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A New Approach to Target Region Based
Multi-objective Evolutionary Algorithms

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MASTER'S THESIS

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Abstract

In this paper, an approach to target region based multi-objective evolutionary algorithms is proposed to incorporate preference before the optimization process. It aims at finding a more fine-grained resolution of a target region without exploring the whole set of Pareto optimal solutions. It can guide the search towards the regions on the Pareto Front which are of real interest to the decision maker. The algorithm framework has been combined with SMS-EMOA, R2-EMOA, NSGA-II to form three preference based multi-objective evolutionary algorithms: T-SMS-EMOA, T-R2-EMOA and T-NSGA-II. In these algorithms, three ranking criteria are applied to achieve a well-converged and well-distributed set of Pareto optimal solutions in the target region. The three criteria are: 1. Non-dominated sorting; 2. indicators (hypervolume or R2 indicator) or crowding distance in the new coordinate space (i.e. target region) after coordinate transformation; 3. the Chebyshev distance to the center of target region.

Moreover, by introducing a parameter ϵ in the algorithms to improve the diversity and allocating a proportion of population to each target, the proposed algorithms have been enhanced to support multiple target regions and preference information based on a target point or multiple target points.

On some benchmark problems, including continuous problems and discrete problems, experimental results show that the new algorithms can handle the preference information very well and find an adequate set of Pareto-optimal solutions in the preferred region(s) or close to the preferred point(s). In the paper, rectangular and spherical target regions have been tested, while target regions in other shapes are also possible.

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List of Abbreviations

DF	**D**esirability **F**unction
DF-SMS-EMOA	**D**esirability **F**unction-based **SMS-EMOA**
DM	**D**ecision **M**aker
DTLZ	**D**eb-**T**hiele-**L**aumanns-**Z**itzler (Benchmark)
EA	**E**volutionary **A**lgorithm
EMO	**E**volutionary **M**ulti-objective **O**ptimization
HV	**H**yper**V**olume
MCDM	**M**ulti-**C**riteria **D**ecision **M**aking
MOEA	**M**ulti-objective **O**ptimization **E**volutionary **A**lgorithm
MOP	**M**ulti-objective **O**ptimization **P**roblem
NE	**N**umber of **E**valuations
NSGA-II	**N**on-dominated **S**orting **G**enetic **A**lgorithm **II**
PF	**P**areto **F**ront
PMOEA	**P**reference-based **M**ulti-objective **E**volutionary **A**lgorithm
R-NSGA-II	**R**eference point-based **NSGA-II**
ROI	**R**egion **O**f **I**nterest
SBX	**S**imulated **B**inary **C**rossover
SMS-EMOA	**S** **M**etric **S**election-based **EMO** **A**lgorithm
ZDT	**Z**itzler-**D**eb-**T**hiele (Benchmark)

Chapter 1

Introduction

The aim of solving an optimization problem is to find the optimal solution(s). Usually in the real world, it is difficult to find the optimal solution for multi-objective optimization problems (MOPs) because they involve in optimizing two or more conflicting objectives simultaneously. For a solution, if none of the objective can be improved without degrading some of the other objectives, then this solution is called Pareto optimal. When solving MOPs, exact methods are not often usable because the search space can be too large and too complex. On the contrary, evolutionary algorithms (EAs) have been proven to be particularly suitable for approximately solving MOPs. As a result, numerous multi-objective evolutionary algorithms (MOEAs) have been proposed and the research domain of evolutionary multi-objective optimization (EMO) has received a great deal of attention.

Most MOEAs approximate the entire Pareto Front (PF). However, the final goal of EMO is to help the decision maker (DM) to find solutions which match his/her preferences most. The DM may only pay attention to a smaller set of Pareto optimal solutions, instead of the entire PF. Under this condition, approximation of the whole PF is neither computationally efficient nor requested. Therefore, integrating preferences in solving MOPs has become the subject of intensive studies of EMO (Deb and Miettinen, 2008). Many preference-based MOEAs in which the DM incorporates his/her preferences before (a priori), after (a posteriori), or during (interactively) (Purshouse et al., 2014) the optimization process have been proposed in literature (Deb and Sundar, 2006) (Molina et al., 2009) (Said, Bechikh, and Ghédira, 2010) (Brockhoff et al., 2013) (Tanigaki et al., 2014) (Yang et al., 2016) and a review of the preference-based multi-objective evolutionary algorithms (PMOEAs) field is given in (Li et al., 2016).

Among the well-known MOEAs, the Non-dominated Sorting Genetic Algorithm II (NSGA-II) (Deb, Pratap, et al., 2002) is a Pareto dominance-based approach which finds approximation fronts as close to the PF and as diverse as possible; S-Metric Selection EMOA (Beume, Naujoks, and Emmerich, 2007) and R2-EMOA (Trautmann, Wagner, and Brockhoff, 2013) are indicator-based approaches which use performance measures (indicators) on the quality of the PF approximations to guide the search. The Hypervolume (HV) (Zitzler and Thiele, 1998) used in SMS-EMOA and the R2 indicator (Hansen and Jaszkiewicz, 1998) used in R2-EMOA are two main approaches measuring both convergence and diversity of a PF approximation. In the thesis, a group of algorithms have been proposed to include preference information within MOEAs before the start of the search in order to find solutions within a more fine-grained resolution in a predefined region of interest (ROI) (Adra, Griffin, and Fleming, 2007) on the PF. The group of algorithms consists of three algorithms: T-SMS-EMOA, T-R2-EMOA and T-NSGA-II. In these algorithms, non-dominated sorting, hypervolume (in T-SMS-EMOA), R2 indicator (in T-R2-EMOA), crowding distance (in T-NSGA-II) and the Chebyshev distance work together to attract the population to target region and maintain diversity simultaneously.

Besides the basic algorithms to find a preferred set of solutions in a target region, more abilities have been added in the algorithms to make them more powerful. Inspired by some ideas from (Deb and Sundar, 2006), by introducing a parameter ϵ in the algorithms to improve the diversity, the target point can also be used to generate a subset of preferred Pareto optimal solutions; by allocating a proportion of population to each target, the enhanced algorithms can handle multiple target regions or points.

1.1 Overview

The remainder of this thesis is organized as follows: Section II introduces some background knowledge and Section III presents some related works. In Section IV, the proposed algorithms are described and the structures of them are given. The experimental results on a target region are reported in Section V. The details and graphical results of enhanced algorithms are presented in Section VI and Section VII concludes the work with the summary and outlook.

Chapter 2

Background

2.1 Multi-Objective Optimization Problem

A MOP is formulated as

$$Min \quad f(\mathbf{x}) = (f_1(\mathbf{x}), f_2(\mathbf{x}), \cdots, f_m(\mathbf{x}))^T \tag{2.1}$$

subject to

$$\begin{aligned} g_i(\mathbf{x}) \leq 0 \qquad & i = 1, \cdots, P, \\ h_j(\mathbf{x}) = 0 \qquad & j = 1, \cdots, Q, \end{aligned} \tag{2.2}$$

where $\mathbf{x} = (x_1, x_2, ..., x_n) \in S$ is decision vector in search space, f_k $(k = 1, ..., m)$ are the objective functions, $f(\mathbf{x}) \in Z$ is the objective vector and g_i, h_j $(i = 1, ..., P; j = 1, ..., Q)$ are the constraint functions of the problem. In the following, some widely known definitions are provided to understand the solutions of 2.1.

