Identity and uncertainty in social influence: An agent-based approach

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***Abstract*—Computer simulations have been used to model psychological and sociological phenomena in order to provide insight into how they affect human behavior and population-wide systems. In this study, three agent-based simulations (ABSs) were developed to model opinion dynamics in an online social media context. The main focus was to test the effects of 'social identity' and 'certainty' on social influence. When humans interact, they influence each other's opinions and behavior. In recent years attention has been paid to how polarization and extreme opinions emerge within populations, including on social media. It was hypothesized that the influence of other agents based on ingroup/outgroup perceptions can lead to extremism and polarization under conditions of uncertainty. The first two simulations isolated social identity and certainty respectively to see how social influence would shape the attitude formation of the agents, and the opinion distribution by extension. They were designed to address the flaws of previous models of opinion dynamics, which were remedied to some extent, but not fully resolved. The third combined the two to see if the limitations of both designs would be ameliorated with added complexity. The combination proved to be moderating, and while stable opinion clusters form, extremism and polarization do not develop in the system without added forces.**

1. Introduction

Social influence on social media websites like Facebook has become a prominent topic as it seems to be a factor in swaying political opinion. Notably the injection of 'fake news' has been argued to contribute to the development of information disorders, such as the spread of disinformation, and the emergence and persistence of polarization and of extremism (Lazer et al. 2018, Lewandowsky et al. 2012). On Facebook in particular, information is disseminated differently from traditional media outlets, as it is negotiated by a network of "friends" with whom the user has some kind of relationship, so that one's social network will affect what information they are exposed to. Some studies have shown that Facebook networks, like real-world networks, can be highly segregated (Hofstra et al. 2017), contributing to the formation of small groups who communicate among each other with little or no exposure to contrasting opinions (so-called echo chambers), which compound the problem of the spread and circulation of misinformation. There is reason to believe, then, that social influence itself is a major factor in how information is distributed in this context.

Social influence is the process by which people adjust their opinions in some capacity based on their interactions with other people (Moussaïd et al. 2013). This study aims to explore social influence insofar as social identity and uncertainty contribute to it. The key aim of this study is to see if these two factors in a social media communication structure will affect the extent to which agents are vulnerable to social influence, which will in turn will affect how they form their attitudes[1]. Attitude formation is the process by which an individual goes from unstable, ambivalent or ambiguous attitudes about a certain subject to a stable opinion. Once an attitude is formed, it becomes the standard by which an individual uses to evaluate the attitudes of others (Sherif & Cantril 1945).

The way attitudes are distributed across a population can vary. Polarization occurs when two, often conflicting, attitudes are highly represented. Brexit is an important example of polarization, showing how these system-wide dynamics have real-world, large-scale societal implications. Conversely, consensus is when the population agrees on one attitude. An example of near consensus, according to the Pew Research Center, is that 95% of Americans think open and fair elections are at least somewhat important.[2] Heterogeneous distributions occur when people have many different opinions spanning across the spectrum, resulting in a diverse set of attitudes. For example, while many Americans believe in health care reform, there are a myriad of opinions as to the best way to enact it, according to a number of recent polls.[3] These system-wide dynamics are what will be examined in this paper as possible attitude distributions.

Humans form groups based on their social identity. In this study, social identity is operationalized as the set of groups an individual subscribes to, and includes demographic traits like gender, race, and nationality but also cultural traits such as ethnicity, religion, and political affiliation (cf. Abrams et al. 2006). It is assumed that group structures affect how information is distributed. Therefore social identity is used as a variable to see what effect it has on system-wide opinion dynamics. Uncertainty refers to the confidence with which an agent holds an opinion, and it is shown to be affected by group membership (Smith et al. 2007, Hogg et al. 2007, Hogg et al. 2012). Group membership is an important concept driving social influence, because people are more likely to be influenced by those who they consider to have the same group membership as themselves, or their ingroup. Conversely those who identify as a different social category are considered

[1] In the literature, 'attitude' and 'opinion' are often used interchangeably.

[2] Pew Research Center, March, 2017, "Large Majorities See Checks and Balances, Right to Protest as Essential for Democracy",https://www.people-press.org/2017/03/02/large-majorities-see-checks-and-balances-right-to-protest-as-essential-for-democracy/

[3] Gallup, May, 2019, "Americans' Mixed Views of Healthcare and Healthcare Reform", https://news.gallup.com/opinion/polling-matters/257711/americans-mixed-views-healthcare-healthcare-reform.aspx

outgroup members and are less influential (Abrams et al. 1990).

While some models have combined uncertainty and social identity (Grow et al. 2011), the context of their social interactions are dyadic (an interaction between two agents), unlike online social networks. Multiadic communication (one agent communicating to many other agents at once), which is how information is shared on Facebook, has not been extensively studied. While fewer studies have modeled online social networks (Quattrociocchi et al. 2014, Madsen 2018), they have not take into account the specific factors studied here. Furthermore, simulations of extremism and polarization often insert extremist agents into the population, suggesting that extremism does not arise from the same cognitive motivations as the rest of the population (Deffuant 2006, Madsen 2018). In this study, it is assumed that this is not necessarily the case: it is tested if extremism can arise from these models without inserting a few agents who perpetuate it with unique behaviors.

The problem of misinformation is widespread and arguably very dangerous for society. It is estimated that the average American encountered between one and three fake articles daily in the month before the 2016 presidential election, with the vast majority reported being seen on Facebook (Lazer et al. 2018). The fact that Russia has used Facebook as a propaganda tool for political influence demonstrates the severity of the problem and the great need for research into helping to understand the dynamics of how information influences peoples' attitudes. By doing so, we can develop counter strategies to these misinformation campaigns.

The reasons people hold opinions are complex, but it is never in a vacuum. The models developed here focus on social identity driving ingroup/outgroup perceptions of agents, which then influence agents attitudes and attitude certainty (Grow et al. 2011), in order to see how they affect how information is shared on Facebook. Following, Section 2 will provide context for this work in terms of previous agent-based models of opinion dynamics. Subsequently, section 3 will give an overview of the present study while sections 4, 5 and 6 will describe the three models developed for this study in detail. Finally, section 7 is a general Discussion & Conclusion of the findings from these three models.

2. Background Research

2.1. Modelling Opinion Dynamics

The typical way of modelling opinion dynamics in ABMs is using a continuous opinion model, where opinions are represented on a continuous scale (say, between 0 and 1), and the similarity between any two opinions is defined by how close they are on the continuum. This allows for social influence by agent's pulling (or pushing) each others opinions along the spectrum through interaction according to the rules of the model. This continuum represents moderate opinions in the center, and extremist views on either end (Duggins 2014, Flache et al. 2017).

When combining social influence and opinion dynamics, these models have four potentials for distributing opinions: consensus, polarization, strong diversity or weak diversity. As mentioned, consensus is agreement on one opinion, and polarization on two opposing opinions. Strong diversity refers to the representation of many opinions along the spectrum, and weak diversity is so-called "opinion clustering", where only several opinions are represented (Duggins 2014).

2.2. Abelson Diversity Puzzle

The fundamental problem with this type of representation is the so-called Abelson's Diversity Puzzle, which says that social influence represented on a spectrum with opinions being pulled towards each other will always lead to consensus unless there are perfectly separate agents who enact zero influence on one another (Abelson 1964, DeGroot 1974). In a highly connected world it is unreasonable to assume that there are entirely isolated groups of individuals who receive no influence from other groups (Mäs et al. 2010), so there must be another explanation for the persistence of a diversity of attitudes in connected networks like Facebook.

