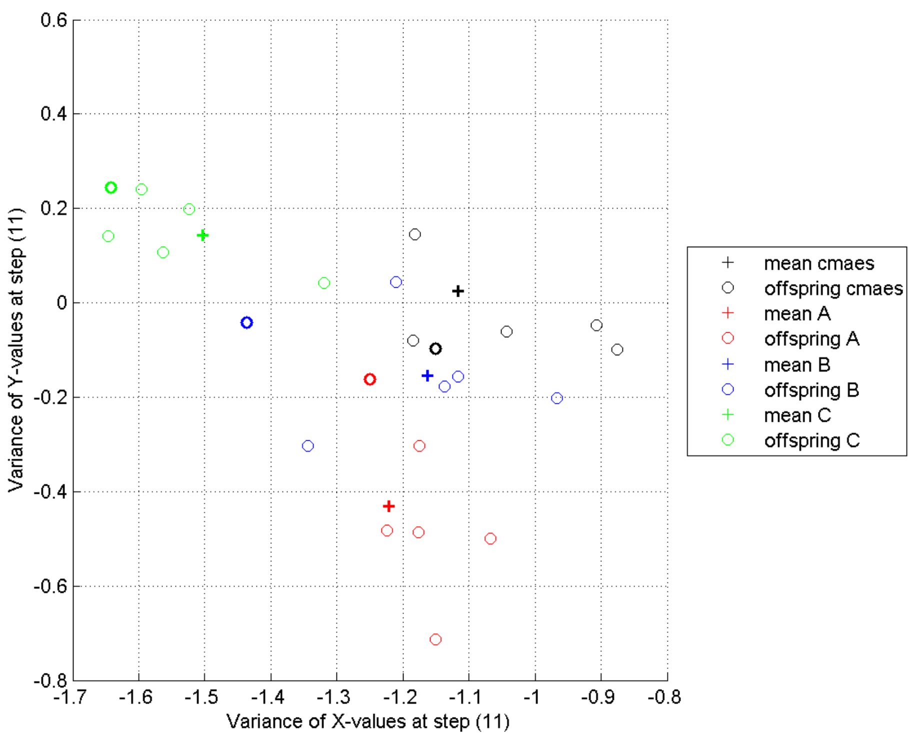

Figure B.11: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 11.

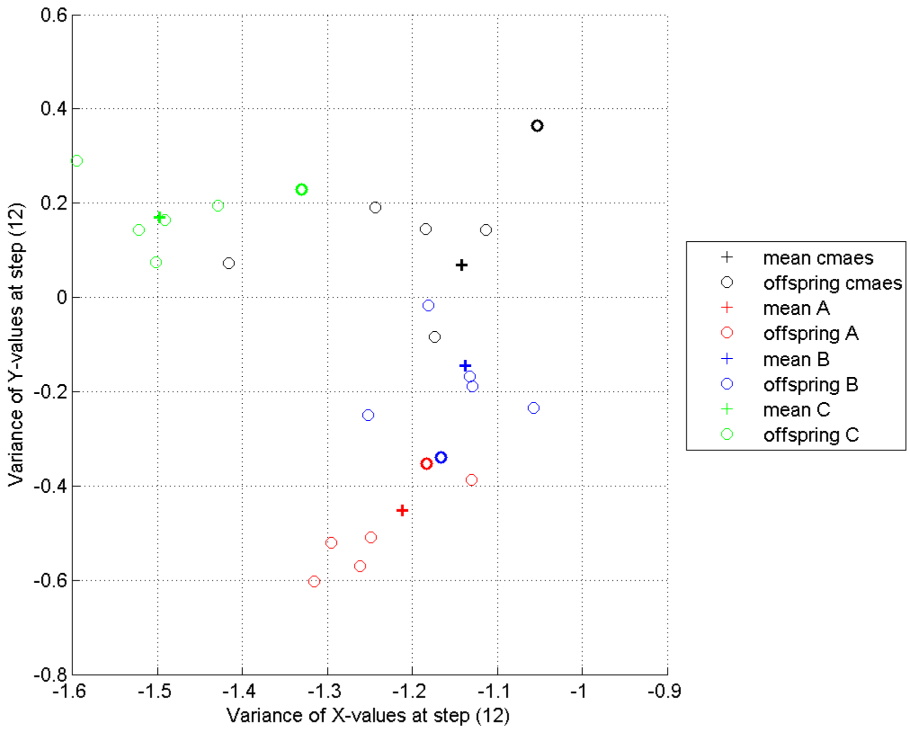

Figure B.12: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 12.

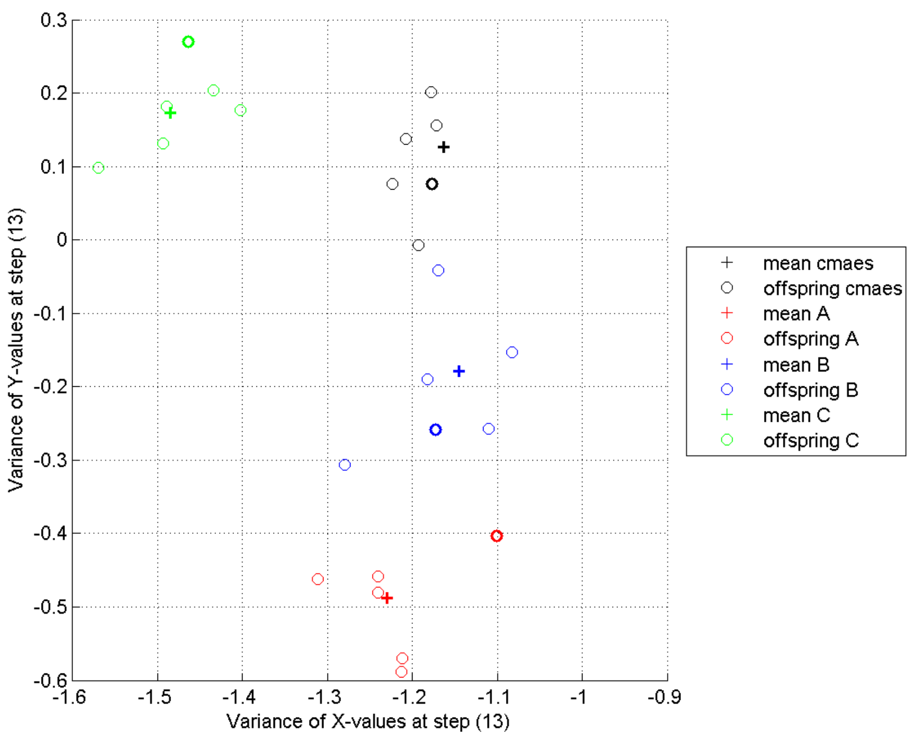

Figure B.13: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 13.

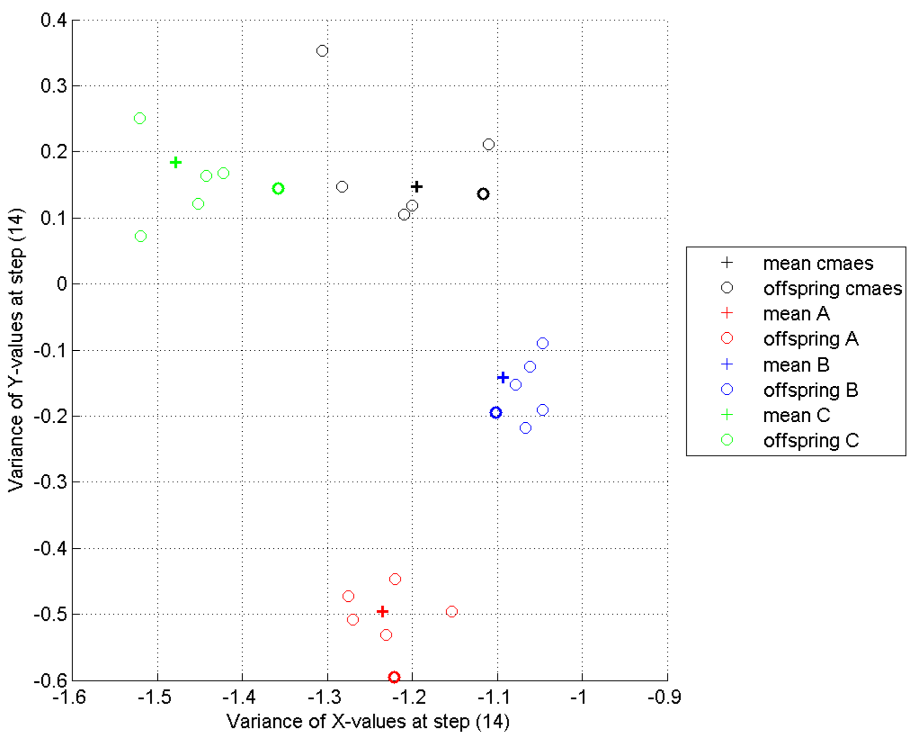

Figure B.14: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 14.

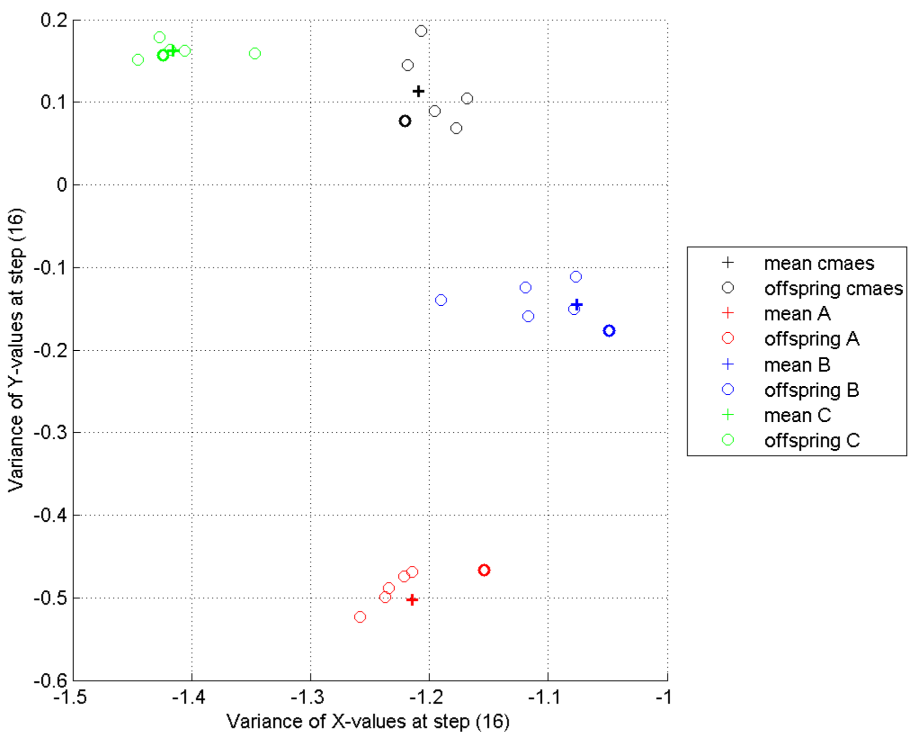

Figure B.15: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 16.