Definition 1 A feasible vector $\mathbf{x}^* \in S$ is said to be efficient iff there does not exist another feasible vector $\mathbf{x} \in S$ such that $f_k(\mathbf{x}) \leq f_k(\mathbf{x}^*)$ for all $k = 1, ..., m$, and $f_l(\mathbf{x}) < f_l(\mathbf{x}^*)$ for at least one index l $(l \in \{1, ..., m\})$. The set S_E of all the efficient points is called the efficient set or Pareto set. If $\mathbf{x_1}$ and $\mathbf{x_2}$ are two feasible points and $f_k(\mathbf{x_1}) \leq f_k(\mathbf{x_2})$ for all $k = 1, ..., m$, with at least one of the inequalities being strict, then we say that $\mathbf{x_1}$ dominates $\mathbf{x_2}$.

Efficiency is defined in the decision space. The corresponding definition in the objective space is as follows:

Definition 2 An objective vector $\mathbf{z}^* = f(\mathbf{x}^*) \in Z$ is said to be non-dominated iff $\mathbf{x}^*$ is efficient. The set Z_N of all non-dominated vectors is called the non-dominated set or Pareto front. If $\mathbf{x_1}$ and $\mathbf{x_2}$ are two feasible points and $\mathbf{x_1}$ dominates $\mathbf{x_2}$, then we say that $f(\mathbf{x_1})$ dominates $f(\mathbf{x_2})$.

It is usually impossible for one single solution to minimize all objectives functions simultaneously because objective functions are often conflicting with each other. Therefore, solving a MOP means obtaining the whole efficient set and its corresponding Pareto front, and the most common approach to solve MOPs is the use of MOEAs.

2.2 NSGA-II

NSGA-II is a widely applied Pareto dominance-based MOEA. Its fitness evaluation is based on an elitist strategy composed of a Pareto dominance based rank assignment mechanism called non-dominated sorting and a secondary measure for diversity maintenance called crowding distance. In NSGA-II, the offspring population is created from the parent population by selection, crossover and mutation[1]. Thereafter, the two populations of equal size (the specified population size) are merged to form a new population which is classified into different groups according to the ranks assigned by non-dominated sorting. The population of the next generation are generated by choosing the 50% best solutions from the merged population and the new generation is filled by each front subsequently until the population size exceeds the specified population size: starting with solutions in the first non-domination front, and continuing with solutions in the second non-domination front, and so on. If by adding all solutions in one front, the population size exceeds the specified population size, then picking solutions in the descending order of crowding distance.

2.3 SMS-EMOA

SMS-EMOA (where SMS stands for S-metric selection) is a hypervolume indicator-based EMO algorithm with a $(\mu + 1)$ selection scheme. The hypervolume of a set is the total size of the space dominated by the solutions in the set and is measured relative to a reference point, usually the worst possible point in the objective space. It is an important quantitive measure to estimate the closeness of the estimated points to the true PF and can be computed without the knowledge of the true PF, therefore be used also in guiding algorithms towards the PF.

In SMS-EMOA, starting with an initial population of μ individuals, a single solution is generated by selection, crossover and mutation. The worst solution is to be removed from the merged population with the size of $\mu + 1$ in order to maintain the population size. SMS-EMOA selects the worst solution by using the non-dominated sorting as the first ranking criterion and the hypervolume contribution as the second ranking criterion. The removed solution is the one that contributes the least to the hypervolume of the worst ranked front. Imaging $x^{(i)}$ is one solution in the solution set S. The hypervolume contribution of $x^{(i)}$ to the hypervolume of the set S is defined as:

$$HC(x^{(i)}|S) = HV(S) - HV(S \backslash \{x^{(i)}\}). \tag{2.3}$$

2.4 R2-EMOA

R2-EMOA modifies SMS-EMOA by replacing the hypervolume indicator with the R2 indicator. The Chebyshev distance between two vectors is the greatest of their differences along any coordinate dimension. In case the standard weighted Chebyshev utility function with ideal point $\mathbf{i}$ and the objective number d, the R2 indicator is defined as

$$R2(A, \Lambda, \mathbf{i}) = \frac{1}{|\Lambda|} \sum_{\lambda \in \Lambda} \min_{a \in A} \{ \max_{j \in \{1,\ldots,d\}} \{\lambda_j |\mathbf{i}_j - a_j|\}\} \tag{2.4}$$

for a solution set A and a given set of weight vectors $\lambda = (\lambda_1, ..., \lambda_d) \in \Lambda$. Usually, the weight vectors are chosen uniformly distributed over the weight space, for example for m = 2 objectives,

$$\Lambda_k = (0, 1; \frac{1}{k-1}, 1 - \frac{1}{k-1}; \frac{2}{k-1}, 1 - \frac{2}{k-1}; ...; 1, 0) \tag{2.5}$$

denotes k uniformly distributed weights in the space $[0, 1]^2$.

[1] By default, NSGA-II uses binary tournament selection, Simulated Binary Crossover (SBX) and polynomial mutation.

Chapter 3

Related Works

It is believed that the first PMOEA was proposed in 1993 (Fonseca, Fleming, et al., 1993) and the main idea of this work is to give higher priority to objectives in which the goal is not fulfilled. The 2004 and 2006 Dagstuhl seminars were the key points for the development of PMOEAs when researchers in EMO and MCDM fields gathered together to get to know each other and stimulate cooperation. Since then, large numbers of methods and algorithms were proposed and published.

Besides the categorization by the timing the DM expresses his/her preferences: a *priori*, a *posteriori* or *interactively*, in (Li et al., 2016), PMOEAs are also be classified to Reference point-based approaches (Wierzbicki, 1980), Reference direction-based approaches (Korhonen and Laakso, 1986) (Jaszkiewicz and Słowiński, 1999) (Deb and Kumar, 2007), Preference region-based approaches (Cheng et al., 2016), Trade-off-based approaches (Miettinen, Ruiz, and Wierzbicki, 2008) (Shukla, Hirsch, and Schmeck, 2010), Objective comparison-based approaches (Rachmawati and Srinivasan, 2010) (Brockhoff et al., 2013), Solution comparison-based approaches (Phelps and Köksalan, 2003), Outranking-based approaches (Fernandez et al., 2011), Knee point-based approaches (Bechikh, Ben Said, and Ghédira, 2010).

The classical reference point method was first proposed in 1980 (Wierzbicki, 1980). In a reference point method, the DM supplies a reference point, which represents the DM's desired values for each objective and therefore guides the search toward the desired region, and/or a weight vector which provides further information about what Pareto optimal point to converge to. Deb et al. combined the classical reference point method with NSGA-II and proposed the preference based NSGA-II called R-NSGA-II (Deb and Sundar, 2006) which searches the ROIs according to a user-provided reference point set. A modified crowding distance operator based on the distance (the normalized Euclidean distance) between preference points and solutions in the objective space is used in place of original crowding distance to choose a subset of solutions from the last front which cannot be entirely chosen to maintain the population size of the next population. An extra parameter ε was used to control the extent and the distribution of the final obtained solutions. R-NSGA-II has demonstrated good results on two to five objective test problems. However, the diversity is not well-maintained when using a single reference point. Moreover, the ε parameter setting is also a difficulty.