Furthermore, consensus on many issues in large, diverse groups is empirically not how it works. While consensus can happen, and is important in certain situations (such as discourse among experts), when a large diversity of perspectives is involved, it can be very difficult to come to (Bohman, 2007). We can see this with the difference between the scientific consensus and the public opinion on evolution in the United States. According to Pew Research, in 2009, while 97% of scientists agreed on the theory of evolution, only 1/3 of the public did, with the other 2/3 believing in some form of creationism.[4] Different motivations and perspectives can yield different beliefs about scientific fact, which if everyone was motivated by rationalism, would lead to a consensus among the population. Consensus can even be undesirable if it was not formed based on mutual interest and rational deliberation, because people are subject to cognitive errors. Heterogeneous systems can eliminate cognitive errors in attitude formation by drowning out possible biases of judgement (Bohman, 2007). This is particularly salient in the communication structure of social media, where people receive information from many different sources whose reliability is questionable. Because these models are trying to simulate Facebook, where people of different social identities often do not come to a consensus on their opinions about a particular issue, the Abelson problem is one which must be reckoned with.

2.3. Solutions in Modelling

The most prominent and perhaps successful solution to this problem is the bounded confidence model (Hegselmann & Krause 2002, Deffuant et al. 2000). Bounded confidence models assign 'boundaries' between what agents can be influenced by who and in what direction. Agents have an opinion and a threshold (the 'bound of confidence') on either side of their opinion, where if another agent's opinion is within this threshold, then it can be influenced, if it is outside, it can no longer be influenced. Relative Agreement Models are

[4] Pew Research Center, July, 2009, "Evolution, Climate Change and Other Issues", https://www.people-press.org/2009/07/09/section-5-evolution-climate-change-and-other-issues/

an augmentation on this, where the amount of agreement between agents will determine the extent of the influence, and agents with lower thresholds (equated with less "uncertainty" surrounding their opinion) will proportionately have more influence in the model (Deffuant et al. 2000, Meadows et al. 2012). This is taken to be a more faithful representation of real influence, because influence is proportional to the certainty of that agent (and not a binary only taking account the distance of opinion), so that confident agents can be more convincing despite how different their opinion is from a less certain agent (Meadows et al. 2012).

There are two major issues with these models. Firstly, if there is even a slight probability that an agent will influence another agent outside of its bound of confidence, the system degrades to consensus (Figure 1) (Kurahashi-Nakamura et al. 2016). Secondly, the clustering of agents are a mathematical necessity determined by their initialized distance from each other and agents only interact on the basis of this distance, which is unrealistically oversimplified even for a reductive model of human behavior.

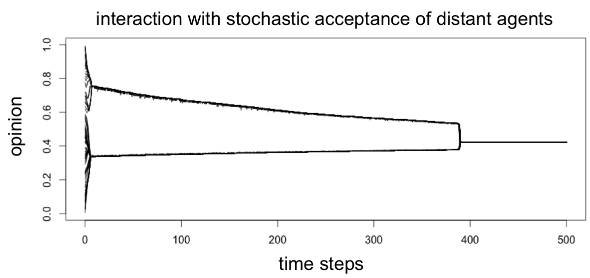

Fig. 1. Probability of acceptance outside of bounds of confidence of .0001 will eventually lead to consensus (from Kurahashi-Nakamura et al. 2016).

Another approach is to add a disintegrating force which pulls agents away from assimilating towards one another. Two theories of why this could occur are social distancing, where agents move their attitudes away from dissimilar others (Mäs et al. 2014), and optimal distinctiveness, where agents strive to be unique when there are high levels of similarity in a population (Smaldino et al. 2012). Individuation is often modelled using noise, whereby an agent will change its attitude from other agents with a certain probability. If these are totally randomized (Kurahashi-Nakamura et al. 2016), stable clusters will evolve over time. However, while the clusters themselves are stable, no particular agent holds a stable opinion, because they are all vulnerable to random opinion change (Figure 2). As attitude stability is related to attitude strength, this model presumes all agents attitude strength be dynamic, and does not account for social factors which could affect attitude stability (Petty 2014, Tormala 2016).

Following the idea of individuation, optimal distinctiveness is modelled where agents change their opinion as a function of the amount of agents who hold a certain opinion. Here, clustering occurs but they are dynamic (Mäs et al. 2010, 2014). While agents maintain their opinion consistently until the pressure to individuate is great enough, which in a micro (individual) level is more consistent than the previous system, the distribution of opinions is constantly changing. Macro-level attitude stability is not represented in this construct, and the clustering amounts to a macro random-walk (Figure 3).

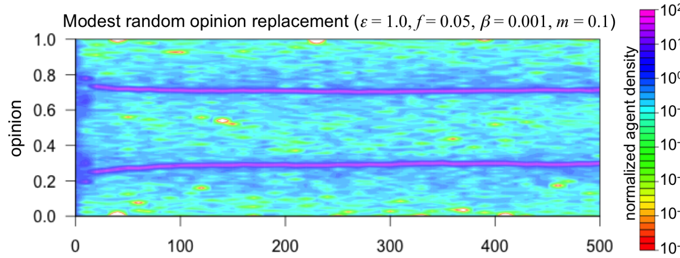

Fig. 2. Opinion Noise: Stable clusters result but any agent has probability (m) of changing it's opinion, which is the same for all agents (from Kurahashi-Nakamura et al. 2016).

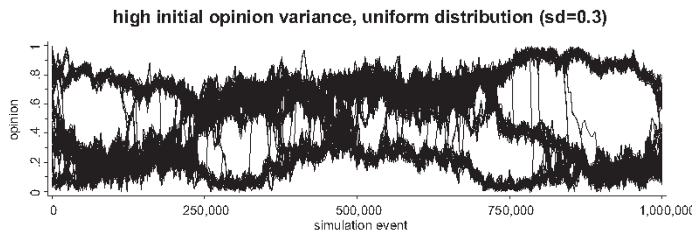

Fig. 3. Opinion noise: sufficient pressure (too many agents hold similar opinions) will trigger disintegration, leading to dynamic opinion changes which never stabilize (from Mäs et al. 2014).

3. Present Study

The models described here are also models of social influence, but social influence is mediated by social identity and certainty. This is conceptually driven by the idea that attitudes are embedded in a social context, and that people base their attitudes around their social ties (Hogg et al. 2007). Furthermore, their susceptibility to influence is mediated by how certain they are, with less certain agents being more vulnerable to changing their opinion (Tormala 2016), and social cohesion facilitating certainty (Petty et al. 2014).

Three models were developed for experimentation. In the first model, instead of agents forming groups because of attitude proximity (as with the BC model), they will form groups based on similarity of social identity, following the identity repertoire construct (Lustick 2000). The second model takes the BC model as is, but uses certainty as a negotiator for group formation as well as stochastic noise, to see if this affects the mathematical rigidity of the original model. Finally, the two models are combined to see if a combination of them creates a more faithful representation of attitude formation, and see what tweaking the parameters of this system results in. If it is possible for stable opinion clustering to form (that is, a heterogeneous distribution) given the Abelson Diversity Problem, can extremism or polarization be modelled by the design of these models given the variables in question? Each model is discussed separately in sections 4, 5 and 6, including their design choices, assumptions and results of experimenting

TABLE I
MODEL FEATURES

	Bound of Confidence	Noise	Meaningful Inter-Agent Ties	Micro Cluster Stability	Macro Cluster Stability
Hegselmann & Krause	yes	no	no	yes	yes
Kurahashi-Nakamura	no	yes	yes	no	yes
Mäs	no	yes	yes	yes	no
Model 1	no	no	yes	yes	yes
Model 2	yes	yes	no	yes	yes
Model 3	yes (random)	yes	yes	yes	yes

Comparing the models presented here with previous work described. Micro cluster stability refers to the consistency of each individual agent, so that the overall clusters are not stable but agents are not subject to randomly changing their opinion across the spectrum. Macro cluster stability refers to the stability of the clusters themselves over time, despite that some agents may change their opinion at random. Meaningful inter-agent ties refers to group formation occurring outside of mathematical necessity which is the fallback of the Hegselmann Krause model (discussed above). Other models remove the bound of confidence but fail to achieve micro and macro cluster stability. Model 3 does use a bound of confidence, but as it is random it allows for clustering which would not be possible under the typical bounded confidence conditions (Figure 1).

with each, followed by an overall Discussion & Conclusion in section 7.