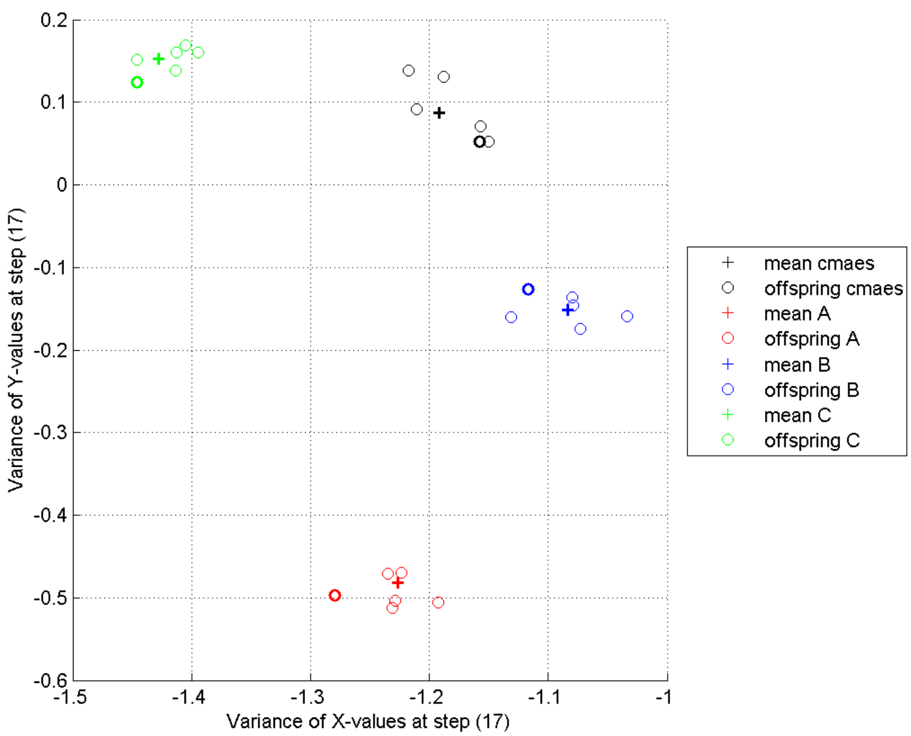

Figure B.16: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 17.

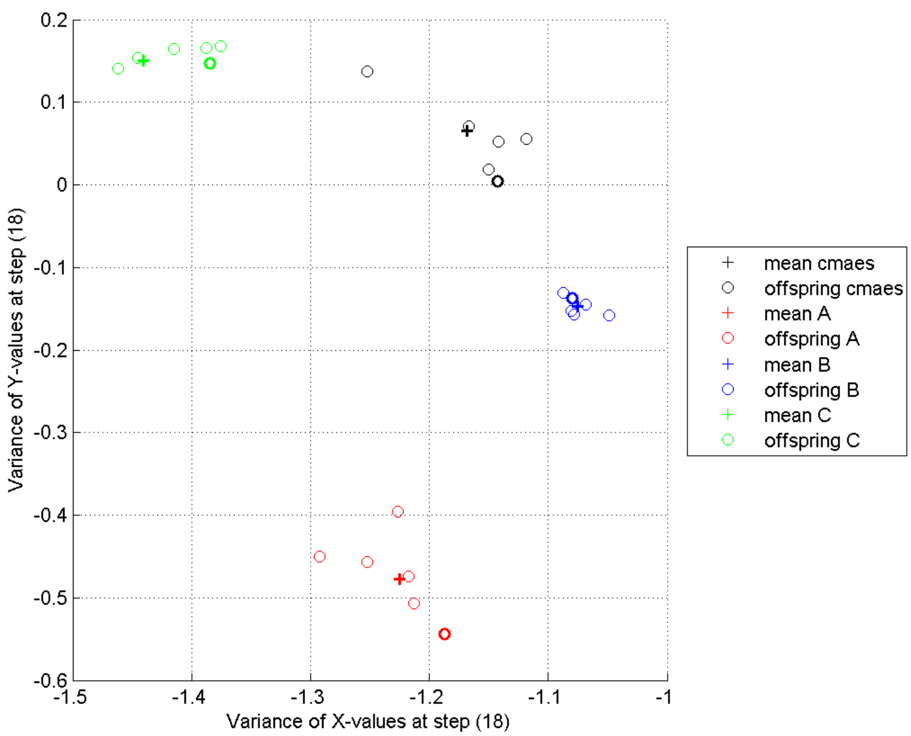

Figure B.17: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 18.

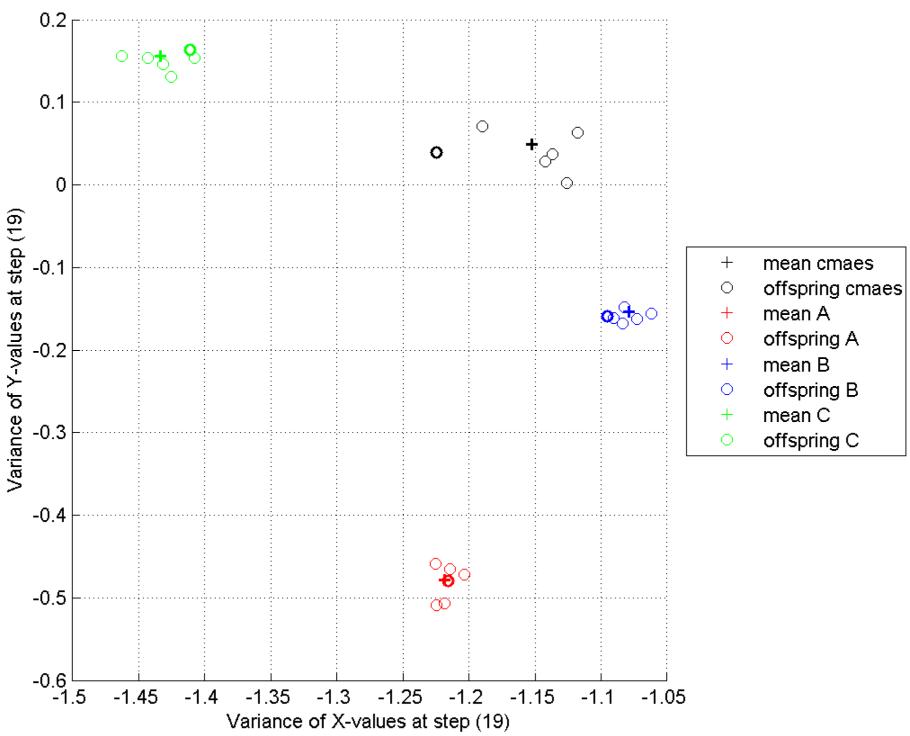

Figure B.18: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 19.

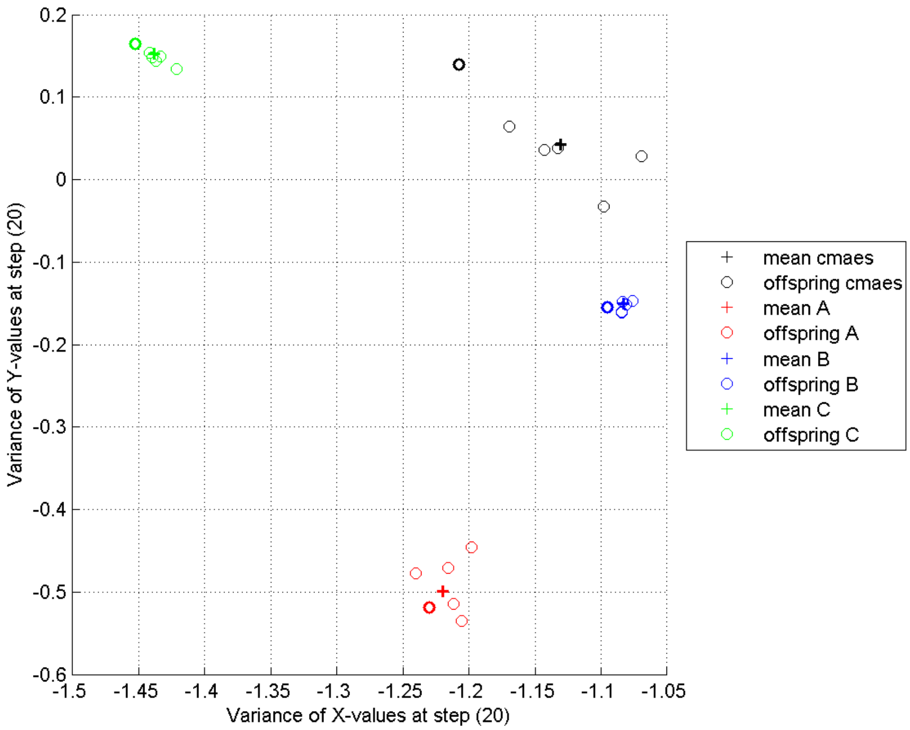

Figure B.19: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 20.

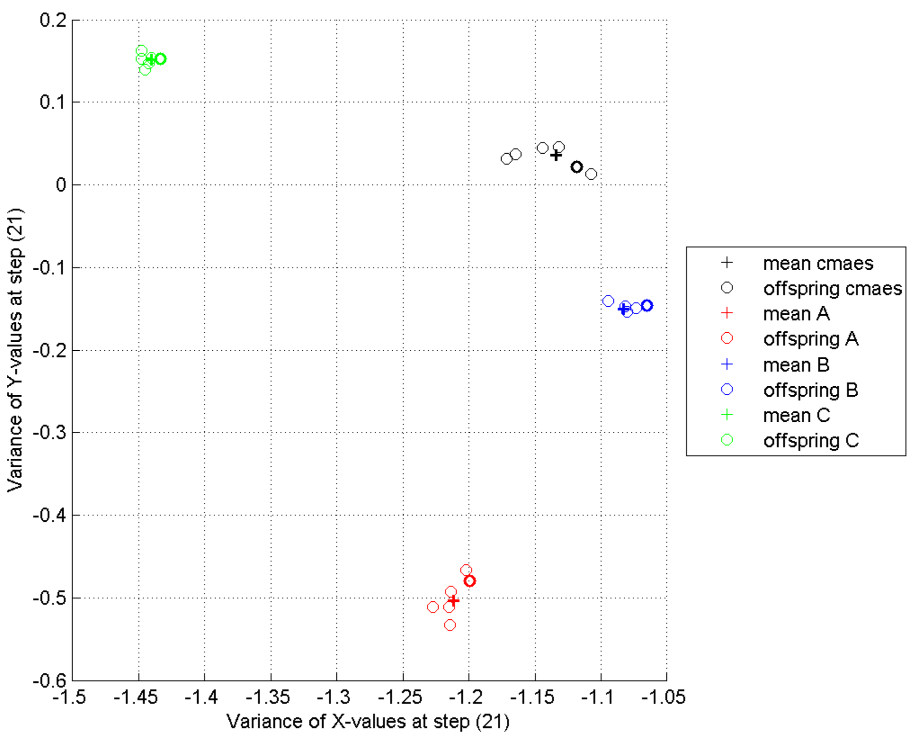

Figure B.20: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 21.

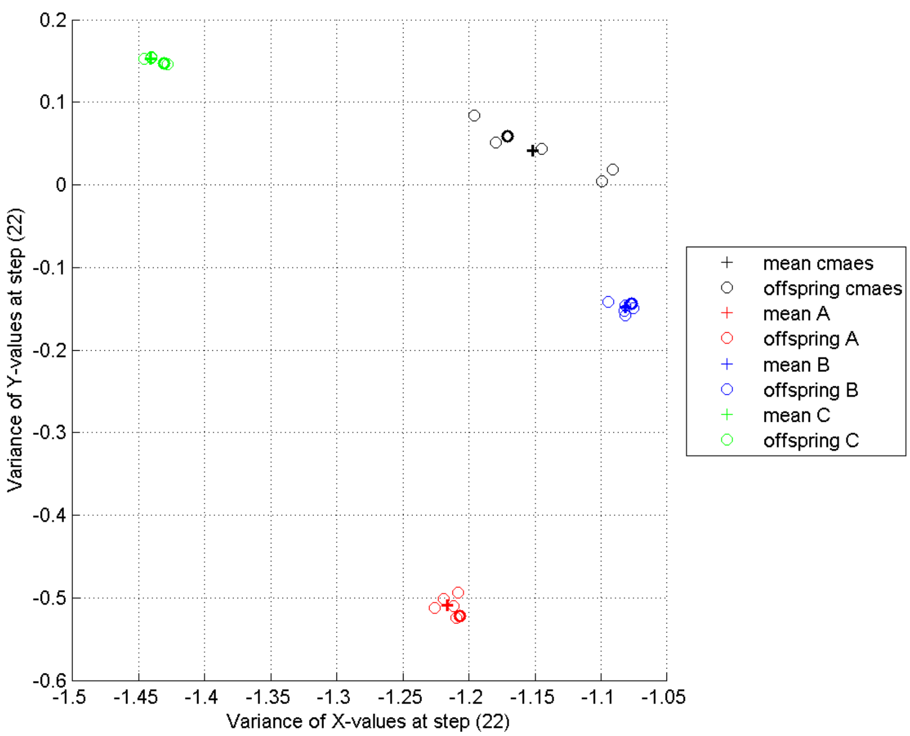

Figure B.21: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 22.