A preference region in the objective space is another widely used method in PMOEAs. Desirability Functions are a common way to reflect the objective values and the DM's degree of satisfaction in preference region based approaches. Wagner and Trautmann integrated DFs into SMS-EMOA and proposed the Desirability Function-based SMS-EMOA (DF-SMS-EMOA) (Wagner and Trautmann, 2010). The main idea of DF-SMS-EMOA is to convert the objective function of the original MOP into DFs and then optimizing these DFs instead of the original objectives. The DF-SMS-EMOA has demonstrated its ability to bias the search towards the DM's preferred region on the two objective ZDT test functions and the five objective turning process problem (Biermann, Weinert, and Wagner, 2008). However, when the number of objectives increases, the number of border solutions outside the specified limits of DFs increases and the hypervolume computational effort also increases. In fact, when $P = NP$, it grows at a polynomial rate with the increase in the number of objectives.

A DF-based coordinate transformation is introduced in the proposed approach to transform the objective values based on the preference region predefined by the DM. I combine the DF-based coordinate transformation with SMS-EMOA, R2-EMOA and NSGA-II and propose the target region based SMS-EMOA, R2-EMOA and NSGA-II called T-SMS-EMOA, T-R2-EMOA and T-NSGA-II. An important feature of new algorithms compared with DF-SMS-EMOA is that new algorithms can deal with multiple target regions simultaneously. Reference points can also be handled by considering it as a special case of target region. Furthermore, the method of controlling the extent and distribution of solutions in R-NSGA-II is used in new algorithms to allocate a proportion of population to each target.

Chapter 4

Proposed Approach

The proposed target region based multi-objective evolutionary algorithms work reliably when the DM wants to concentrate only on those regions of the PF which are of real interest to him/her. In the proposed algorithms, i.e., T-SMS-EMOA, T-R2-EMOA and T-NSGA-II (where T stands for target region), three ranking criteria (1. non-dominated sorting; 2. performance indicator (Hypervolume in T-SMS-EMOA or R2 in T-R2-EMOA) or crowding distance in T-NSGA-II; 3. the Chebyshev distance to the target region) work together to achieve a well-converged and well-distributed set of Pareto optimal solutions in the target region using preference information provided by the DM. Non-dominated sorting is used as the first level ranking criterion, performance indicator or crowding distance as the second and the Chebyshev distance as the third level ranking criterion. The Chebyshev distance speeds up evolution toward the target region and is computed as the distance to the center of the target region.

The hypervolume, R2 indicator or crowding distance is chosen as the second level ranking criterion, which is used as a diversity mechanism and is measured based on coordinate transformations using desirability functions (DFs). The concept of desirability was introduced by Harrington (Harrington, 1965) in the context of multi-objective industrial quality control and the approach of expressing the preferences of the DM using DFs is suggested by Wagner and Trautmann (Wagner and Trautmann, 2010). DFs map the objective values to desirabilities which are normalized values in the interval [0,1] where the larger the value, the more satisfying the quality of the objective value. The Harrington DF (Harrington, 1965) and Derringer-Suich DF (Suich and Derringer, 1977) are two most common types of DFs and both of them result in biased distributions of the solutions on the PF through mapping the objective values to desirabilities based on preference information. In the proposed algorithms, a simple type of DFs is used and it classifies the domain of the objective function into only two classes, “unacceptable” and “acceptable”. For this approach we have:

$$D(x) = \begin{cases} 1 & x \text{ is in the target region,} \\ 0 & x \text{ is not in the target region.} \end{cases} \tag{4.1}$$

The desirability here is for a solution. It is not necessary to consider desirability by each objective because the goal of new algorithm is to *zoom in* the target region. Therefore, we treat solutions out of the target region as unacceptable solutions and assign their desirabilities to be 0; at the same time, we assume that all solutions inside the target region are of equal importance, (i.e. acceptable) and assign their desirabilities to be 1. There is no further bias on the points in the target region in our algorithms, however, if other types of DFs are integrated in the algorithms, it is possible to generate solutions of different distributions in the target region with respect to the specified preferences.

For solutions with desirability 0, their second level ranking criterion is assigned to be 0 and for solutions with desirability 1, their second level ranking criterion need to be calculated further. Because only solutions in the target region are retained, a way is derived to simplify the calculation of the indicator values or to realize a reference point free version of indicators (M. T. Emmerich, Deutz, and Yevseyeva, 2014), which is coordinate transformation. The target region is treated as a new coordinate space of which the origin being the lower bound. For the maximization problem in T-SMS-EMOA or the minimization problem in T-R2-EMOA, a coordinate transformation is performed for the i-th objective as:

$$Ct_i(x) = f_i(x) - LB(f_i) \tag{4.2}$$

For minimization problem in T-SMS-EMOA or the maximization problem in T-R2-EMOA, coordinate transformation is performed for the i-th objective as:

$$Ct_i(x) = UB(f_i) - (f_i(x) - LB(f_i)) \tag{4.3}$$

where $LB(f_i)$ and $UB(f_i)$ are the lower bound and upper bound of the i-th objective in the target region which is predefined by the DM.

The reason of distinguishing the maximization and minimization problem in coordinate transformation is that the origin of the new coordinate space (i.e. the lower bound of the target region) is used as the reference point when calculating the indicator values. In T-SMS-EMOA, the worst point in the target region is chosen as the reference point when calculating hypervolume. On the contrary, the ideal point is chosen as the reference point when calculating R2 indicator in T-R2-EMOA. After coordinate transformation, the calculation of the second ranking criterion is implemented only in the target region instead of the whole coordinate system. It does make sense because the target region is the desired space to the DM. No reference point is needed in the calculation of crowding distance, therefore, any of the two formulas of coordinate transformation can be chosen in T-NSGA-II. Figure 4.1 shows an example of obtaining solutions in target region by the proposed approach.

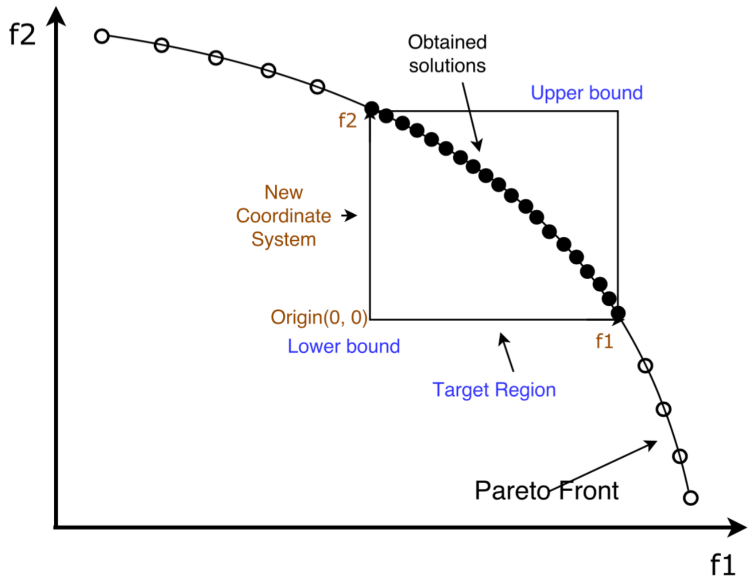

FIGURE 4.1: An example of obtaining solutions in the target region by proposed approach.

The shape of the target region does not necessarily need to be rectangular, it could as well be a circle, an ellipse or in other shapes as long as it can be confirmed efficiently whether or not a solution is in the target region. For instance, if the DM wants the solutions to be restricted to a sphere, s/he can specify the center point and radius of the sphere and new algorithms can obtain the set of the PF in the sphere.

4.1 T-SMS-EMOA

The details of T-SMS-EMOA are given in Algorithm 1.