4. MODEL 1: SOCIAL IDENTITY

This model relies conceptually on the idea of in-group/outgroup perceptions, where an agent can only be influenced by another agent if they are perceived of as their ingroup. The major challenge in this construct is that, on average, if the agent's ingroups are too broad, and contain too many agents, there is too much influence and consensus will occur. Conversely, if the agent's ingroups are too small, no agent will be influenced and anomie (each agent maintains their own opinion and it does not change) will result. Furthermore, if the agent's criteria for someone being a member of their ingroup is that they share all of their traits in common, consensus will occur among those agents who share all traits. In other words, each 'type' of agent has a singular opinion.

What is being manipulated here is how many identity dimensions agents are comparing themselves on (e.g. they only consider gender, or gender and religion, when determining their ingroup), and how many possible identities exist within these dimensions (e.g. three possible religions). The combination of these two factors determines the composition of the population, and therefore how diverse it is.

The goal here is to see if there is some combination where ingroup sizes will facilitate clustering, but not into groups of agents who share all traits. The major questions here are:

- What population composition will result in stable opinion clustering? That is, what are the diversity requirements for the population as a whole?
- What ingroup requirements (if any) will result in stable opinion clustering? How closed/open must agents be in determining their ingroup?

4.1. Design

Each agent has a set of identity traits referred to here (and in the literature) as their 'identity repertoire' (Lustick 2000). In this experiment, this repertoire is a set of arbitrary length, and the length of the set affects the composition of the population. Larger identity repertoires, and more options within each identity dimension will lead to a more diverse population. This repertoire is the same length for every agent, and it corresponds to the identity dimensions mentioned previously (the set of identities agent use to compare each other). If the identity repertoire length is 3, this could theoretically correspond to gender, race, and religion.

Within each identity an agent has a corresponding category (e.g. Christian/Muslim/Jewish), which is indicated as a discrete integer. This means that if two agents share an integer on one dimension, they are of the same category on this dimension, but for the purposes of this model, they cannot switch. The larger the repertoire and the more possible 'types' within each repertoire, the more possible combinations for an individual agent. For example, consider a population which has an identity repertoire of 2 (they compare themselves on 2 dimensions) and each dimension has 2 categories (0 or 1). This basic combination means that there are 4 possible types: 00, 01, 10, 11. Agents in this construct may share no traits in common (00 and 11), one trait in common (00 and 01), or all traits in common (00 and 00). Whether or not an agent considers another agent their ingroup is defined by how many traits they share in common, which is also a variable named the 'similarity threshold'.

The model is fully connected to the extent that each agent is exposed to the attitude of any other. On each time step, a random agent is chosen to 'broadcast' it's opinion, which is then received by all agents in the network. If this agent is in a particular agent's ingroup, it will be influenced by this agent to some degree, k_u, the 'influence factor'. If x is an agents attitude and x' is the influencing agents attitude, the change in the agents attitude, Δx, is calculated as follows:

$$\Delta x = x + k_u|x' - x| \quad (1)$$

Where x moves towards x' by the difference between x and x' times k_u.

The influence factor k_u is a modified version of Deffuant et al. (2006) which includes the uncertainty of the influencing agent (which will be used in Model 2) and is calculated as follows:

$$k_u(x, x', u, u') = (1 - u')(e^{-(x - x'u)^2}) \quad (2)$$

Where u is the agents uncertainty and u' is the influencing agent's uncertainty.

This equation moderates the degree to which an agent will go towards another agent's opinion. If the agent is very certain, k_u will be smaller, and the more quickly the graph of possible influence given the difference between the two attitudes will go to zero. Also, the larger the distance between the two agent's attitudes, the faster the equation goes to 0 generally.

This basic formula will be used throughout the models, however as mentioned this particular model does not take uncertainty into account as a variable. For these simulations, both u and u' will be set to .5 for all agents and will not be subject to change as a result of influence. The equation is then:

$$k_u(x, x') = .5(e^{-(x-x'.5)^2}) \tag{3}$$

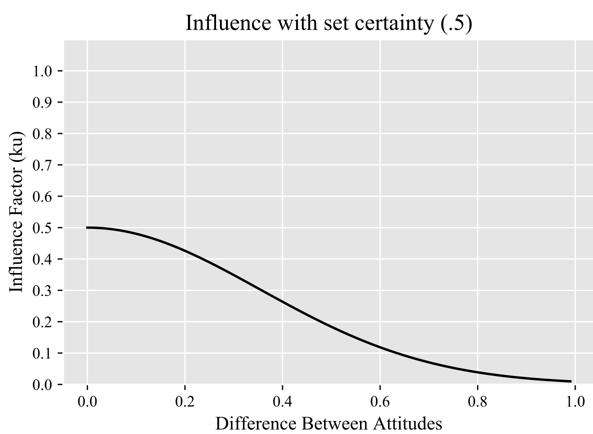

Fig. 4. Influence when certainty is set to .5.

Figure 4 shows that agents will go half the distance towards the other agent as their opinion differences approach 0. As certainty never changes, this functions to mediate the distance towards another agent's opinion an agent will go on any interaction, where agents who are farther away on the opinion spectrum will be influenced less, which is consistent with principles of opinion formation like social judgement theory (Sherif et. al 1965) and other models of opinion dynamics, including bounded confidence (Hegselmann & Krause 2002). This is also consistent but a somewhat modified version of many influence models, who either use an average of the two opinions or a coefficient which mediates the degree of influence proportionately (see Deffuant et al. 2006).

In order to maintain the integrity of the model conceptually (in terms of the Abelson Diversity Puzzle), agents who are under no chance of influence are altered. That is, if an agent does not share enough similarities with any agent to consider them the ingroup (and therefore are immune to social influence), their similarity threshold is lowered until they are ensured to have at least one ingroup member. The result is a shift in the distribution of agent similarities from a normal distribution, which happens with a single similarity parameter, to a linear distribution (Figure 5).

This is important for two reasons. First, the distribution of nodes in social networks affects how influence is distributed (Stocker 2002, León-Medina 2019). Many social networks are known to have a power relationship (Madsen 2018). That is, there are a few well connected nodes, but the majority of nodes have much fewer connections on average, known as a scale-free network (Figure 7). Given the constraints of the design, this study was unable to duplicate the conditions by which a real power distribution can be achieved. However, the linear distribution is an approximation which is facilitated by the identity repertoire model. Because of the model design, as long as the relationship between what agents consider their ingroup reflects the power distribution more than the normal distribution, opinion clusters result while still maintaining a connected network. Secondly, this redistribution results in a network which looks more like an actual online social network (Mislove et al. 2007). Figure 6 shows a typical Facebook network which results from scale-free networks, where may people have a small number of connections and a few have a dense number.