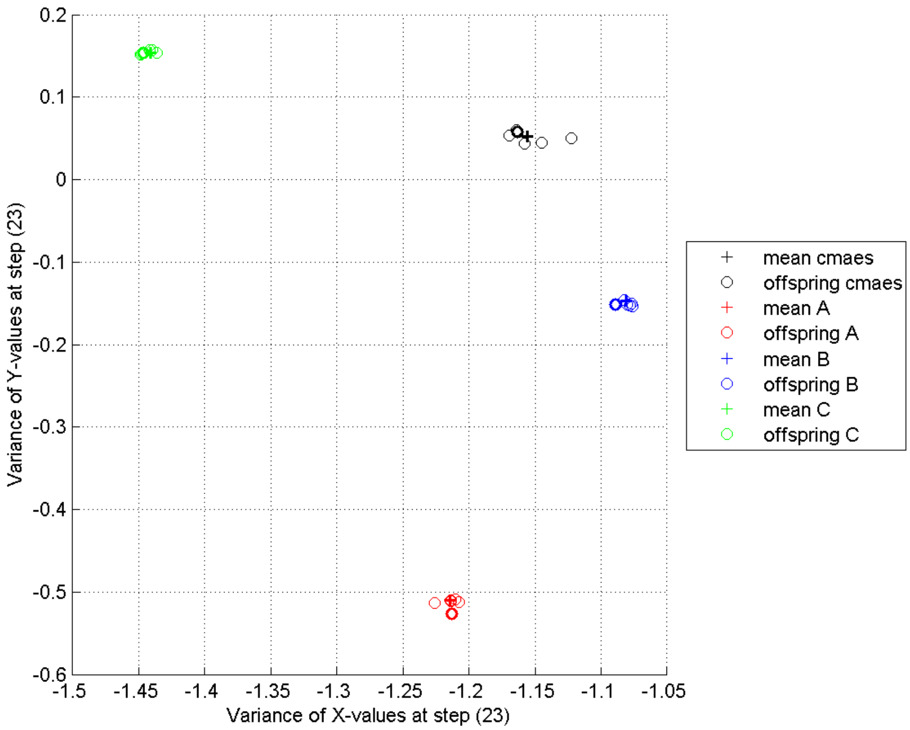

Figure B.22: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 23.

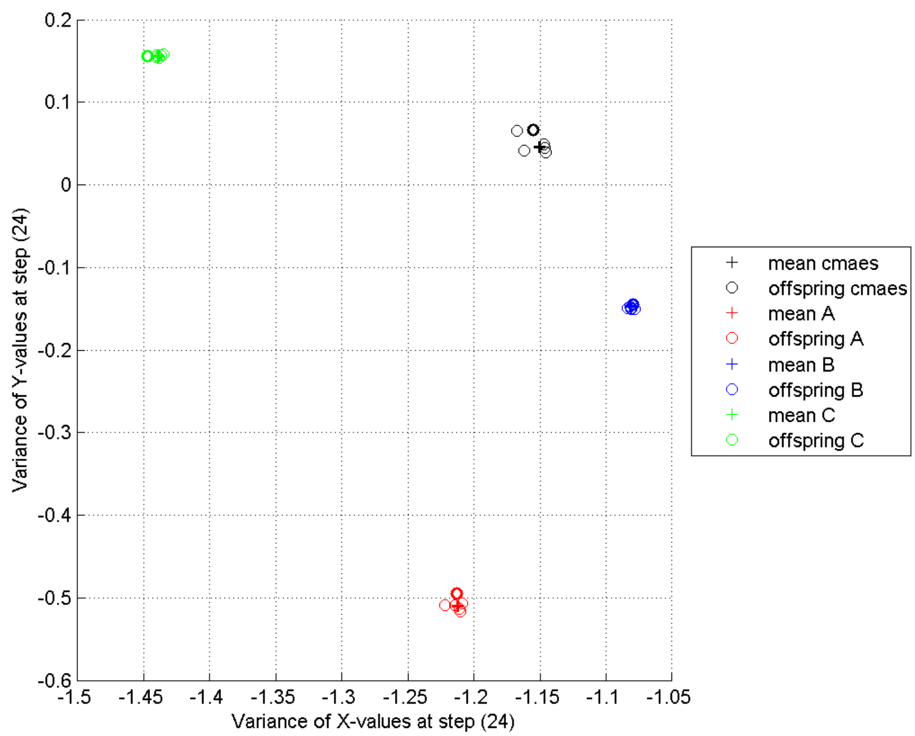

Figure B.23: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 24.

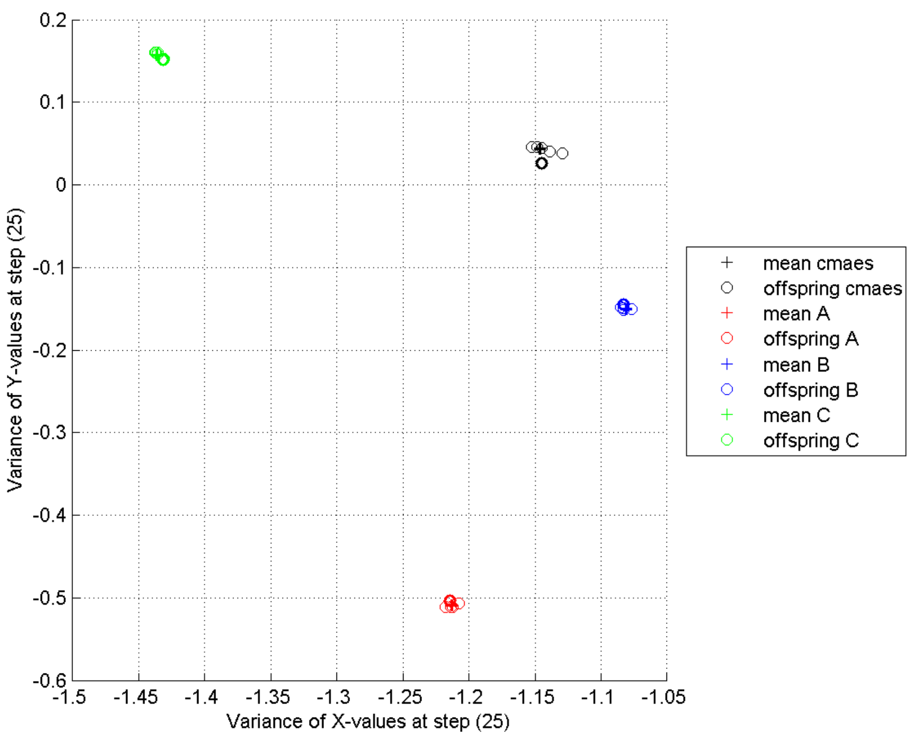

Figure B.24: 2D plot of the variance of mean and offspring solutions. The picture shows the results at step 25.

Appendix C

Population Variance Results

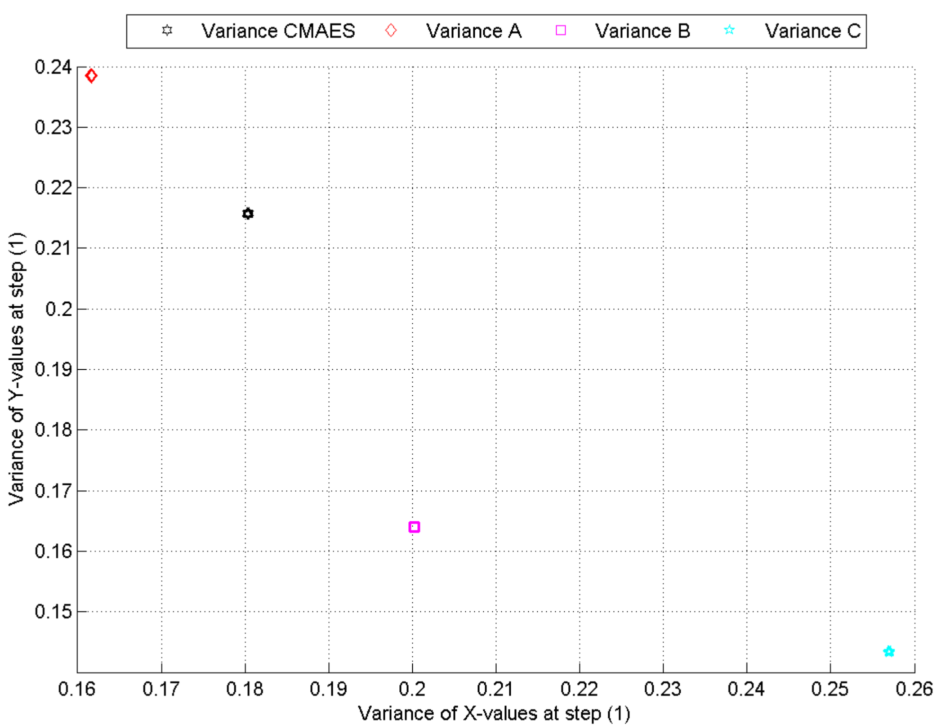

Figure C.1: 2D plot of the population variance at step 1.

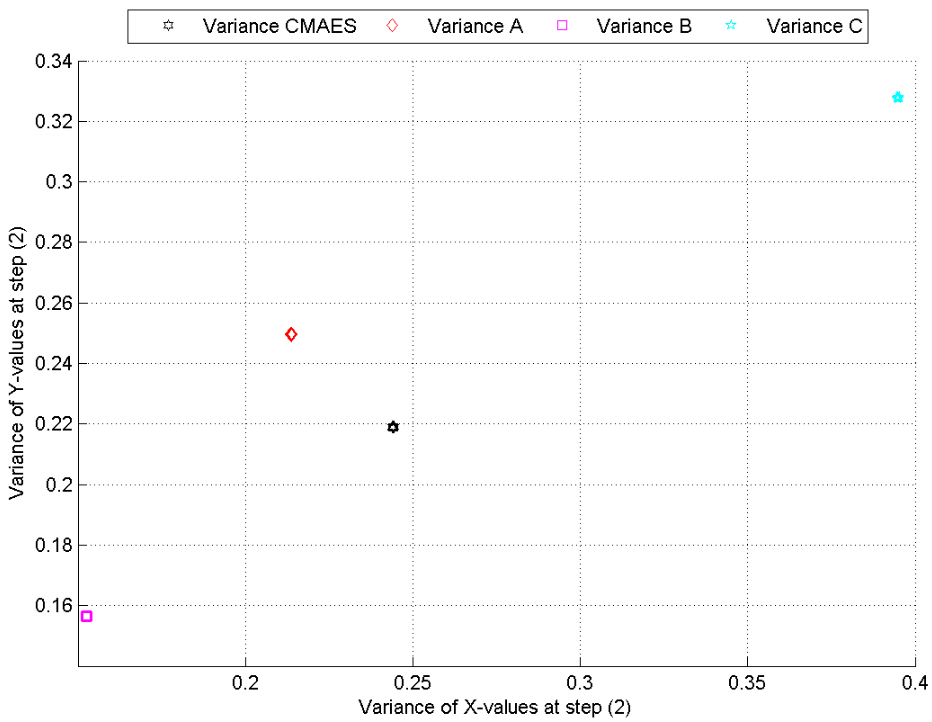

Figure C.2: 2D plot of the population variance at step 2.