The framework of T-SMS-EMOA is from SMS-EMOA. However, after fast non-dominated sorting, all the solutions in the worst ranked front are separated into two parts (acceptable and unacceptable) by the DF. Solutions in the first part have desirability 0 and their hypervolume contributions are assigned to be 0; solutions in the second part have desirability 1 and coordinate transformation is conducted on each objective of each solution in this part. After that, their hypervolume contributions are calculated in the new coordinate system and the origin in the new coordinate system is adopted as the reference point. The other difference between T-SMS-EMOA and SMS-EMOA is the involvement of the Chebyshev distance. In the early iterations, the existence of individuals in the target region is unlikely, the Chebyshev distance works on attracting solutions towards the target region.

4.2 T-R2-EMOA

The details of T-R2-EMOA are given in Algorithm 2.

R2-EMOA is extended to T-R2-EMOA in the same way SMS-EMOA is extended to T-SMS-EMOA. The formula of coordinate transformation used in T-R2-EMOA, however, is opposite to the formula used in T-SMS-EMOA for the same problem since the origin of the new coordinate system is used as the reference point in the measure of both hypervolume indicator in T-SMS-EMOA and R2 indicator in T-R2-EMOA. In T-R2-EMOA, the number of weight vectors is set to 501 for two objective

Algorithm 1 T-SMS-EMOA

$P_0 \leftarrow$ init( ) /$*$ Initialise random population $*$/
$t \leftarrow 0$
repeat
  $q_{t+1} \leftarrow$ generate(P_t) /$*$ generate offspring by variation $*$/
  $P_t = P_t \cup \{q_{t+1}\}$
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  $\forall x \in R_v$: compute $D_{Ch}(x)$/$*$ Chebyshev distance to the center of the target region $*$/
  $R_{v1} \cup R_{v2} \leftarrow R_v$/$*$solutions not in the target region $\rightarrow R_{v1}$; solutions in the target region $\rightarrow R_{v2}$ $*$ /
  $\forall x \in R_{v1} : HC(x) = 0$
  $R_{v2} \leftarrow$ Coordinate Transformation(R_{v2})
  $\forall x \in R_{v2} : HC(x) = HV(R_{v2}) - HV(R_{v2} \backslash x)$
  if unique $argmin\{HC(x) : x \in R_v\}$ exists
    $x^* = argmin\{HC(x) : x \in R_v\}$
  else
    $x^* = argmax\{D_{Ch}(x) : x \in R_v\}$/$*$ in case of tie, choose randomly $*$/
  $P_{t+1} = P \backslash \{x^*\}$
  $t \leftarrow t + 1$
until termination condition fulfilled

Algorithm 2 T-R2-EMOA

$P_0 \leftarrow$ init( ) /$*$ Initialise random population $*$/
$t \leftarrow 0$
repeat
  $q_{t+1} \leftarrow$ generate(P_t) /$*$ generate offspring by variation $*$/
  $P_t = P_t \cup \{q_{t+1}\}$
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  $\forall x \in R_v$: compute $D_{Ch}(x)$/$*$ Chebyshev distance to the center of the target region $*$/
  $R_{v1} \cup R_{v2} \leftarrow R_v$/$*$solutions not in the target region $\rightarrow R_{v1}$; solutions in the target region $\rightarrow R_{v2}$ $*$ /
  $\forall x \in R_{v1} : r(x) = 0$
  $R_{v2} \leftarrow$ Coordinate Transformation(R_{v2})
  $\forall x \in R_{v2} : r(x) = R2(P \backslash \{x\}; \Lambda; i) / * \, i$: ideal point$*$/
  if unique $argmin\{r(x) : x \in R_v\}$ exists
    $x^* = argmin\{r(x) : x \in R_v\}$
  else
    $x^* = argmax\{D_{Ch}(x) : x \in R_v\}$/$*$in case of tie, choose randomly$*$/
  $P_{t+1} = P \backslash \{x^*\}$
  $t \leftarrow t + 1$
until termination condition fulfilled

problem and 496 for three objective problem as former experiments in (Trautmann, Wagner, and Brockhoff, 2013) and (Wagner, Trautmann, and Brockhoff, 2013).

4.3 T-NSGA-II

The details of T-NSGA-II are given in Algorithm 3.

Algorithm 3 T-NSGA-II

$P_0 \leftarrow$ init( ) /$*$ Initialise random population $*$/
$t \leftarrow 0$
repeat
  $Q_t \leftarrow$ generate(P_t) /$*$ generate offsprings by variation $*$/
  $P_t = P_t \cup Q_t$
  $\forall x \in P_t$: compute $D_{Ch}(x)$/$*$ Chebyshev distance to the center of the target region $*$/
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  for i = rank 1,...,v do
    $R_{i1} \cup R_{i2} \leftarrow R_i$/$*$solutions not in the target region $\rightarrow R_{i1}$; solutions in the target region $\rightarrow R_{i2} *$ /
    $\forall x \in R_{i1} : D_c(x) = 0/ * D_c$: crowding distance$*$/
    $R_{i2} \leftarrow$ Coordinate Transformation(R_{i2})
    $\forall x \in R_{i2}$: compute $D_c(x)$
  $P_{t+1} \leftarrow$ half the size of P_t based on rank, D_c and then D_{Ch}
  $t \leftarrow t + 1$
until termination condition fulfilled

In T-NSGA-II, the size of the offspring population is the same as the size of the parent population, which is the specified population size. The next population is generated by choosing the best half solutions from the merged population: starting with points in the first non-domination front, continuing with points in the second non-domination front, and so on; if by adding all points in one front, the population size exceeds the specified population size, picking points in the descending order of crowding distance; if by adding all points with the same crowding distance, the population size still exceeds the specified population size, picking points in the ascending order of the Chebyshev distance. Unlike T-SMS-EMOA and T-R2-EMOA, no reference point is needed in T-NSGA-II.

Chapter 5

Experimental Study

5.1 Experimental Setup

All experiments in the thesis are implemented based on the MOEA Framework (version 2.11, available from `http://www.moeaframework.org/e`). The MOEA Framework is Java-based framework for multi-objective optimization and it supports a number of MOEAs, test problems and search operators. It is also easy to be extended to introduce new problems and algorithms.

In this section, simulations are conducted to demonstrate the performance of the proposed algorithms. In all simulations, we use the SBX operator with an index of 15 and polynomial mutation with an index 20 (Deb, 2001). The crossover and mutation probabilities are set to 1 and $1/N$, where N stands for the number of objectives.

We conduct experiments on some benchmark problems, including ZDT, DTLZ and knapsack problems, to investigate performance of the new algorithms. All experiments were run on a personal laptop with i5-5257U @ 2.7 GHz and 8GB RAM. The population size and the number of evaluation are chosen to be dependent on the complexity of the test problem. Table 5.1 shows the population size and the number of evaluations (NE) we use on different test problems.