Note that the information being exchanged is random and equal, that is, no particular agent takes precedence or has more opportunity to broadcast, so that the resulting clusters could be compared to an unbiased system. This is important for further experimentation in injecting other variables, but also in seeing to what extent ingroup/outgroup perceptions contribute to clustering on their own without positive feedback loops.

4.2. Results

When agents must share all traits in common to be considered an ingroup, stable opinion clusters occur. They are essentially small consensus islands whereby each type of agent is excluded from influence from any agent who does not share all of their traits. This is not only an artificial barrier which does not contribute anything new to the opinion dynamic problem, but also empirically people do not agree categorically with others who share all of their identity traits. That is, the overall opinion landscape is not divided into subgroups who entirely agree with people who are exactly like them, and are unable to be influenced by those who are not, the reality is a bit more nuanced. However, as with the Abelson Diversity Problem, in populations which are not sufficiently diverse, if agents consider anything less than sharing all traits in common, the population will converge to consensus (Figure 8).

The solution to this, then, is to increase the identity repertoire and the complexity of each dimension, and to find the optimum number of traits by which agents compare each other and see what the resulting opinion clusters are. There are only a few scenarios which create any semblance of a reasonable amount of clustering, or a balance between consensus and complete anomie (Figure 8, 9). The diversity has to be large enough whereby there are no 'types' for agents to separate into, so that they form groups with others based on overlapping, uncorrelated traits.

The problem with this system is that it is not realistic. Having one similarity threshold for basically the entire population

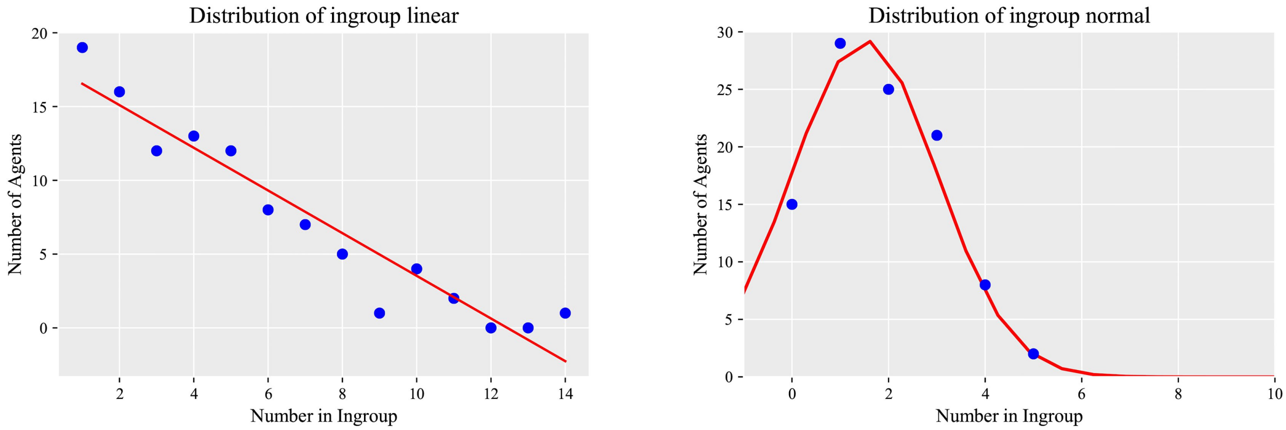

Fig. 5. Distribution of the amount of connections for each node.

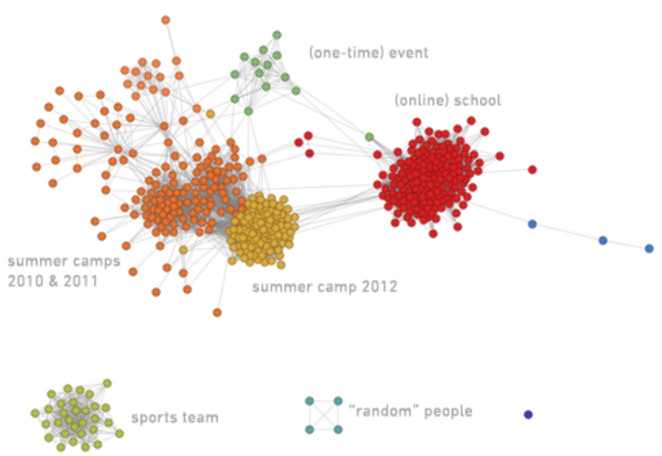

Fig. 6. One particular Facebook network, has clusters of nodes with small connections to other clusters of nodes.

is not how people identify their ingroups, some people are more or less open than others. There are no 'rigid' rules as to how people choose to identify with each other, and on what grounds. If the amount of similarities is loosened in either direction, or the threshold is randomized, the result is either anomie if it is too rigid, or consensus if it is too open or random.

It is hoped that by adding other variables, the rigidity can be loosened and a more realistic system develop. There are many other factors which could affect how influence works are not taken into account in this model. For example, conformity is the tendency for humans to be influenced by large groups of people who agree with each other (Asch 1956). Here, agents will be influenced equally by anyone in their ingroup, whether or not other members of their ingroup also agree with them. If agents were influenced proportionally to the amount of their ingroup who hold an attitude, this would make this model less simplistic. Therefore, it is encouraging that at least under very limited circumstances, identity and affiliation itself can have some effect on stable opinion clusters, supporting the assumption that social identity explains some of the many possible factors in attitude formation.

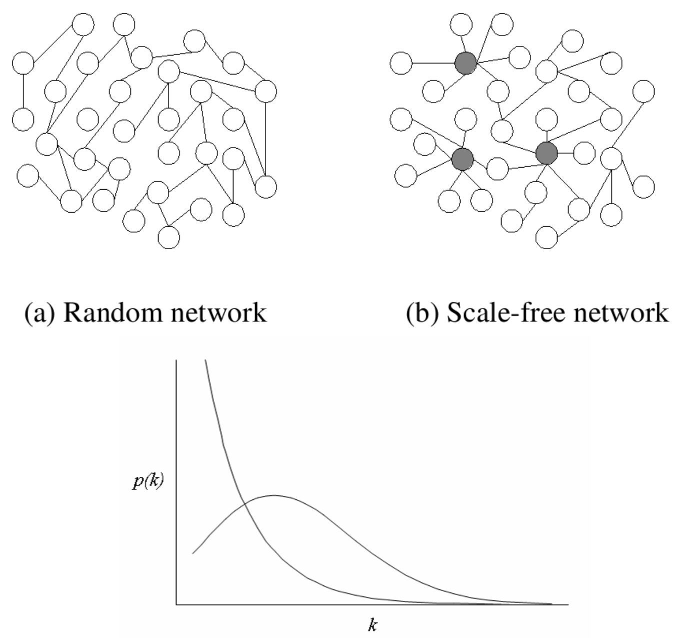

Fig. 7. (Top) Scale free network vs random network. (Bottom) Node distribution of scale free (power) and random (normal) networks, compare with Figure 5.