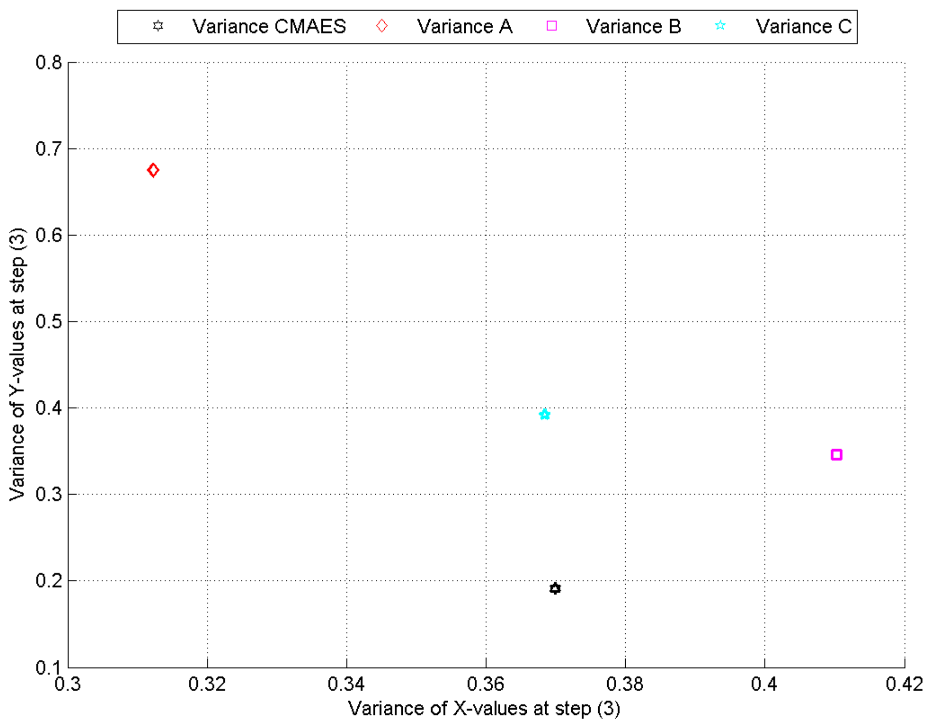

Figure C.3: 2D plot of the population variance at step 3.

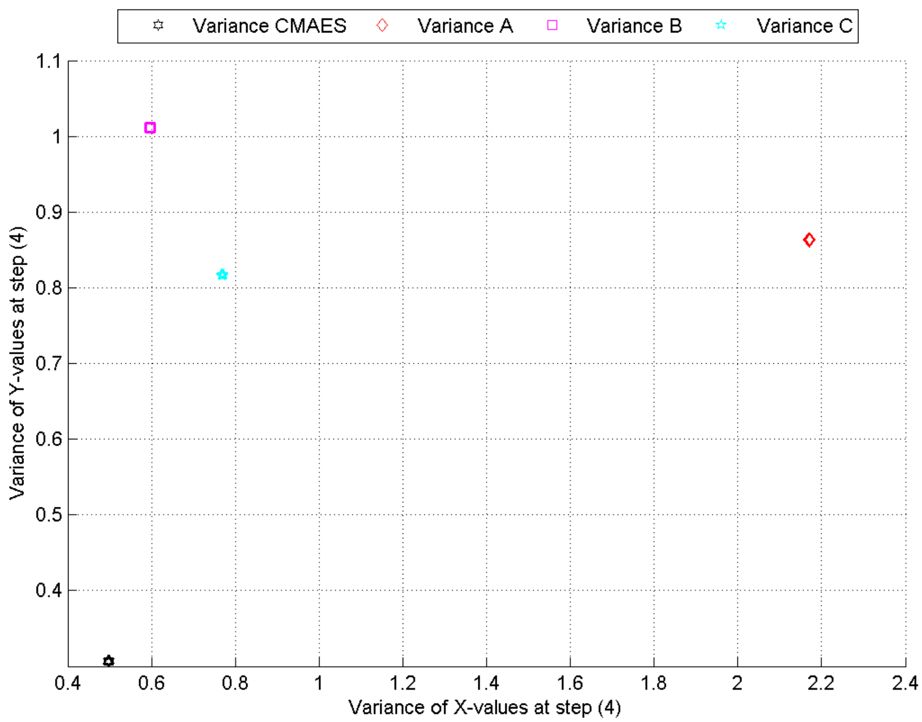

Figure C.4: 2D plot of the population variance at step 4.

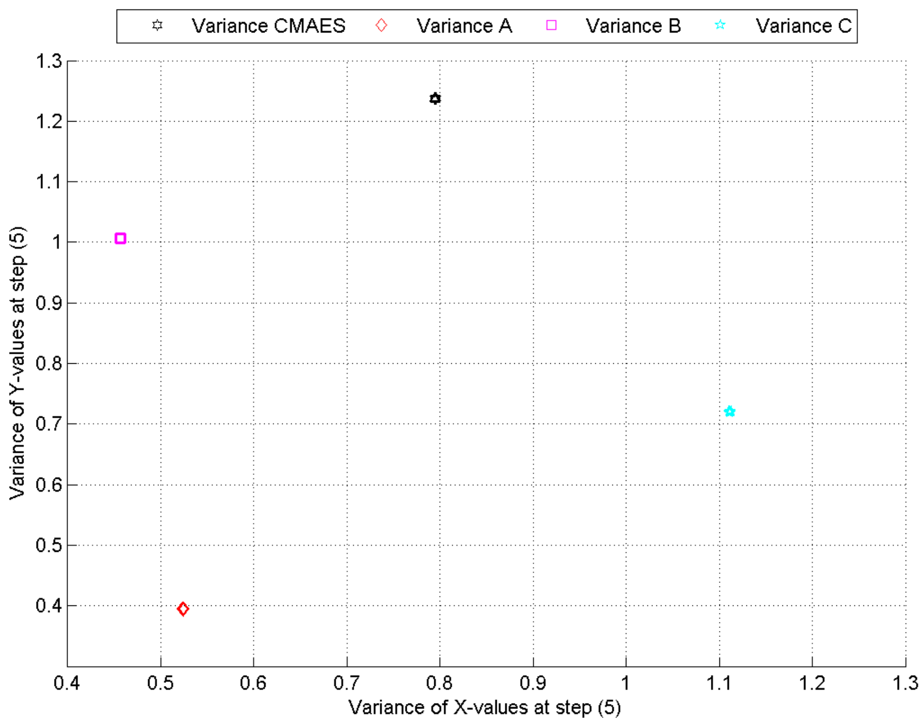

Figure C.5: 2D plot of the population variance at step 5.

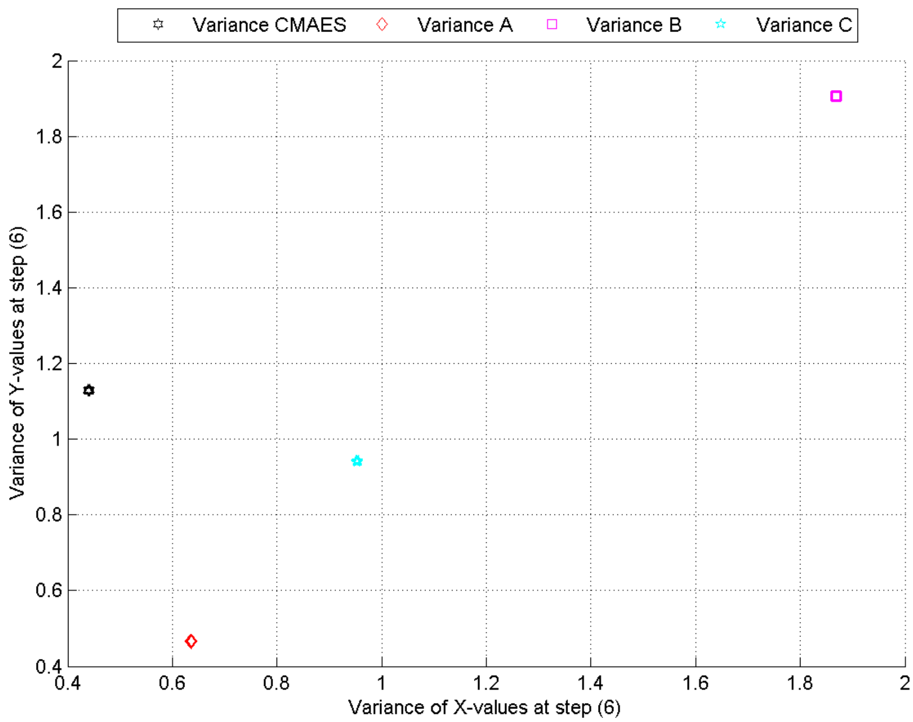

Figure C.6: 2D plot of the population variance at step 6.

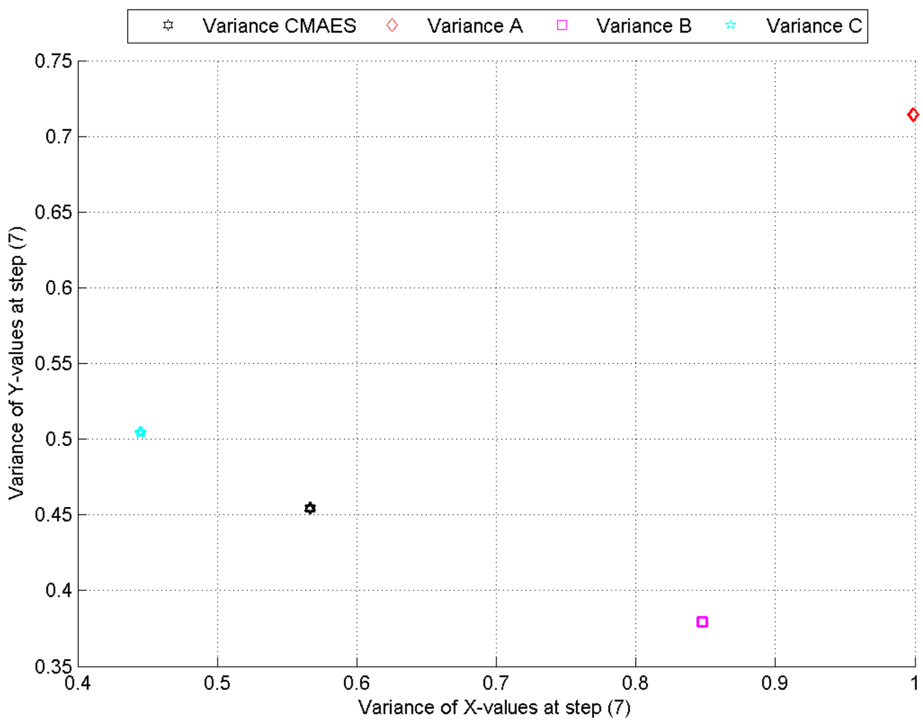

Figure C.7: 2D plot of the population variance at step 7.

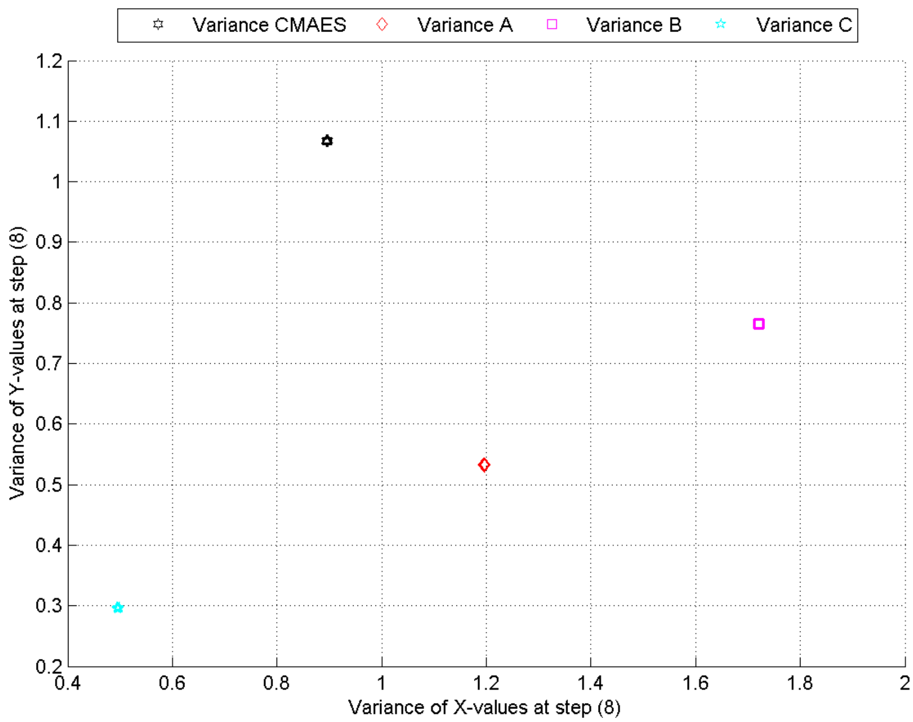

Figure C.8: 2D plot of the population variance at step 8.