TABLE 5.1: Population Size and Number of Evaluation

Problems	Population Size	NE
ZDT1	100	10000
ZDT2-3	100	20000
DTLZ1-2	100	30000
knapsack-250-2 knapsack-500-2	200	200000
knapsack-250-3 knapsack-500-3	250	500000

5.2 Two-Objective ZDT Test Problems

In this section, we consider three ZDT test problems. First, we consider the 30-variable ZDT1 problem. This problem has a convex Pareto optimal front which is a connected curve and can be determined by $f_2(x) = 1 - \sqrt{f_1(x)}$. The true PF spans continuously in $f_1 \in [0, 1]$. Four different target regions are chosen to observe the performance of T-SMS-EMOA, T-R2-EMOA and T-NSGA-II. The first target region covers the entire PF with the lower bound (0,0) and the upper bound (1,1). The second target region restricts preferred solutions to the central part of the PF and its lower bound is (0.1,0.1), upper bound is (0.5,0.5). The third and fourth target regions take two ends of the PF respectively and have their lower bounds to be (0,0.6) and (0.6,0), upper bounds to be (0.3,1) and (1,0.3).

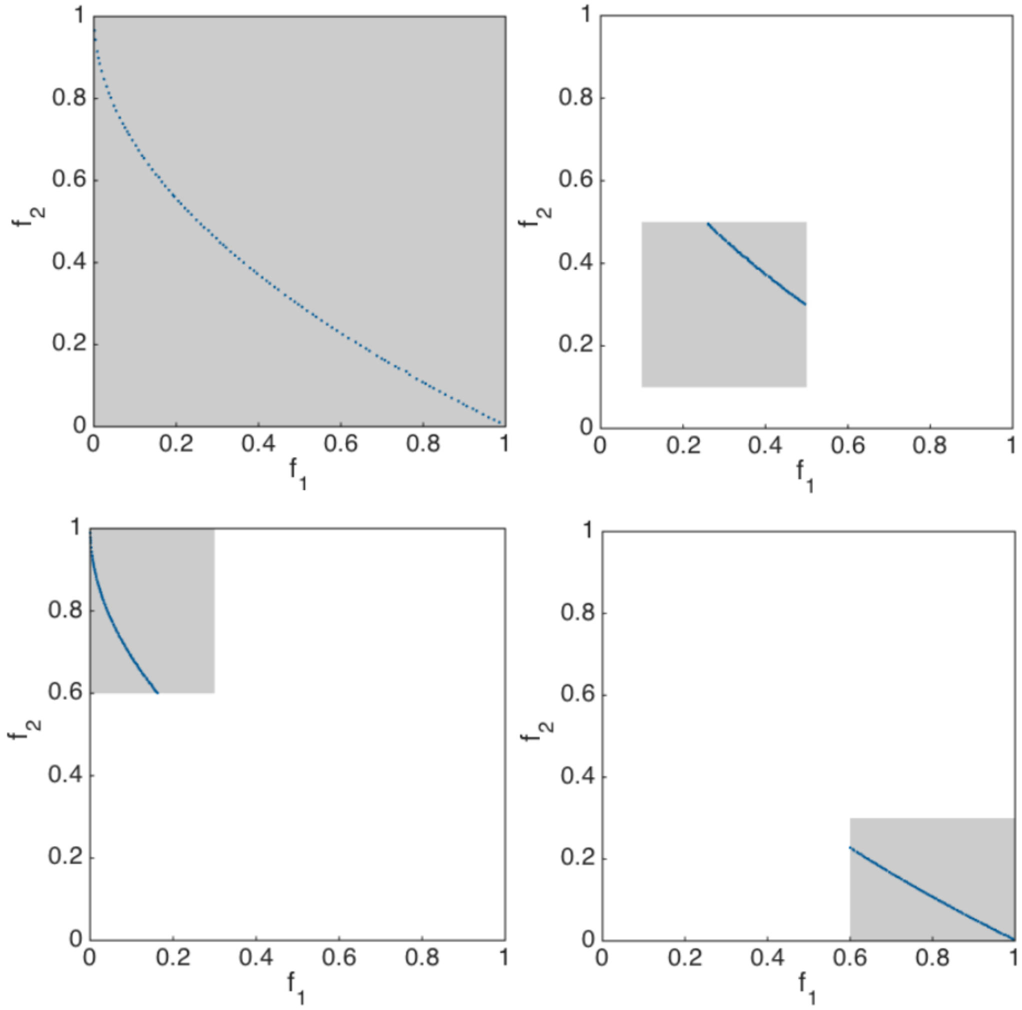

FIGURE 5.1: Representative PF approximations of T-SMS-EMOA on ZDT1.

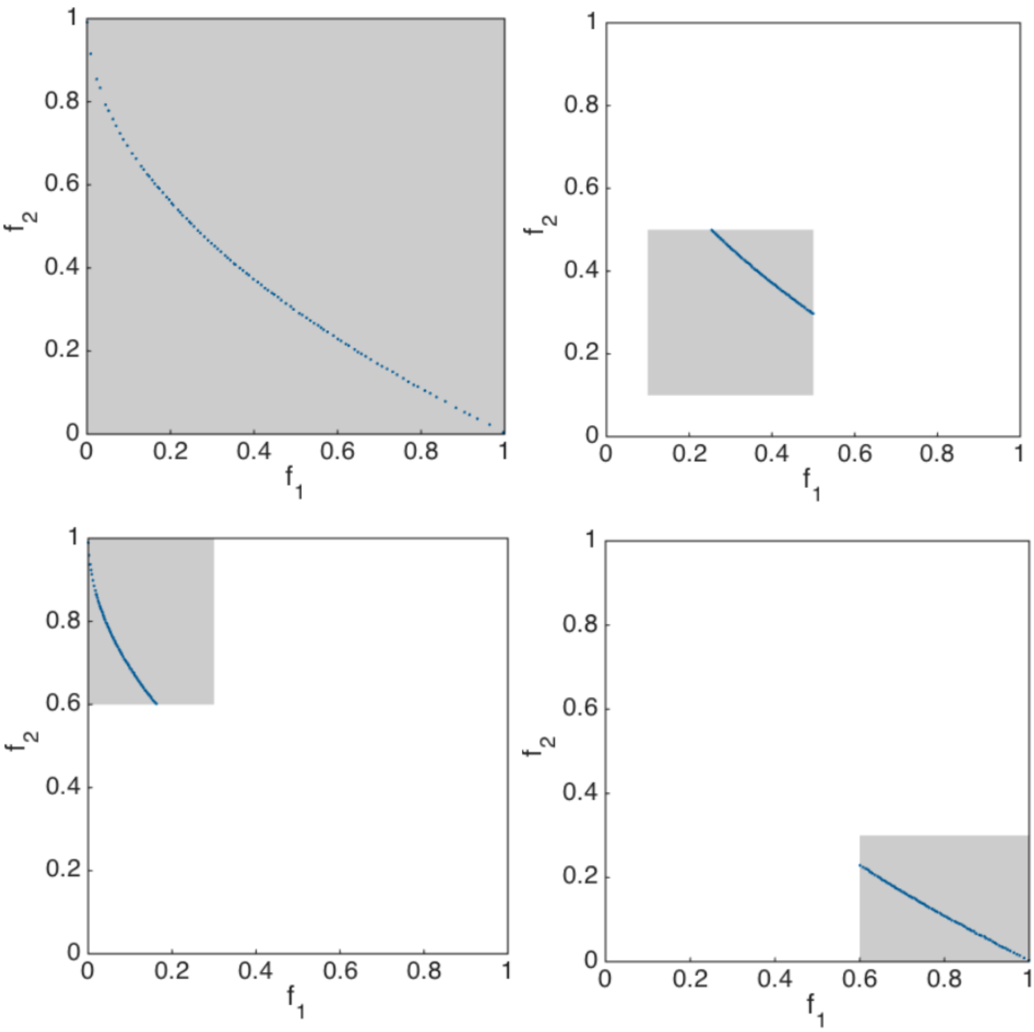

FIGURE 5.2: Representative PF approximations of T-R2-EMOA on ZDT1.