5. Model 2: Certainty

This model is based directly on the bounded confidence model, but this study does not claim to resolve the problems with the BC system, where small random amounts of acceptance outside the threshold creates consensus, as with the Abelson problem. Instead, it is to modify the Bounded Confidence construct, which by design deterministically has agents cluster by nearest 'acceptable' neighbors, creating stable opinion clusters as a mathematical necessity. By introducing certainty, it is hoped that the diversity of sources of information circulating in the system will affect the quality of these clusters to create a more realistic set of opinion dynamics. "More realistic" means specifically:

- A system where the diversity of information being circulated affects the overall certainty of the system, and the

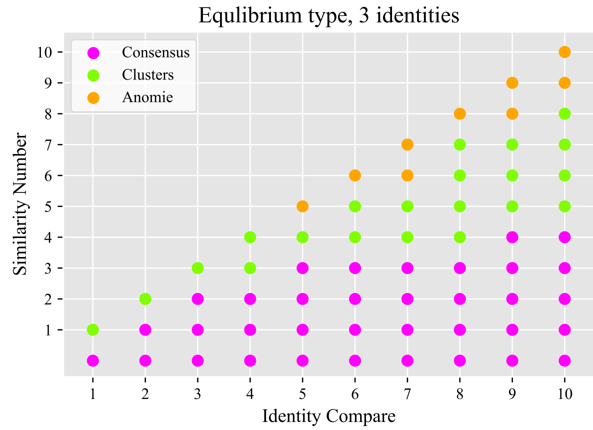

Fig. 8. Depending on the amount of identities, the similarity number must be a bit higher than 50% to avoid monoculture.

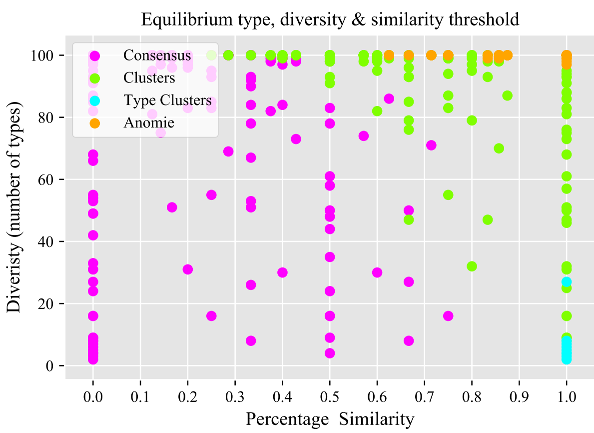

Fig. 9. Clustering occurs when the diversity is higher, and the requirement for similarities is relatively high. Type clusters occur at strict similarity requirements (100%), with low levels of diversity. Typically, similarity requirements below 50% will lead to consensus, although requirements as high as 75% can lead to consensus in low diversity populations.

length of time for the system to stabilize.
- A system which agents do not cluster according to their "uniform" distribution as with BC models.

The certainties of the agents will be negotiated by the source of the information being broadcast (whether it is from their ingroup or their outgroup), so that it is the exposure to information which makes an agent more or less certain (Visser et al. 2004). The reason this is important in studying social influence in identity is that in moments of uncertainty, people default to the opinions of others (Smith et al. 2007).[5] This tendency facilitates misinformation, because when an individual defaults without question, their beliefs can be reinforced by others regardless of the validity of that attitude, or the consequences of believing it (Moussaïd et al. 2013).

[5]Classical studies in psychology have also long confirmed this tendency. See Sherif(1936) for social norms, Festinger(1954) for social comparison theory, Asch(1956) for conformity, Schacter(1959) for affiliation and Turner & Hogg(1987) for social categorization theory. For a summary see Smith et al. (2007, pg 770).

While there have been many opinion models based on certainties of the agents and how this affects influence, the results are not emergent properties of individual agents who have the same "cognitive" motivations, but manufactured results from specifically designed populations. For example, many studies on extremism seed their populations with "moderates" and "extremists" (Deffuant 2006, Madsen 2018). These extremists have a higher certainty than moderates, which ultimately ends in polarization. This logically follows from the rules of the diversity problem, because agents with higher certainties will draw less certain agents, and those who are closer to this agent on the spectrum will categorically be influenced by it, resulting in polarization. So finally, this model hopes to see if a system where all agents have the same set of tools for influence (i.e. no agents are "more certain" by makeup) can result in stable majority/minority opinion clusters, or indeed extremism or polarization.

5.1. *Design*

In this model, agents still broadcast their opinion at random, but their opinions can change randomly based on their certainty. Certainty is a number between 0 and 1 which describes how committed the agent is to the opinion it holds. Low certainties allow for a greater likelihood of random opinion change, or noise.

Two principles are borrowed from Grow (Grow et al. 2011) which are drawn from psychological research and used in their model on certainty and social influence:

1) Certainty is inversely related to the ability to be influenced.
2) Certainty is directly related to the amount of agreement among peers (social consensus).

Equation 1 ensures that agents who are more certain will be less influenced by agents whose opinion is farther from them on the spectrum, thereby fulfilling principle 1 (Figure 10).

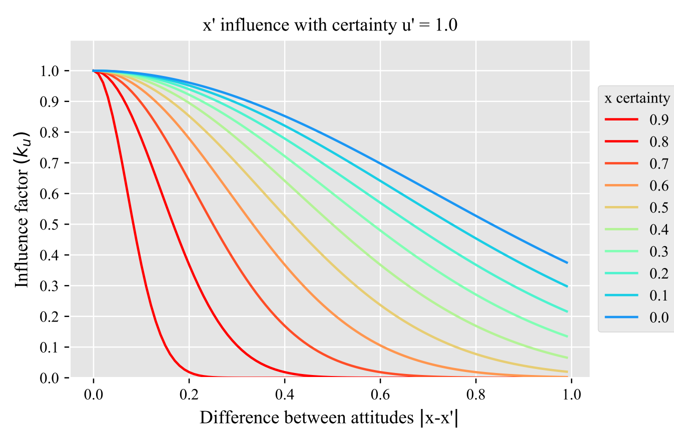

Fig. 10. As the certainty of x increases, the influence factor drops quickly to zero as the difference between their opinions ($|x - x'|$) increases. For lower certainties of x (0.0 -0.4), even if the difference between them is 1.0, they will still be influenced to some degree.

Principle 2 describes the process of certainty changing as a result of the (non-linear) interactions among agents. Therefore, it was fulfilled using a series of coefficients which change the certainty of the agent depending which agent is broadcasting at a particular time step:

- If an agent broadcasts, its certainty increases. This means agents will not broadcast an attitude held in uncertainty, and that voicing an opinion is related to being more certain (Tormala 2016, pg 6).
- If an agent receives information (i.e. is not broadcasting), its certainty changes relative to the agent who is broadcasting:
  - If the broadcast comes from an outgroup, certainty will decrease by a small amount. This supports ambivalence coming from contrary information. This model does not take into account distancing forces, which suggest negative influence (Mäs et al. 2014), or moving away from opinions of the outgroup.
  - If the broadcast comes from the ingroup, certainty will increase if the agent agrees with this information, or the agent changes their mind and adjust their attitude with respect to this information. If the agent does not adjust their attitude, their certainty will decrease with contrary information. This is consistent with, when confronted with contrary information from a trusted source (the ingroup), it will make one less certain of their own ideas, but social consensus will make one more certain (Johnson 1940, Tormala 2016).

TABLE II
RECEIVING BROADCAST WEIGHTS

	Ingroup		Outgroup
Change Attitude	(1)$\mu = +1$		(2)$\mu = -.01$
	Agree	Disagree	(5)$\mu = -.01$
	(3)$\mu = +1$	(4)$\mu = -1$	

Values of μ for each possible scenario of receiving information. (1) If an agent changes its mind it can only do so if the broadcast is from the ingroup. (2)(5) A small change happens from not agreeing with your outgroup which makes the system less stable the more opinions are broadcasted. (3)(4) The weight of not changing an attitude is equal but opposite whether you agree or disagree. This supports agreement, and means groups will have a collective raise in attitude the more closely they agree with one another. Groups are punished if they do not agree, so the larger majority is dismantled if there are more opinions within the ingroup(4).