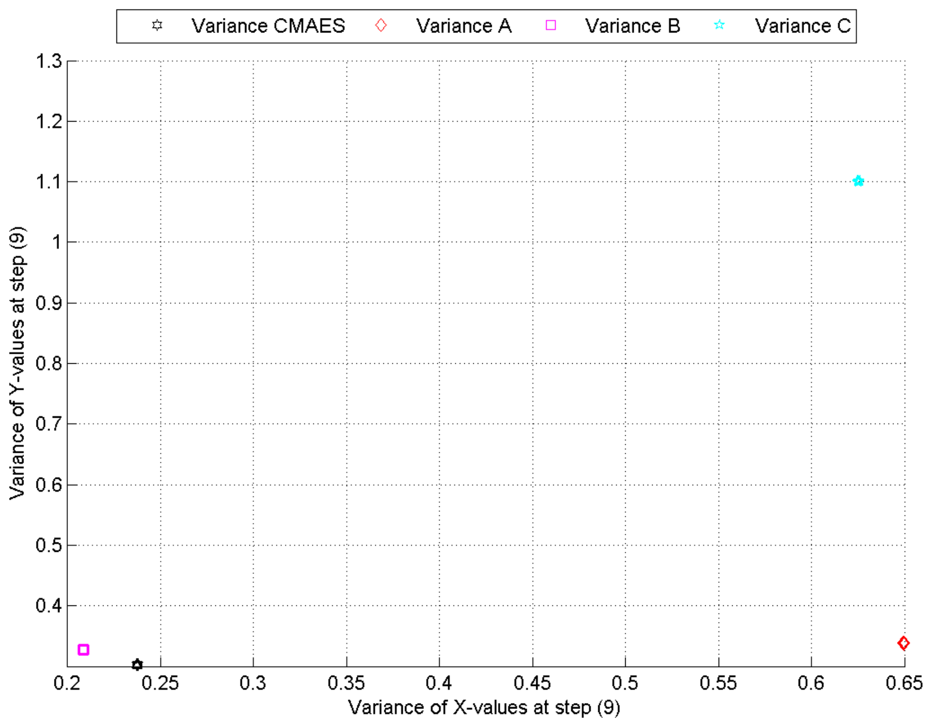

Figure C.9: 2D plot of the population variance at step 9.

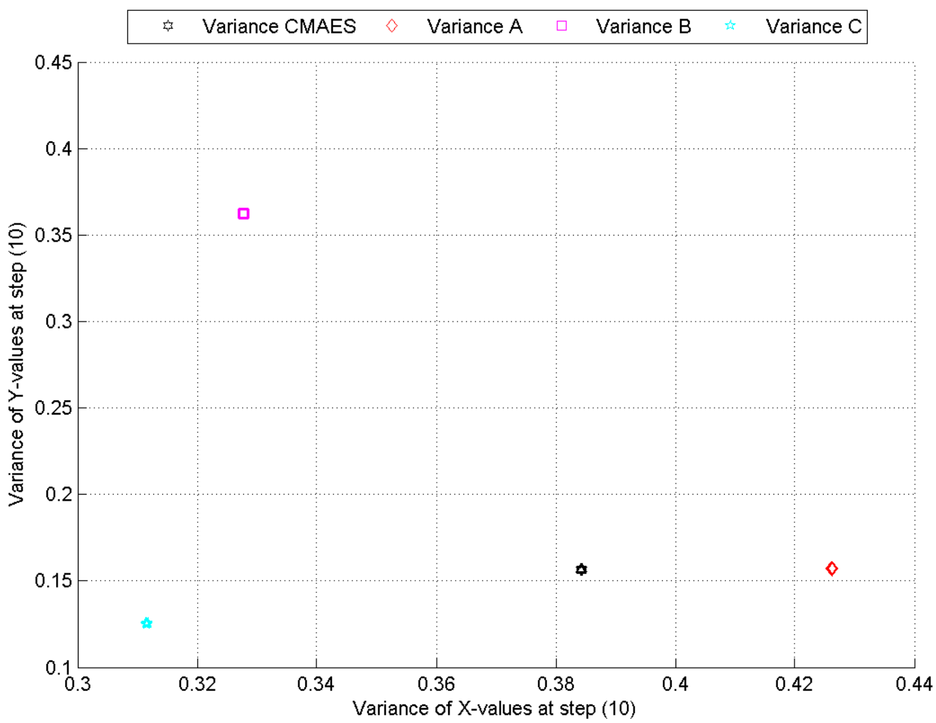

Figure C.10: 2D plot of the population variance at step 10.

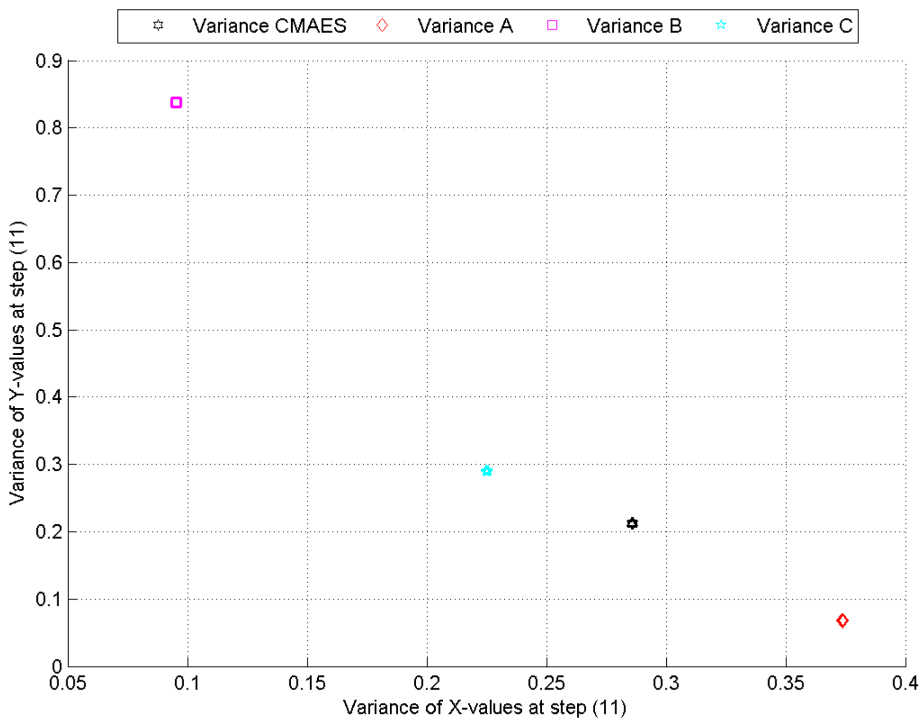

Figure C.11: 2D plot of the population variance at step 11.

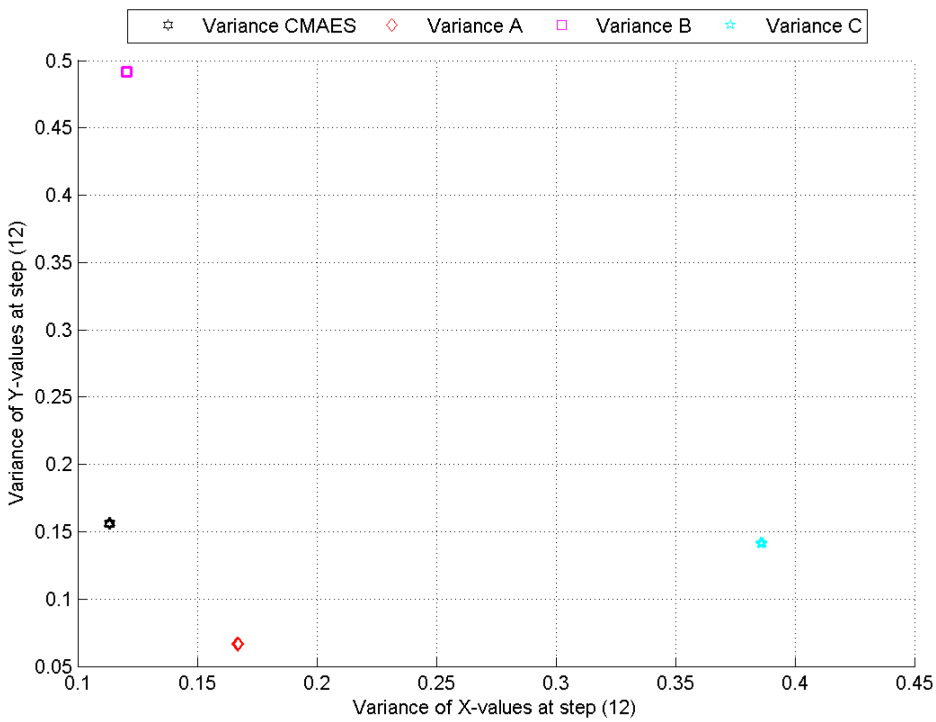

Figure C.12: 2D plot of the population variance at step 12.

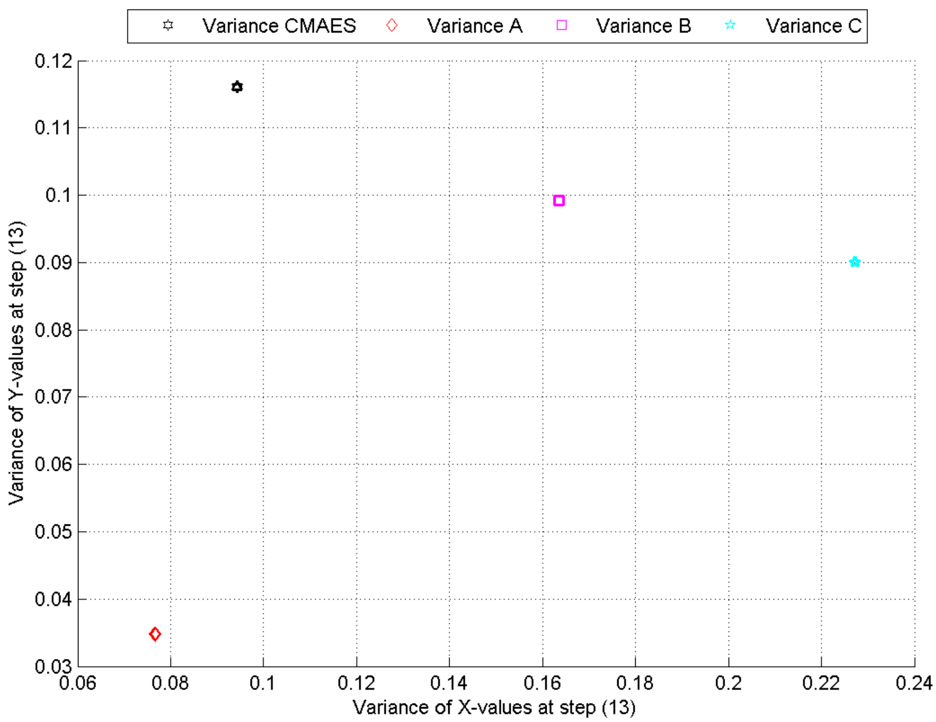

Figure C.13: 2D plot of the population variance at step 13.