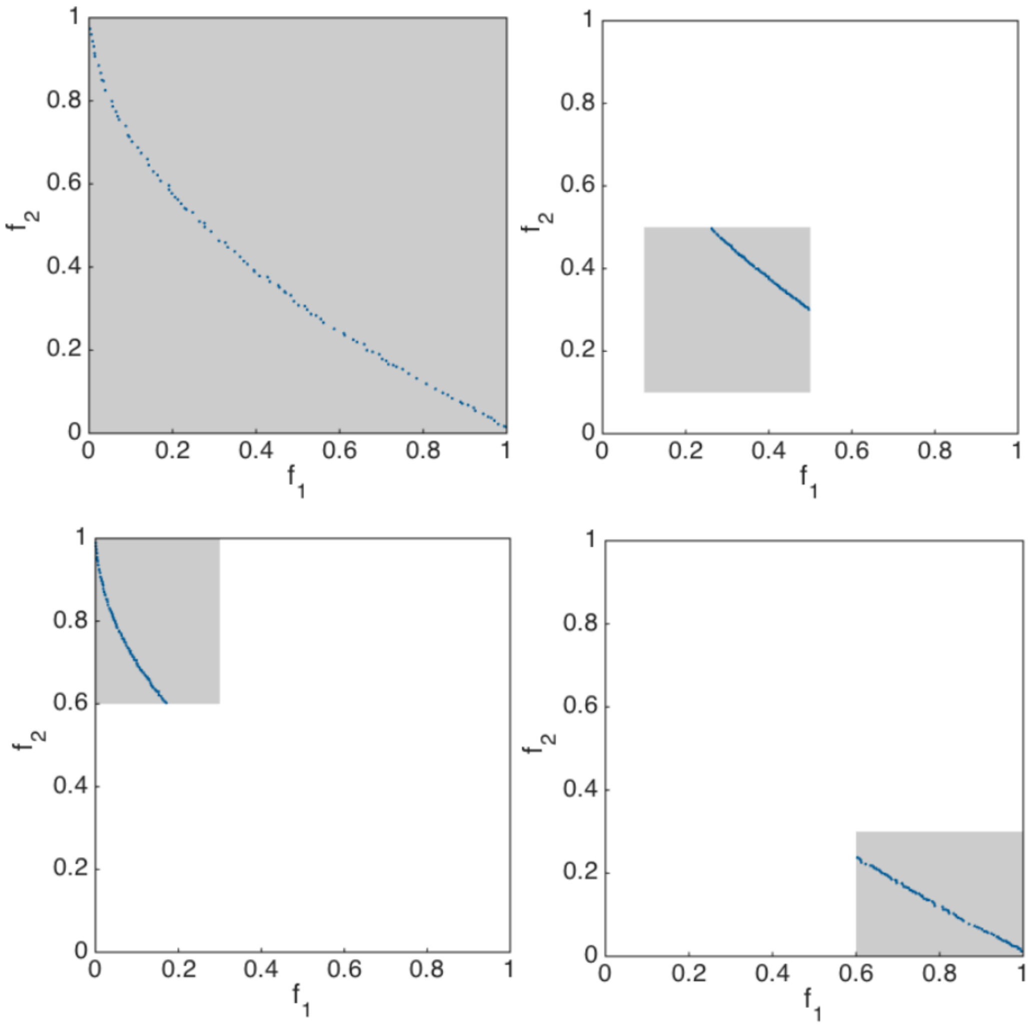

FIGURE 5.3: Representative PF approximations of T-NSGA-II on ZDT1.

Figure 5.1 $\sim$ Figure 5.3 show PF approximations obtained from new algorithms on the four different target regions in a random single run. The different target regions are highlighted by gray boxes and their lower and upper bounds are: upper left graph:(0,0)(1,1), upper right graph:(0.1,0.1)(0.5,0.5), lower left graph:(0,0.6)(0.3,1), lower right graph:(0.6,0)(1,0.3). It is observed that all three algorithms can find well-distributed and well-converged solutions on the PF in the target regions and no outliers exist. The solution set obtained by T-SMS-EMOA is more uniform than the solution sets obtained by the other two algorithms. It is also observable from upper left graph in Figure 5.2 that the R2 indicator has a bias towards the center of the PF.

We examine the performance of the new algorithms using the hypervolume metric. The hypervolume is calculated within the target region by normalizing the values of each objective to the values between 0 and 1 and using the lower bound of the target region as the reference point for the maximization problem and the upper bound of target region as the reference point for the minimization problem. Table 5.2 shows the median and variance of hypervolume over 30 runs. The statistical results correspond to the observation that T-SMS-EMOA outperforms T-R2-EMOA and T-NSGA-II slightly. The original SMS-EMOA, R2-EMOA and NSGA-II are also involved in the comparison and the results of the original MOEAs are obtained by firstly, presenting constraints in the description of problem, and secondly, presenting no constraints in the description of problem. It is demonstrated that the new algorithms obtain higher hypervolume value than original MOEAs with no constraint descriptions in the problem. Although the results of the proposed algorithms are not better than original MOEAs with constraints on the range of objectives, experiments show that the proposed algorithms can reduce computation time dramatically on this problem.

In the table, the symbol of "*" on the values for the same target region means the medians of these algorithms are significantly indifferent. The Mann-Whitney U test (also called the Mann-Whitney-Wilcoxon (MWW), Wilcoxon rank-sum test, or Wilcoxon-Mann-Whitney test) is used to determine if the medians of different algorithms for the same problem are significantly indifferent. The chances that the medians of T-SMS-EMOA and T-R2-EMOA are indifferent have been observed.

Box plots are used to visualize the distribution of hypervolume indicators of original MOEAs without constraints and new algorithms over 30 runs, as shown in Figure 5.4. In a box plot, the bottom and top of the box are the first and third quartiles. The line inside the box is the median of the data set. Two lines extend from the front and back of the box are called whiskers. The front whisker goes from quartile 1 to the smallest non-outlier in the data set, and the back whisker goes from quartile 3 to the largest non-outlier. Outliers are plotted as points.

TABLE 5.2: The median, variance of hypervolume and average computation time (Sec.) on ZDT1 with respect to different target regions and different algorithms