All of these results have a population of 100 agents and are measured first with a uniform starting certainty of .5. The reason for this is twofold: first, if agents all begin with the same certainty the resulting groupings will not be affected by the initial state and second, .5 certainty will ensure the system begins in a state of enough certainty that noise will not take over and equilibrium can be reached.

To adjust certainty as described above, agent x with uncertainty u adjusts its certainty at each time step as follows:

$$u_{(t+1)} = u + \varepsilon\mu \tag{4}$$

TABLE III
BROADCAST WEIGHTS

Change Attitude	$\mu = 1$
Do Not Change Attitude	$\mu = .01$

Broadcasting has a higher weight when the agent changes their mind, supporting the idea that to express a contrary opinion means one is more certain. Attitudes which are expressed generally get a small change, meaning certainty increases over time.

Where $\varepsilon = .01$, and μ varies depending on the communication. ε is a measure of the speed of certainty change, and has been chosen as .01 for practical purposes of simulation duration (ε varies with the number of agents and is calculated by the percent of the population of a single agent, with a population of 100, this is 1% or .01). μ is a weight value that when varied promotes different dynamics in the simulation (Tables 2 and 3).

Finally, agents with low certainty can change their opinion at random with a probability defined by the following equation, which is a function of the agent's uncertainty u:

$$p(u) = (ue^{-(1-u)})^2 \tag{5}$$

Which results in Figure 11 and is determined by a probability event.

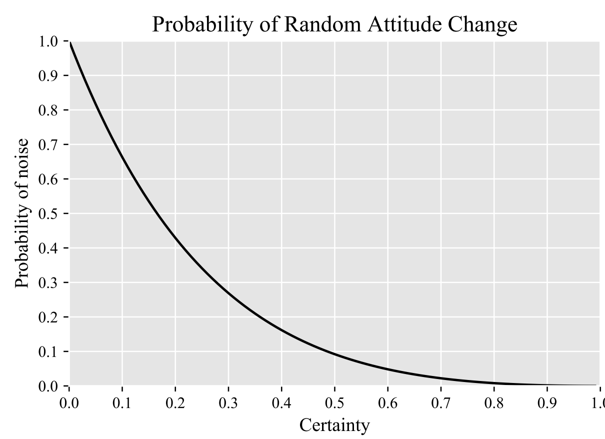

Fig. 11. The probability of change goes towards 0 as certainty increases. Probability is .17 when the certainty is .5 (initial certainty). For clarity certainty is used here, which is equal to 1-u in the equation above.

5.2. *Results*

The resulting system is one where the "pressure to conform" is high enough that extremism, and indeed small groups in general, can only persist in situations which have a diverse enough opinion cluster that majority pressures do not overcome small ingroup stability. That is, since large groups of agents are consistently confirming each others opinions, if they are large enough they will destabilize small groupings. The stability of cluster formation, then, is related to the number and population of each opinion group, which is consistent with the literature on social groups and attitude certainty (Petrocelli 2007).

First, an information space where certainty (on average) is less given the amount of information being circulated is demonstrated in Figure 12 and 13. To start, Figure 12 shows simply the more clusters the longer the system takes to stabilize, with a Pearsons correlation of .49 (moderately positive). Figure 13 shows that average certainty after 100 stable runs is significantly smaller given a larger amount of clusters, which demonstrates that more information in the system leads to less certain agents overall (more clusters = more attitudes). This trend diminishes after longer runs, but this is because for a cluster to be stable, the average certainty is always increasing, if the average certainty were always decreasing, the cluster would be vulnerable to random opinion change and would no longer remain stable. Furthermore, the certainty increasing over time when unchallenged is considered a feature of certainty under normal conditions (Petrocelli 2007, Tormala et al. 2009). Given this stage is a random expression of opinions which eventually stabilizes, it suffices to use the first 100 runs as an indication of the uncertainty a diversity of information causes in the system.

Fig. 13. Average certainty of clusters over the course of the simulation. Each dot represents one simulation run.

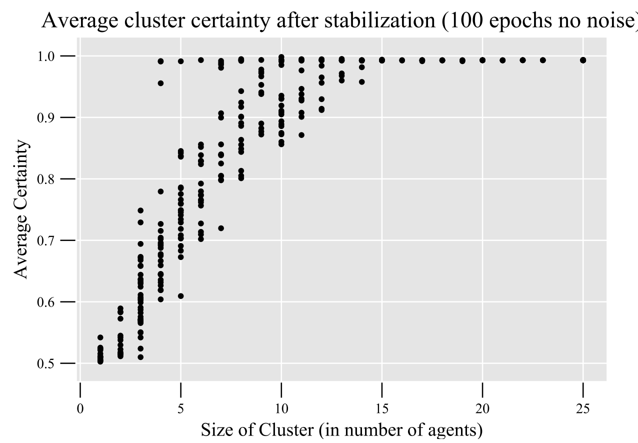

Fig. 14. Average certainty at the moment of stabilization. Each dot represents one cluster.

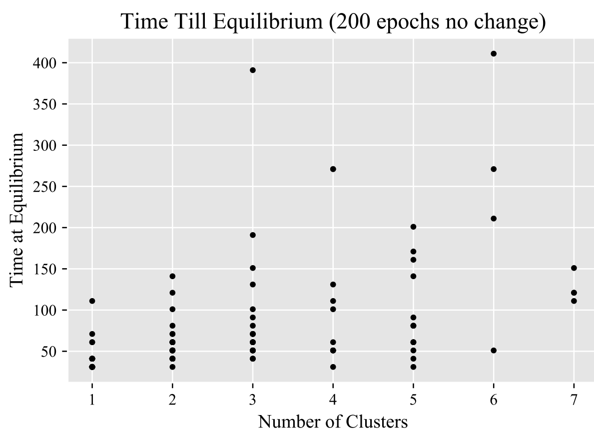

Fig. 12. Time until equilibrium is reached and number of clusters at equilibrium. Each dot represents one simulation run.

Figure 14 may seem to contest Figure 13, because the average certainty for large clusters (meaning less clusters overall), is higher than smaller ones, therefore small numbers clusters would have high certainty average and it is not a feature of the information being circulated. However, Figure 14 is the average of each cluster based on it's population at the moment the simulation has stabilized for 100 epochs, while Figure 13 is average certainty over the course of 100 epochs. Figure 14 shows that the certainty of agents on average is based on the size of the cluster, which is desirable for this model, because we want to reinforce group formation, and then see what happens when we disrupt it.

6. Model 3: Combination

This model is a combination of the two former models. It is hoped that combining both can resolve issues with the previous by virtue of its complexity, and produce a less rigid model by employing both certainty and identity.

6.1. Design

This model uses all of the former methods, running essentially in parallel. Here, however, the similarity threshold was able to be lowered to less than 50% similarities, and the difference tolerance (essentially the 'bound of confidence'), will also be randomized between 0 and 1. This creates a heterogeneous population of more and less 'open' agents who nevertheless operate by the same basic rules as the previous implementations. Heterogeneity is a desirable feature in agent-based models generally in that it is more reflective of human populations (Epstein 2006). Also, 'relaxing' the strict parameters required in the first models addresses the limitations of those models in hopes that this simulation will produce clustering with less rigid restrictions.

Due to this heterogeneity, however, an asymmetry of ingroup relationships occur, which it did not in the previous two models. This is because each agent has different requirements for what it considers its ingroup, based on its 'openness'. The result, when using the basic parameters of μ listed above, is the inability for the system to reach equilibrium for any agent who has an asymmetrical ingroup construct. While exposure to a diversity of opinions does produce more attitude ambivalence (Visser 2004), having a large amount of agents constantly shifting their opinion towards any possible influence demonstrates the limitations of the design.