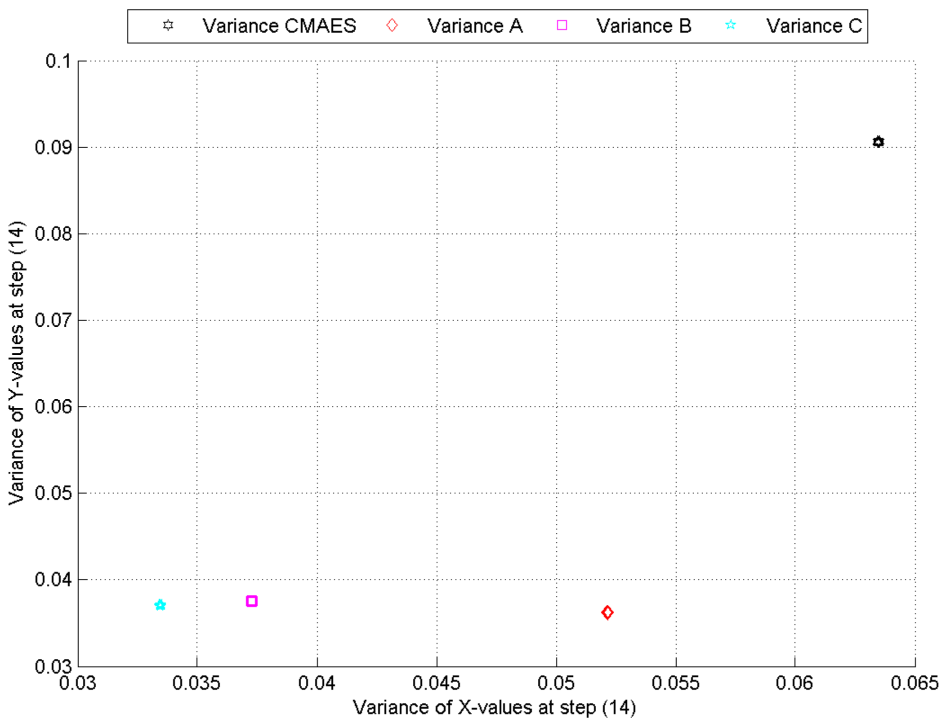

Figure C.14: 2D plot of the population variance at step 14.

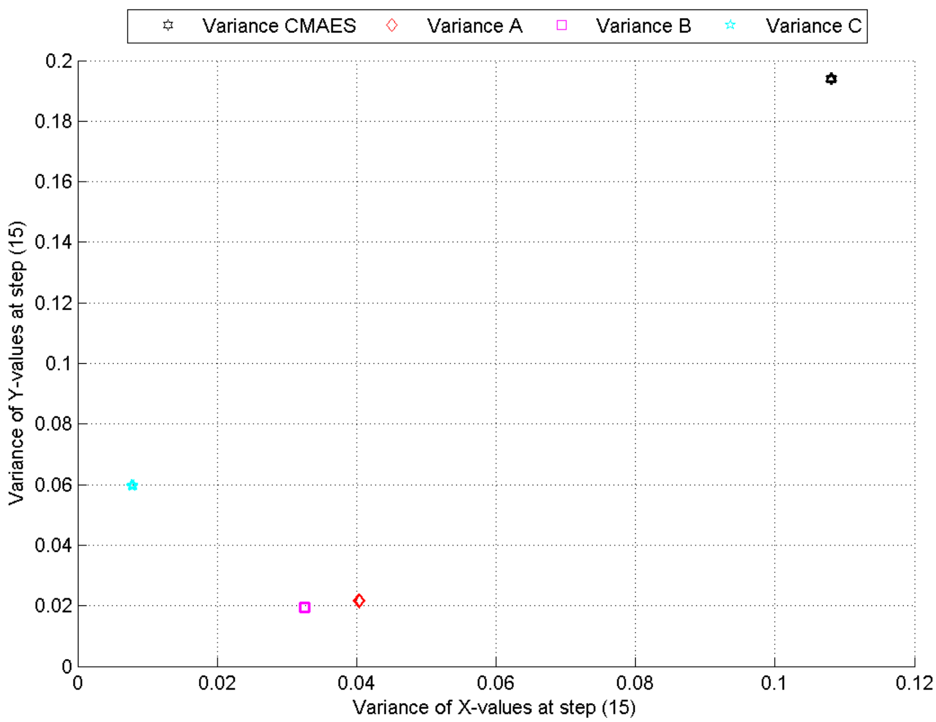

Figure C.15: 2D plot of the population variance at step 15.

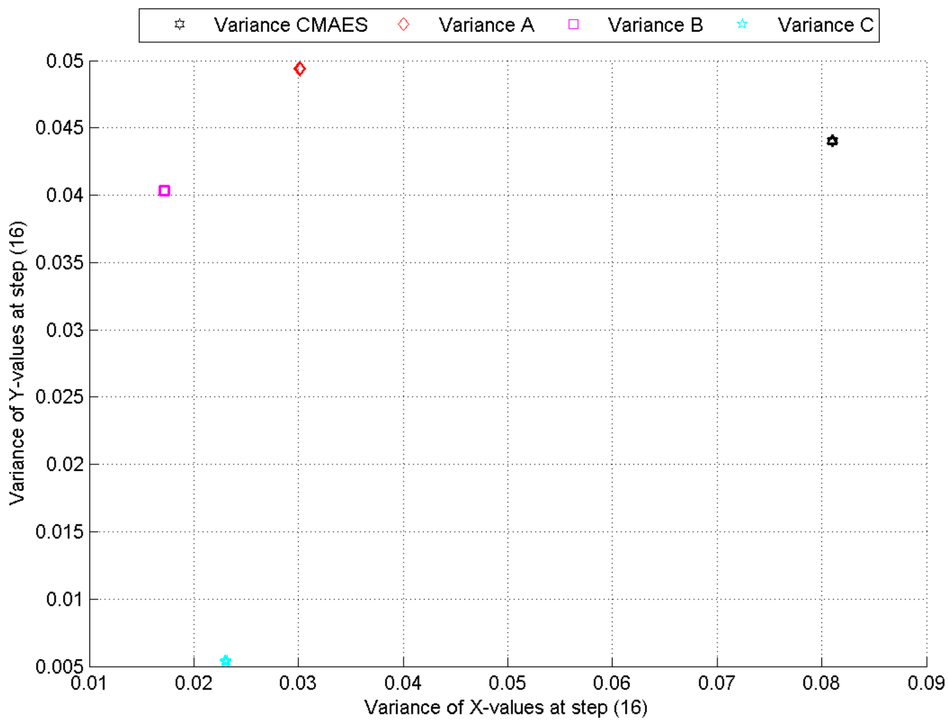

Figure C.16: 2D plot of the population variance at step 16.

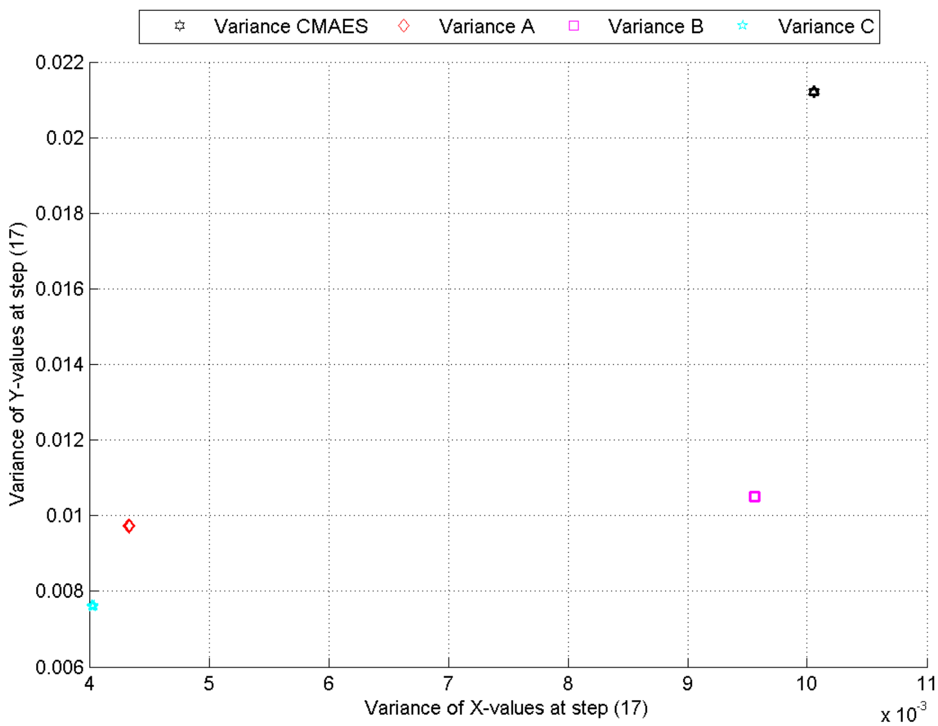

Figure C.17: 2D plot of the population variance at step 17.

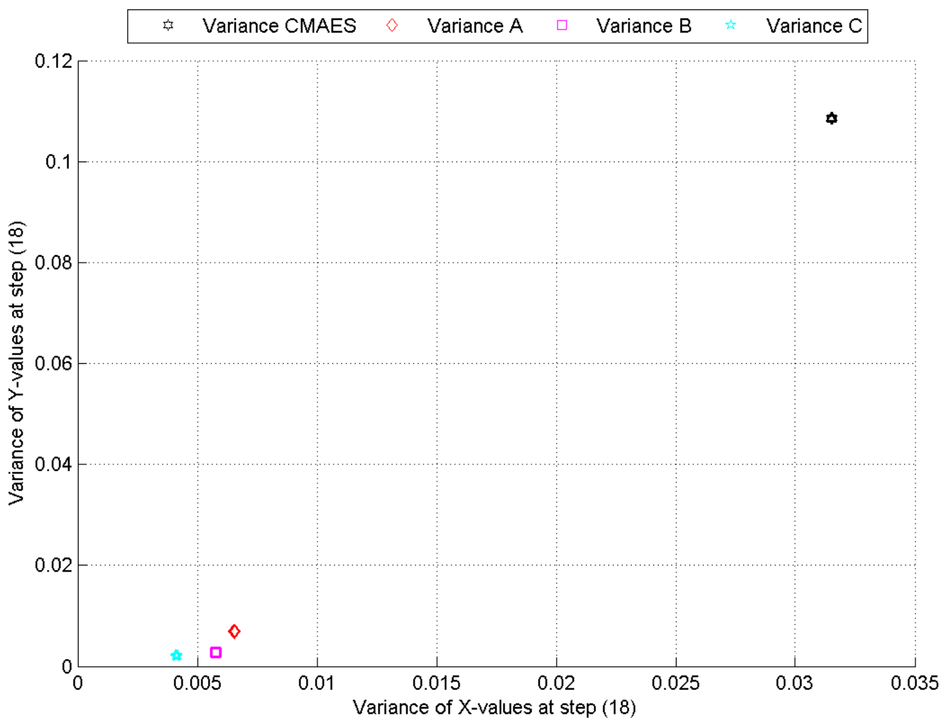

Figure C.18: 2D plot of the population variance at step 18.

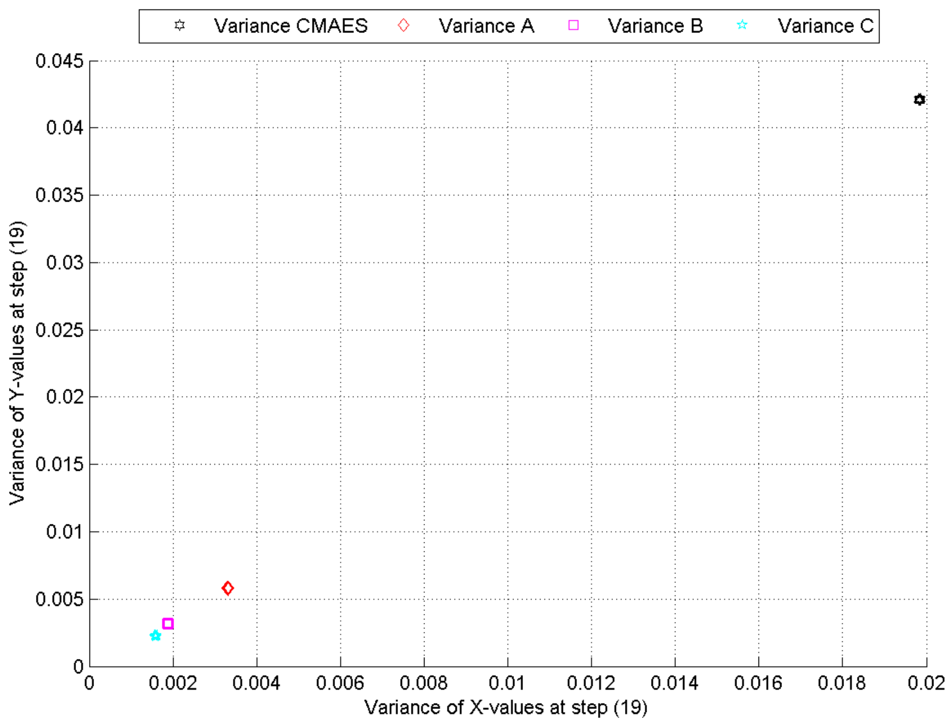

Figure C.19: 2D plot of the population variance at step 19.