New Algorithms		T-SMS-EMOA	T-R2-EMOA	T-NSGA-II
Target Region	Metric			
(0,0)(1,1)	HV(m)	0.6580	0.6566	0.6425
	Variance	6.4e-06	1.4e-06	1.0e-05
	Time	24.99	74.01	0.21
(0.1,0.1)(0.5,0.5)	HV(m)	0.1640*	0.1638*	0.1543
	Variance	1.4e-06	1.5e-06	8.9e-06
	Time	10.30	23.61	0.19
(0,0.6)(0.3,1)	HV(m)	0.8110	0.8103	0.7936
	Variance	5.9e-06	6.0e-06	4.4e-05
	Time	12.86	31.78	0.20
(0.6,0)(1,0.3)	HV(m)	0.6255*	0.6233*	0.6079
	Variance	8.9e-06	6.7e-06	4.5e-05
	Time	11.45	27.92	0.21
Original Algorithms (Constraints)		SMS-EMOA	R2-EMOA	NSGA-II
(0,0)(1,1)	HV(m)	0.6621	0.6610	0.6609
	Variance	8.9e-11	1.2e-08	5.3e-08
	Time	108.57	314.99	0.25
(0.1,0.1)(0.5,0.5)	HV(m)	0.1694	0.1693	0.1690
	Variance	1.6e-11	1.1e-11	6.2e-09
	Time	106.32	274.05	0.23
(0,0.6)(0.3,1)	HV(m)	0.8197	0.8185	0.8191
	Variance	1.6e-08	4.6e-08	2.9e-08
	Time	105.73	271.00	0.21
(0.6,0)(1,0.3)	HV(m)	0.6364	0.6348	0.6356
	Variance	3.2e-09	2.2e-08	3.8e-08
	Time	101.82	283.3	0.22
Original Algorithms		SMS-EMOA	R2-EMOA	NSGA-II
(0,0)(1,1)	HV(m)	0.6558	0.6566	0.6362
	Variance	1.6e-06	8.5e-07	3.5e-05
	Time	26.77	73.34	0.21
(0.1,0.1)(0.5,0.5)	HV(m)	0.1545	0.1585	0.1236
	Variance	4.7e-06	2.2e-06	4.4e-05
	Time	24.17	74.85	0.20
(0,0.6)(0.3,1)	HV(m)	0.8012	0.7972	0.7649
	Variance	6.3e-06	6.4e-06	0.00013
	Time	24.85	71.90	0.20
(0.6,0)(1,0.3)	HV(m)	0.6119*	0.6110*	0.5604
	Variance	2.3e-05	7.4e-06	0.00014
	Time	26.29	78.93	0.20

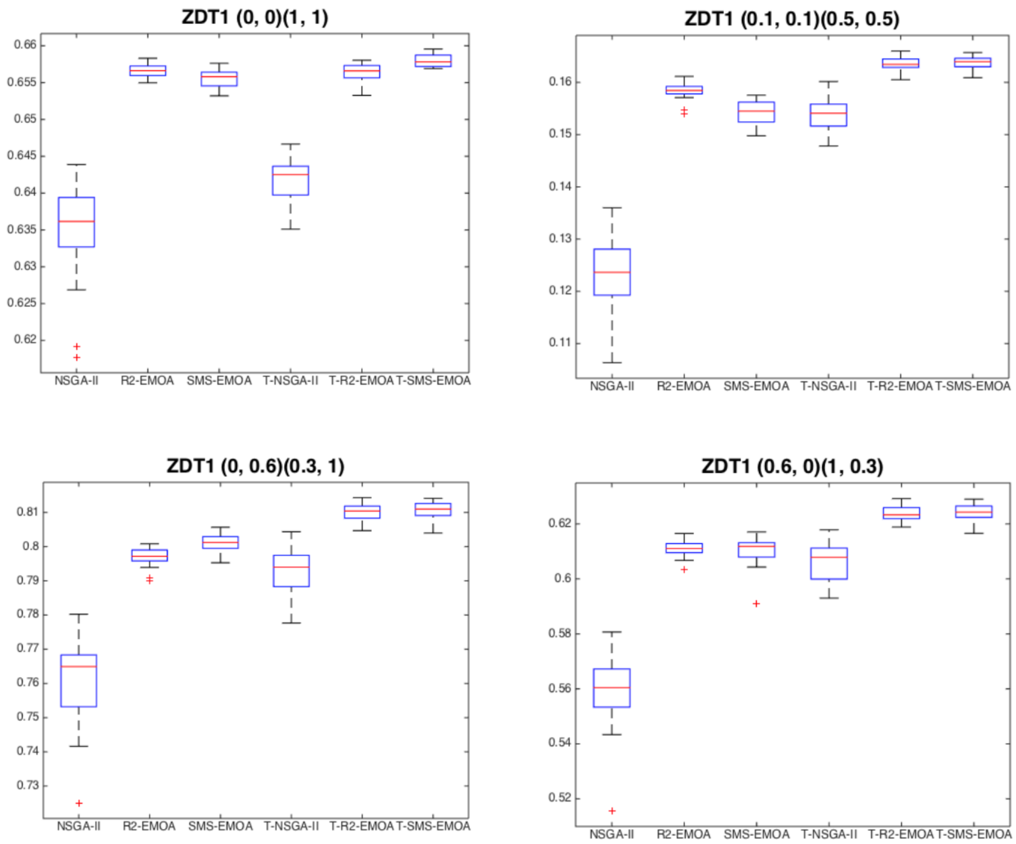

FIGURE 5.4: Boxplots comparing the hypervolume sets of six algorithms on ZDT1 with respect to different target regions.

Next, we consider the 30-variable ZDT2 and ZDT3 problem. ZDT2 has a non-convex Pareto optimal front and ZDT3 has a disconnected set of Pareto optimal front which consists of five non-contiguous convex parts. Circle target regions are adopted on ZDT2 and ZDT3 problems. A circle with a center point (1,0) and radius 0.5 intersects the whole PF of ZDT2 at its one end and a circle with a center point (0.6,0.5) and radius 0.3 intersects the whole PF at its central part. The two different circles are chosen as examples for target regions on ZDT2 problem. Experiments for a circle with a center point (0.3,0.1) and radius 0.3 as target region are conducted on ZDT3 problem.

Figure 5.5 shows PF approximation of T-SMS-EMOA in these target regions. Similar figures can also be achieved by T-R2-EMOA and T-NSGA-II. In the graph, the target regions are purple circles and center points are red points. Orange points denotes the results obtained from T-SMS-EMOA on provided preference information. Approximated optimal PF of ZDT2 problem for 100 blue points are from this page: `http://www.tik.ee.ethz.ch/sop/download/supplementary/testproblems/`. Statistical results of the median of hypervolume for three algorithms in 30 independent runs on each target region are shown in Table 5.3.

5.3 Three-Objective DTLZ Test Problems

In this section, we consider three-objective DTLZ1 and DTLZ2 test problems. The 7-variable DTLZ1 problem has a linear Pareto optimal front which is a three-dimensional,

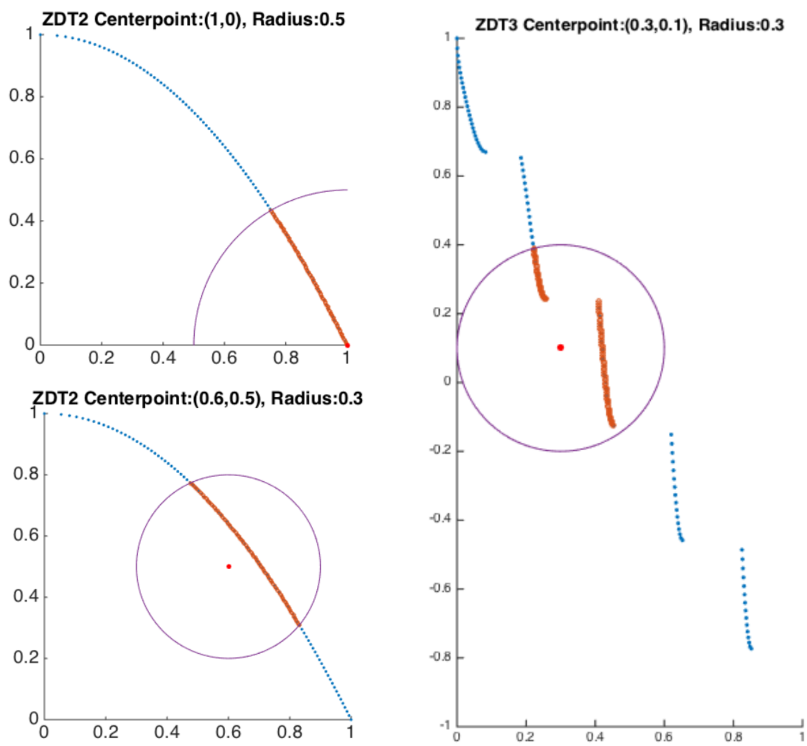

FIGURE 5.5: Representative PF approximations of T-SMS-EMOA on ZDT2 and ZDT3 with respect to different circular target regions.