In order to minimize this, three weight measures were added: conformity, ranking, inertia. The conformity weight is simply a measure of what percentage of the agent's ingroup agree with the broadcast (including the broadcast agent itself). Ranking takes into account the percentage of similarities the agent shares with the broadcasting agent, with the most similar agent in the ingroup having a ranking of 1. Ranking is based on prominence, which is the importance a given agent puts onto each identity (Grant et al. 2012). Each identity has a "prominence", and adding all of these together for one agent is 1, making an identity a percentage of the overall repertoire. Conformity and ranking are multiplied by μ so that the broadcasts with low ingroup agreement and low identity similarity will have less effect, and those with high ingroup agreement and high identity similarity will maintain a positive feedback loop. Finally, inertia is a positive measure of how many times an agent has been influenced, making the probability of influence slightly lower over time. This means agents with large ingroups who change their mind often are encouraged to find equilibrium.

Since this simulation is essentially unbiased, that is, agents broadcast information at random, and no outside influence is injected, it is interesting to see if extremism can emerge in the system itself (endogenously). As described in Model 1, the structure remains important here in the potential emergence of extremism within the system. It is the combination of uncertainty and small connections which facilitate the lack of influence and amount of noise which allows for extremism to develop. Agents with high amounts of connections express higher level of uncertainty (as is consistent within the literature, see Visser 2004), but because of the large amount of influence, the likelihood of extremism is low. On the other hand, agents with low connections are more likely to become certain because of a lack of conflicting messages and are unlikely to change their opinion (possibly towards the extreme) as well as to be influenced by those who do. Therefore, the only kind of agent vulnerable to extremism are those who have less connections (and less likelihood of moderating influence) but who are connected to other uncertain agents, thereby increasing the likelihood of noise and movement of their opinion outside of the moderate majority.

6.2. *Results*

As expected, the relaxation of the parameters from the first two models allows for stable clusters in this iteration. Namely, the amount of similarities required for agents to be considered ingroup members could be lowered to less than half of the repertoire length. Formerly, this would lead to consensus inevitably, however, because of the added difference threshold, this would be resisted. The difference threshold can also be flexible, and is initialized at random between 0 and 1 for each agent, which would have lead to consensus in Model 2. This combination of these two models, then, successfully allows for a relatively more realistic representation of identity and certainty, while still maintaining stable clusters over time. This is significant, because it suggests that adding variables on top of each other can provide solutions to the Abelson Diversity Problem without adding a disintegrating force.

The simulation gives rise to extremism, but by and large only if there are agents which are initialized as extreme. This would imply that a system can become extreme when an extremist is inserted, but does not say anything about the system being able to produce extremism. In order to test this, agents were initialized with attitudes considered moderate (between .2 and .8), and the resulting population of extremists was found once the system arrived at equilibrium (Table 4).

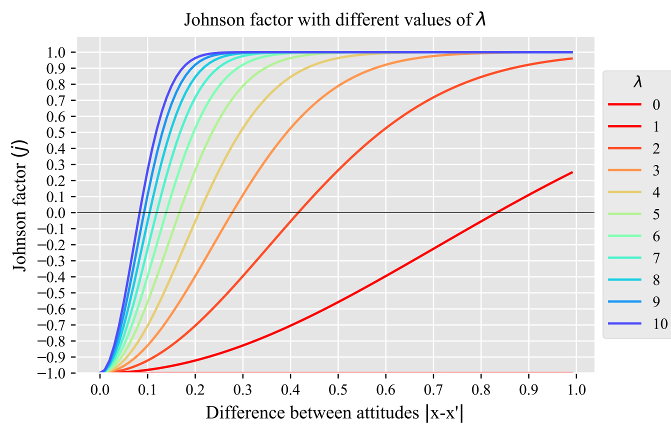

Fig. 15. Johnson factor for different values of λ ($\beta = 1$).

The system in itself, then, does not lead to extremism in any meaningful way due to large pressures towards moderation by the majority of agents. To push the system to its limits and determine if there are conditions whereby polarization or extremism can be produced with an initially moderate population, another parameter was experimented with. Named the Johnson factor, it is based on a theory by Donald Johnson in his 1940 paper *Confidence and the Expression of Opinion*, postulating that extreme attitudes tend to become confident because they are able to reject more opinions which are farther away from their own than those who hold more moderate opinions.

To test this, the Johnson factor was included, which moderates the certainty of agents on any broadcast (see (2)(5) Table 2). Instead of the confidence decreasing by $\varepsilon\mu$ (μ is negative here) in the event of an outgroup broadcast, certainty will decrease by the Johnson factor j, which is defined by the following equation:

$$j(x, x') = \beta(2 * e^{-\lambda(x-x')^2} - 1) \tag{6}$$

Where x is the agent's attitude and x' is the broadcasting agent's attitude, β is a scaling factor determining the magnitude of j and λ is a variable describing at what threshold of

TABLE IV
10 RUN AVERAGES FOR DIFFERENT ATTITUDE RANGES

Initial Attitude Range	Initial Extremist Population	Final Extremist Population	Difference	Final Average Extremist Certainty	Initial Mean/ Standard Deviation	Final Mean/ Standard Deviation	Difference
(1) 0-1	38.3	23.8	-14.5	0.80	0.495/ 0.281	0.510/ 0.215	+.015/-.066
(2) .2-.8	0.0	2.3	+2.3	0.27	0.493/ 0.169	0.496/ 0.124	+.003/-.045

(1) With initial extremists and (2) Without initial extremists (extremists as being defined by attitudes $< .2$ or $> .8$). (1) The population of extremists declines when there is an initial population of extremists, which is evidence for the moderating trend produced by the design. However, some extremists are able to remain at equilibrium, and their average certainty is high enough (.80) whereby noise will not destabilize the clusters. (2) When there are no initial extremists, some will remain at equilibrium, however their average certainty (.27) is low enough whereby noise will destabilize the clusters as the simulation continues. As the mean remains at around .5 for both scenarios extremists are distributed on either end of the opinion spectrum so no extreme as taken precedence, but the standard deviation does diminish slightly, also demonstrating the moderating pressures.

attitude difference there will be zero change in certainty (the x-intercept in Figure 15).

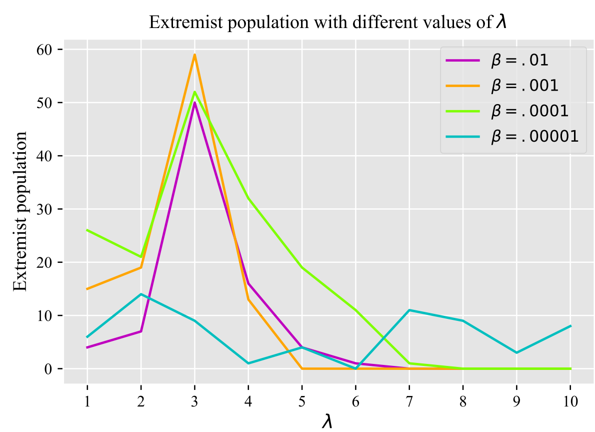

Fig. 16. Extremist population for values of λ. $\lambda = 3$ is the ideal value for producing large amounts of extremists given $\beta > \mu$.