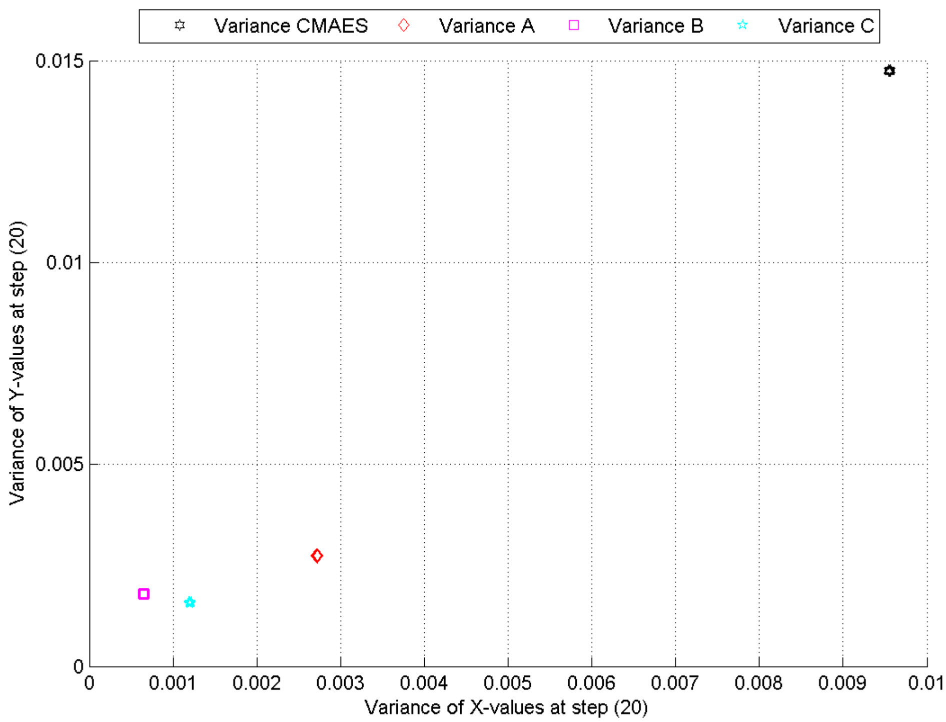

Figure C.20: 2D plot of the population variance at step 20.

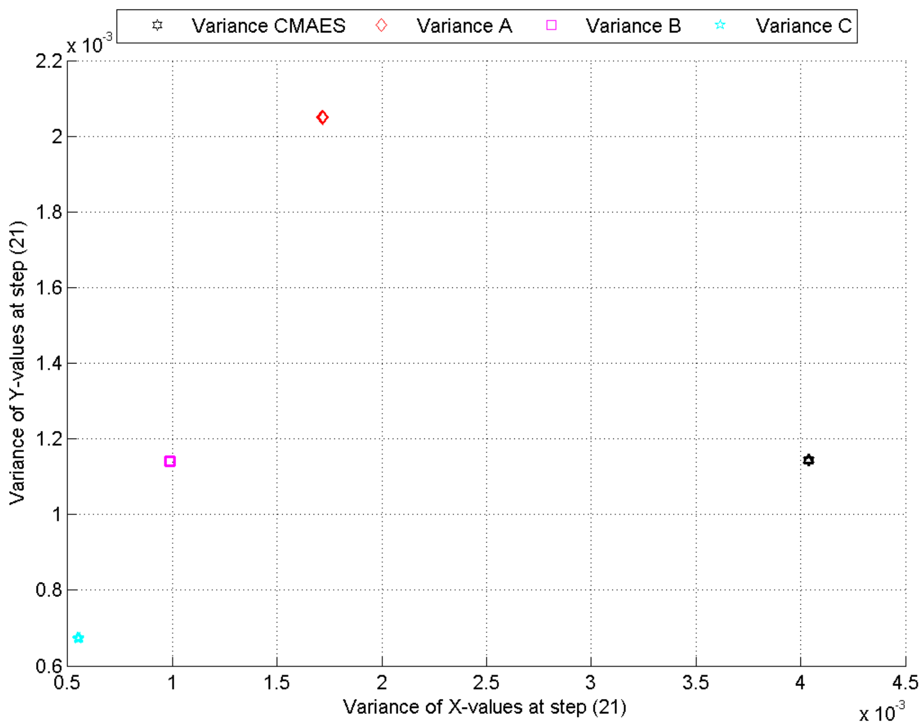

Figure C.21: 2D plot of the population variance at step 21.

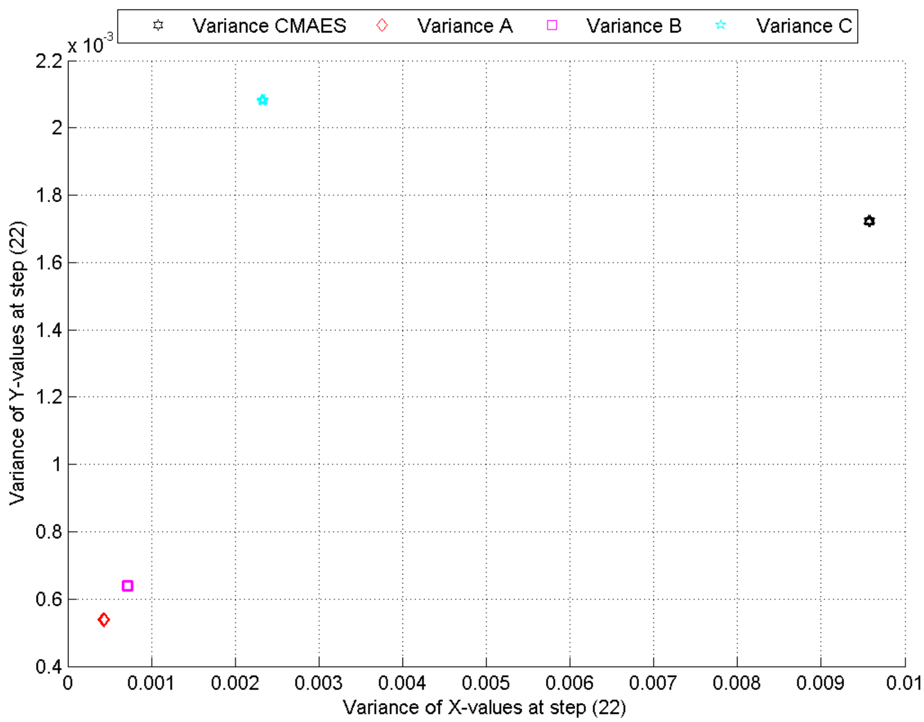

Figure C.22: 2D plot of the population variance at step 22.

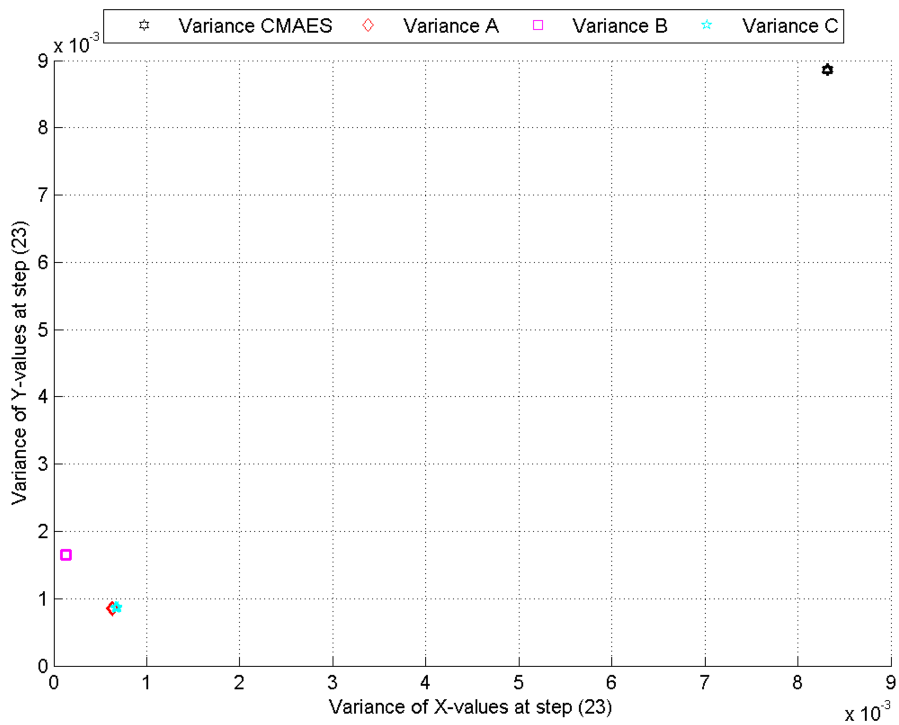

Figure C.23: 2D plot of the population variance at step 23.

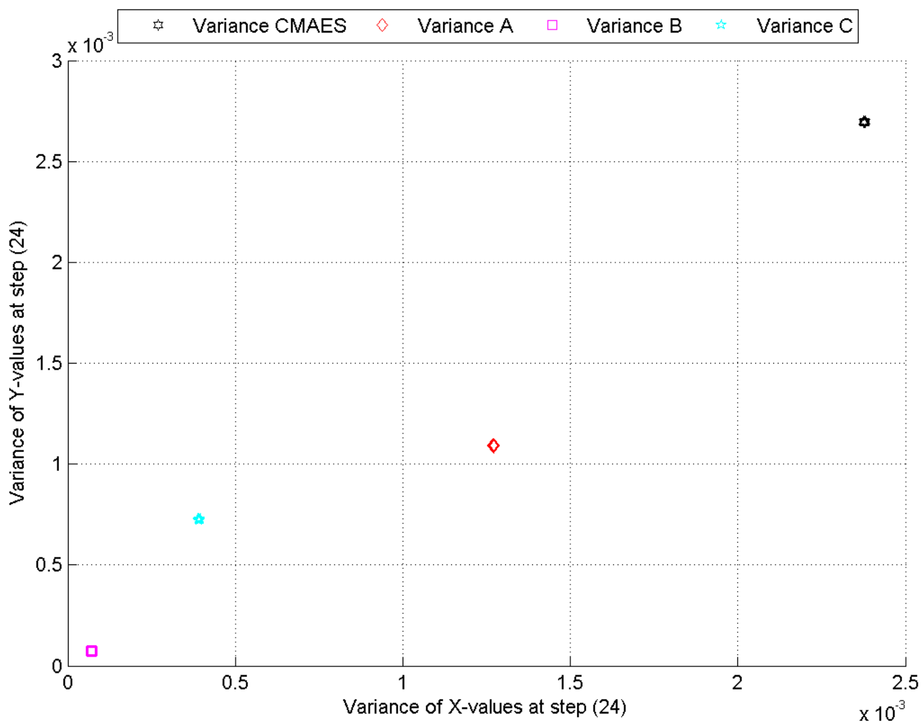

Figure C.24: 2D plot of the population variance at step 24.

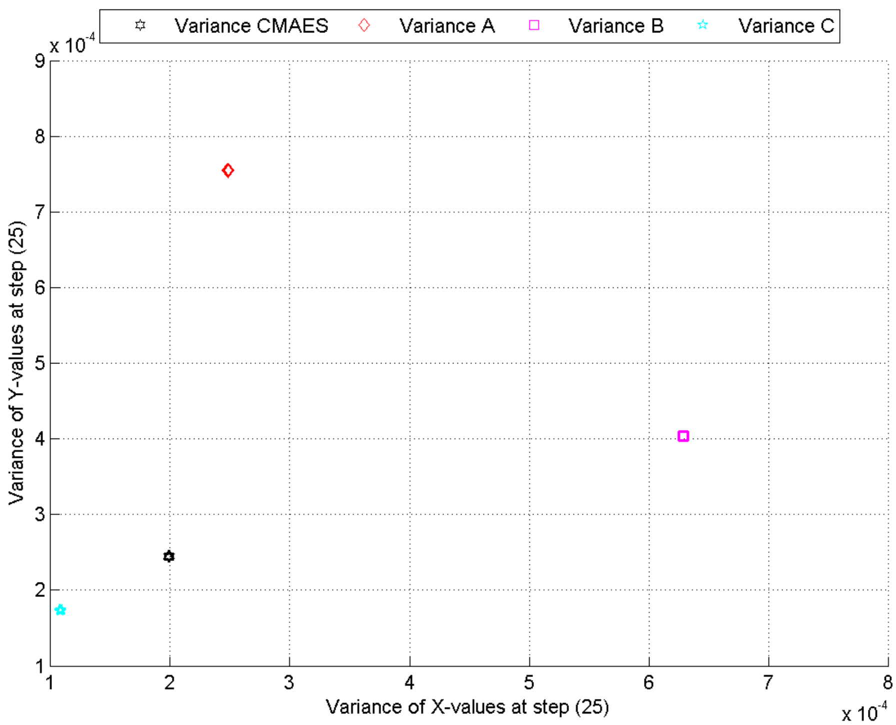

Figure C.25: 2D plot of the population variance at step 25.

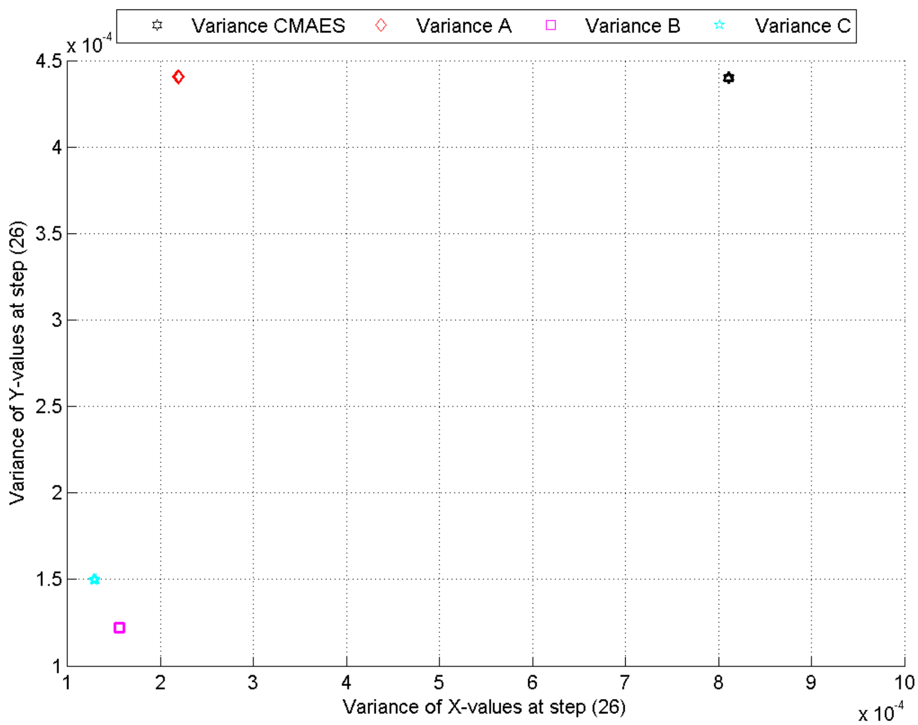

Figure C.26: 2D plot of the population variance at step 26.

Bibliography

[1] Arnold, D. V. Foundations of genetic algorithms: 8th international workshop, foga 2005, aizu-wakamatsu city, japan, january 5 - 9 , 2005, revised selected papers. 215–237.

[2] Auger, A. Benchmarking the (1+1) Evolution Strategy with One-fifth Success Rule on the BBOB-2009 Function Testbed. In *Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers* (New York, NY, USA, 2009), GECCO '09, ACM, pp. 2447–2452.

[3] Auger, A., Brockhoff, D., and Hansen, N. Investigating the Impact of Sequential Selection in the (1,2)-CMA-ES on the Noiseless BBOB-2010 Testbed. In *Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation* (New York, NY, USA, 2010), GECCO '10, ACM, pp. 1591–1596.

[4] Auger, A., Brockhoff, D., and Hansen, N. Mirrored Variants of the (1,4)-CMA-ES Compared on the Noiseless BBOB-2010 Testbed. In *Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation* (New York, NY, USA, 2010), GECCO '10, ACM, pp. 1559–1566.

[5] Auger, A., Brockhoff, D., and Hansen, N. Analyzing the Impact of Mirrored Sampling and Sequential Selection in Elitist Evolution Strategies. In *Proceedings of the 11th Workshop Proceedings on Foundations of Genetic Algorithms* (Schwarzenberg, Austria, 2011), ACM, pp. 127–138.

[6] Auger, A., and Hansen, N. A restart CMA evolution strategy with increasing population size. In *The 2005 IEEE Congress on Evolutionary Computation* (Edinburgh, Scotland, 2005), vol. 2, IEEE, pp. 1769–1776.

[7] Auger, A., and Hansen, N. Benchmarking the (1+1)-CMA-ES on the BBOB-2009 Function Testbed. In *Proceedings of the 11th Annual Conference Companion on Genetic and Evolutionary Computation Conference: Late Breaking Papers* (New York, NY, USA, 2009), GECCO '09, ACM, pp. 2459–2466.

[8] Bäck, T. *Evolutionary algorithms in theory and practice: evolution strategies, evolutionary programming, genetic algorithms*. Oxford university press, 1996.

[9] Beyer, H.-G. *On the asymptotic behavior of multirecombinant Evolution Strategies*. Springer Berlin Heidelberg, Berlin, Heidelberg, 1996, pp. 122–133.

[10] Beyer, H. G., and Deb, K. On self-adaptive features in real-parameter evolutionary algorithms. *Trans. Evol. Comp 5*, 3 (June 2001), 250–270.

[11] Beyer, H.-G., and Schwefel, H.-P. Evolution Strategies - A Comprehensive Introduction. 3–52.

[12] Beyer, H.-G., and Sendhoff, B. Covariance Matrix Adaptation Revisited — The CMSA Evolution Strategy —. In *Proceedings of the 10th International Conference on Parallel Problem Solving from Nature: PPSN X* (Berlin, Heidelberg, 2008), Springer-Verlag, pp. 123–132.

[13] Brockhoff, D., Auger, A., Hansen, N., Arnold, D. V., and Hohm, T. Mirrored sampling and sequential selection for evolution strategies. In *Parallel Problem Solving from Nature, PPSN XI*. Springer, 2010, pp. 11–21.

[14] Chen, J., Emmerich, M. T., Li, R., Kok, J., and Bäck, T. How to Do Recombination in Evolution Strategies: An Empirical Study. In *Methods and Models in Artificial and Natural Computation. A Homage to Professor Mira's Scientific Legacy*. Springer, 2009, pp. 223–232.

[15] Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *Trans. Evol. Comp 6*, 2 (Apr. 2002), 182–197.

[16] El-Abd, M. Black-box Optimization Benchmarking for Noiseless Function Testbed Using Artificial Bee Colony Algorithm. In *Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation* (Portland, Oregon, USA, 2010), ACM, pp. 1719–1724.

[17] Hansen, N. The CMA Evolution Strategy: A Tutorial. ArXiv e-prints, arXiv:1604.00772, 2016, 2005.

[18] Hansen, N. Benchmarking a BI-Population CMA-ES on the BBOB-2009 Function Testbed. In *Workshop Proceedings of the GECCO Genetic and Evolutionary Computation Conference* (Montreal, Quebec, Canada, 2009), ACM, pp. 2389–2395.

[19] Hansen, N., Auger, A., Finck, S., and Ros, R. Real-Parameter Black-Box Optimization Benchmarking 2010: Experimental Setup. Research Report RR-7215, INRIA, Mar. 2010.

[20] Hansen, N., Auger, A., Ros, R., Finck, S., and Pošík, P. Comparing Results of 31 Algorithms from the Black-box Optimization Benchmarking BBOB-2009. In *Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation* (New York, NY, USA, 2010), GECCO '10, ACM, pp. 1689–1696.

[21] Hansen, N., Finck, S., Ros, R., and Auger, A. Real-Parameter Black-Box Optimization Benchmarking 2009: Noiseless Functions Definitions. Research Report RR-6829, INRIA, 2009.

[22] Hansen, N., Finck, S., Ros, R., and Auger, A. Real-Parameter Black-Box Optimization Benchmarking 2009: Noisy Functions Definitions. Research Report RR-6869, INRIA, 2009.

[23] Hansen, N., Müller, S. D., and Koumoutsakos, P. Reducing the time complexity of the derandomized evolution strategy with covariance matrix adaptation (CMA-ES). *Evolutionary Computation 11*, 1 (2003), 1–18.

[24] Hansen, N., and Ostermeier, A. Adapting arbitrary normal mutation distributions in evolution strategies: The covariance matrix adaptation. In *Evolutionary Computation, 1996., Proceedings of IEEE International Conference on Evolutionary Computation* (Piscataway, New Jersey, 1996), IEEE Press, pp. 312–317.

[25] Hansen, N., and Ostermeier, A. Completely Derandomized Self-Adaptation in Evolution Strategies. *Evolutionary Computation 9*, 2 (2001), 159–195.

[26] Hansen, N., and Ostermeier, A. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation 9*, 2 (June 2001), 159–195.

[27] Hansen, N., and Ros, R. Benchmarking a Weighted Negative Covariance Matrix Update on the BBOB-2010 Noiseless Testbed. In *Proceedings of the 12th Annual Conference Companion on Genetic and Evolutionary Computation* (Portland, Oregon, USA, 2010), ACM, pp. 1673–1680.

[28] Igel, C., Hansen, N., and Roth, S. Covariance matrix adaptation for multi-objective optimization. *Evolutionary computation 15*, 1 (2007), 1–28.

[29] Igel, C., Suttorp, T., and Hansen, N. A Computational Efficient Covariance Matrix Update and a (1+1)-CMA for Evolution Strategies. In *Proceedings of the 8th Annual Conference on Genetic and Evolutionary Computation* (Seattle, Washington, USA, 2006), ACM, pp. 453–460.

[30] Jin, Y. A comprehensive survey of fitness approximation in evolutionary computation. *Soft Computing 9*, 1 (2003), 3–12.

[31] Jin, Y. Surrogate-assisted evolutionary computation: Recent advances and future challenges . *Swarm and Evolutionary Computation 1*, 2 (2011), 61–70.

[32] Jin, Y., and Sendhoff, B. A systems approach to evolutionary multi-objective structural optimization and beyond. *Computational Intelligence Magazine, IEEE 4*, 3 (2009), 62–76.

[33] Suttorp, T., Hansen, N., and Igel, C. Efficient covariance matrix update for variable metric evolution strategies. *Machine Learning 75*, 2 (2009), 167–197.

[34] Weisstein, E. W. Mean-value theorem. From MathWorld—A Wolfram Web Resource. Last visited on 14/07/2016.

[35] Weisstein, E. W. Taylor's theorem. From MathWorld—A Wolfram Web Resource. Last visited on 14/07/2016.