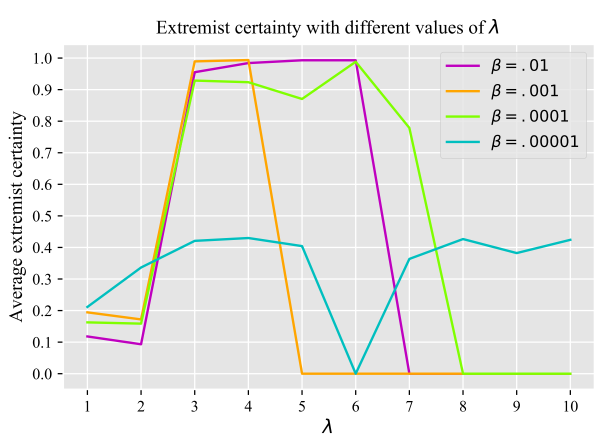

Fig. 17. Certainty of extremists for values of λ. For all values of β, $2 < \lambda < 5$ allows for stable extremist groups.

Higher values of λ result in smaller differences being required to increase confidence, and reaches a limit of about .1 difference (which is relatively small), in order for confidence to be increased. Where $\lambda = 0$, μ remains unchanged and the simulation runs as before. The resulting values for running simulations with each value of λ can be seen in Table 4.

Figure 16 shows that the extremist population increases until $\lambda = 3$ for all values of β which were tested. As λ gets larger than 3, the difference required to increase certainty is much smaller, and the certainty of the population rises proportionally despite whether the agent's opinion resides in the extremes. For $\lambda >= 5$, this is about a difference of 1.5, meaning that many agents will have a difference of opinion which is larger than this. In these cases, certainty increases for all agents and there is not enough uncertainty to produce the noise required for agents to become extreme. Interestingly, larger values of λ actually safeguard against extremism. As β goes towards .0001 it is approaching the original μ, which means it has a very small effect and results in small amounts of extremists due to a slightly lower uncertainty.

While there are extremists for $1 < \lambda < 3$, Figure 17 shows that the average certainty is $< .2$ for all values of β. When $\lambda < 3$, the difference required for an increase in certainty is $> .8$ (Figure 15), which unless agents are on either extreme of the spectrum, will have no positive effect on certainty ($j > 0$). In this case, unless $\beta = \mu$, the system becomes too uncertain and will never reach equilibrium. For all values of β, the optimal value of λ is 3. According to Figure 15, this is a difference of at least .3 in order to increase confidence. Figure 18 shows that all values of β will produce a large standard deviation at $\lambda = 3$: .35 when the mean is about .5 at the start. That suggests the average agent sits at .85 or .15, meaning there are very few moderate agents in this scenario. The moderating forces of the original are present when β approaches μ as the standard deviation falls below zero.

7. Discussion & Conclusion

The main questions of this study were, are the variables of social identity and uncertainty able to affect social influence and result in complex opinion dynamics (including extremism and polarization) as observed in online social networks such as Facebook? Furthermore, given the constraints of the Abelson Diversity Puzzle, do stable opinion clusters form?

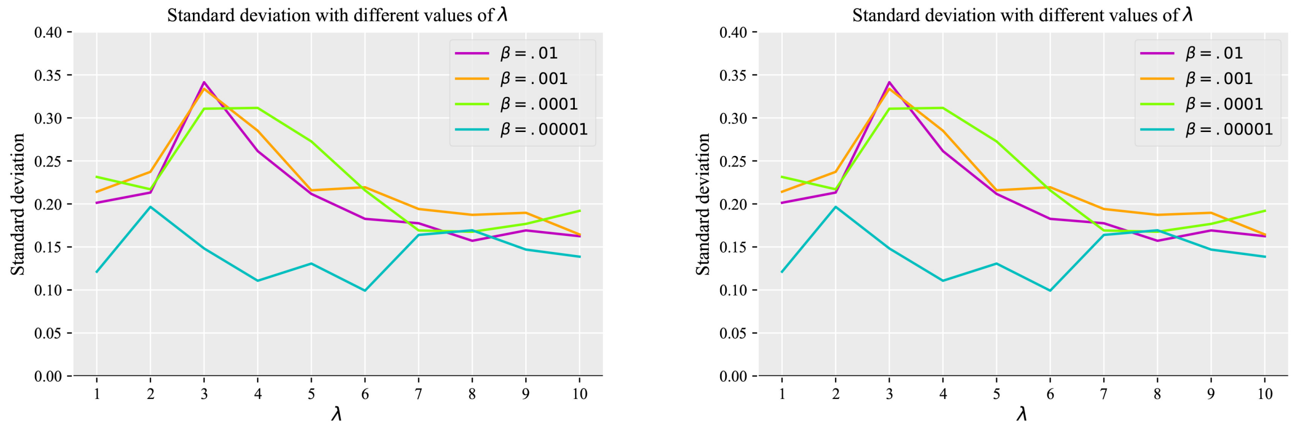

Fig. 18. (left) standard deviation at equilibrium (right) difference between initial and final standard deviation.

Model 1 demonstrated that social identity is able to produce stable opinion clusters as long as the amount of connections is limited and the population is somewhat diverse. Model 2 did successfully allow for certainty to be negotiated by ingroup size, and therefore added a level of complexity to the rigidity of the bounded confidence model. This supports the theory that certainty is a negotiator of group dynamics, as is suggested by the literature, and this basis for a model could be used for further investigation of these concepts (see uncertainty identity theory as described in [**?**] pages 943-45). Model 3 demonstrated that while clustering occurs, moderating forces are strong, and extremism or polarization do not result from the system alone. One option was experimented with to see if extremism resulted, showing the virtues of the design of Model 3 as a testing ground to isolate variables outside of social influence and certainty. The aim of this research is not to systematically test other theories, but it is hoped that the results of this experiment suggests the potentials of the model design.

Ultimately, given the Abelson Problem, these models demonstrate that opinion distributions other than consensus can exist in systems where everyone is connected. That is, since Facebook is not a network where everyone agrees on one opinion, these models are successful to the extent that they were able to reproduce a myriad of opinions on a macro level, while maintaining influence connections between groups of agents. Because of this, social identity and certainty can be considered possible explanations for the formation of social connections, and for how people are influenced by others.

Therefore, these models can tentatively say that if Facebook facilitated an open broadcast of opinions open to all members of the network, it seems to have a moderating effect overall. Encouraging open information exchange, where people are exposed to many diverse opinions, could help to mitigate information disorders, as has been observed in offline social networks [**?**]. As the messages in these models are all weighted equally, that is, no message is more persuasive than any other, it is hard to extrapolate these results to include things like propaganda. Considering these factors would be a fruitful starting point in future research and could be possible contributors in polarization and extremism, as well as other information disorders.

There are several reasons why the design and results are not completely descriptive of the effects of social influence on Facebook. For example, Model 1 does not allow for similarities between agents which are flexible and less than half of the identity repertoire. This is due to the constraints of opinion dynamic models with regard to the Abelson Diversity Problem. Nevertheless, the attempts to reconcile this problem were somewhat successful. The fact that Model 3 allowed for the relaxation of both the bound of confidence principle and the similarity threshold is very encouraging, and suggests that the interaction of these factors is a fruitful starting point both with regards to agent-based model design, and a possible factor in swaying opinion dynamics in the real world.

A key future challenge for all three models is comparison with real-world data. Indeed, the veracity of the models themselves cannot be confirmed without this, even though on an abstract level it can be concluded that they succeeded to reproduce macro-level trends of opinion diversity (i.e. avoiding consensus). A thorough collection of relevant data, either from mining the Facebook API (which is limited due to privacy restrictions) or by gathering it via an application, was beyond the scope of this present study. Given these results, though, follow up research focusing on empirical data and using the modeling methods outlined in this paper would be beneficial to further examining the results and moving forward with more complex models. Nevertheless, this process of building systems and combining them appears to be a sufficient method for exploring the effects of the factors described here in isolation, and could be used to test other possible interacting variables in the psychology of attitude formation.

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