TABLE 5.3: The median of hypervolume on ZDT2 and ZDT3 with a circulr target region

MOEA / Target Region	T-SMS-EMOA	T-R2-EMOA	T-NSGA-II
ZDT2 (1,0) 0.5	0.3168	0.3167	0.3159
ZDT2 (0.6,0.5) 0.3	0.3257	0.3256	0.3234
ZDT3 (0.3,0.1) 0.3	0.3377	0.3375	0.3365

triangular hyperplane. A sphere with the center point (0.3,0.3,0.3) and radius 0.3 is defined as the target region for three objective DTLZ1 problem. The 11-variable DTLZ2 problem has a three-dimensional, non-convex, Pareto optimal front. A box with the lower bound (0.4,0.4,0.2) and upper bound (0.8,0.8,0.8) is defined as the target region for three objective DTLZ2 problem.

Figure 5.6 shows PF approximation of three objective DTLZ1 problem. The center point of the target region is (0.3,0.3,0.3) and radius is 0.3. The graphs in the upper row are solutions of T-SMS-EMOA, graphs in the middle row are solutions of T-R2-EMOA and graphs in the lower row are solutions of T-NSGA-II. Approximated optimal PF for blue points are from the page: `http://jmetal.sourceforge.net/problems.html`. The transparent sphere depicts target region and red points are solutions obtained by T-SMS-EMOA, T-R2-EMOA and T-NSGA-II. We can observe that T-SMS-EMOA behaves best in three algorithms. Statistical results of the median of hypervolume in Table 5.4 are coincident with our observation.

TABLE 5.4: The median of hypervolume on three objective DTLZ1 problem with a spherical target region

MOEA / Target Region	T-SMS-EMOA	T-R2-EMOA	T-NSGA-II
(0.3,0.3,0.3) 0.3	0.8028	0.7992	0.7823

Figure 5.7 shows PF approximation of three objective DTLZ2 problem. The lower and upper bounds of the target region are (0.4.0.4,0.2) and (0.8,0.8,0.8). Approximated optimal PF for blue points are from the page: `http://jmetal.sourceforge.net/problems.html`. The transparent box depicts target region and red points are solutions obtained by T-SMS-EMOA. Statistical results of the median of hypervolume for three algorithms and original MOEAs with constraints description in 30 independent runs on cubic target region are shown in Table 5.5. It is observable that the best result is achieved by T-SMS-EMOA and all new algorithms outperform original MOEAs except for R2-EMOA.

TABLE 5.5: The median of hypervolume on three objective DTLZ2 problem with a cubic target region

MOEA / Target Region	T-SMS -EMOA	T-R2 -EMOA	T-NSGA -II	SMS-EMOA	R2-EMOA	NSGA -II
(0.4,0.4,0.2) (0.8,0.8,0.8)	0.4632	0.4303	0.4189	0.3369	0.4351	0.4185

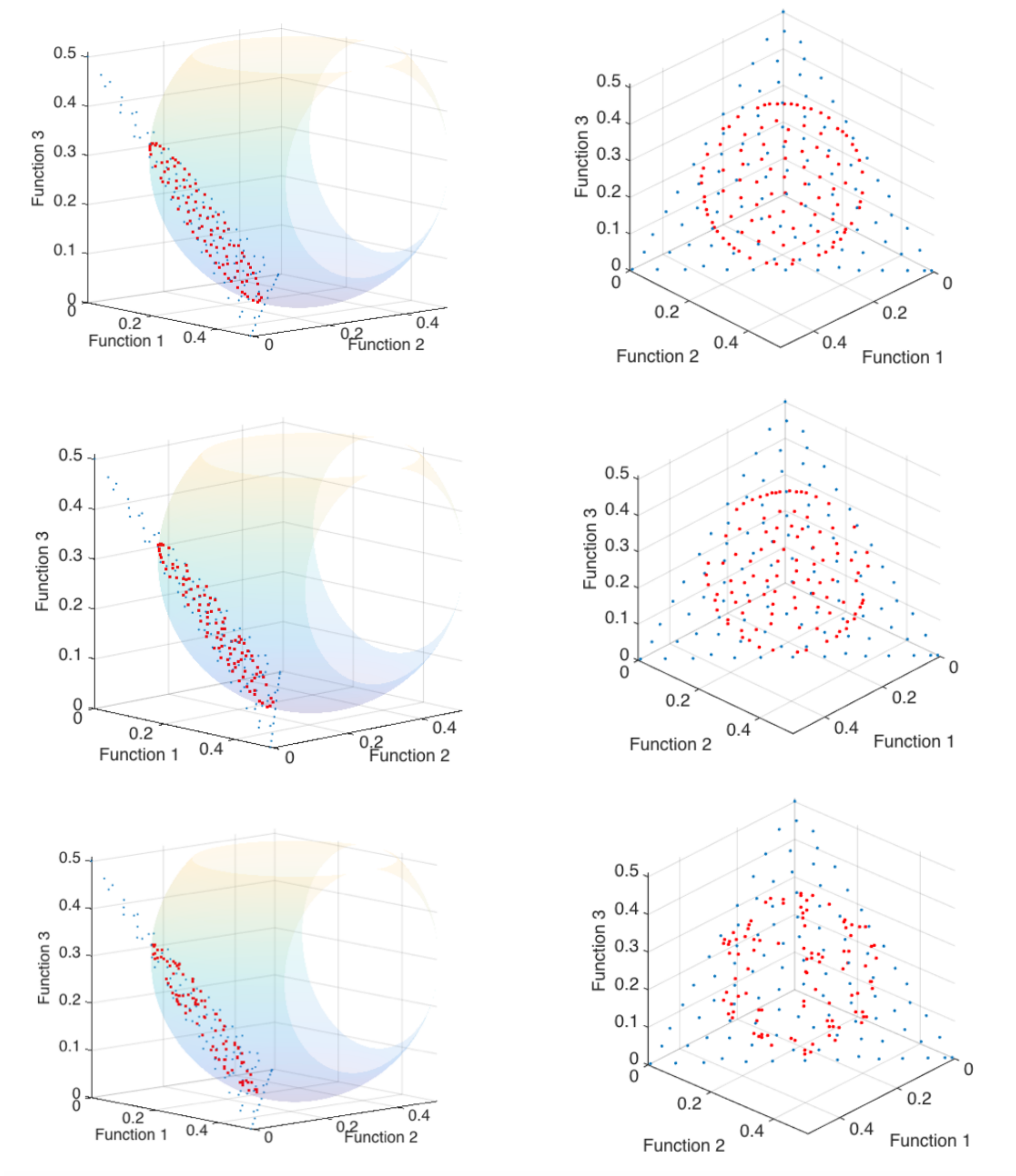

FIGURE 5.6: Representative PF approximations of T-SMS-EMOA(the upper row), T-R2-EMOA(the middle row) and T-NSGA-II(the lower row) in a spherical target region on three objective DTLZ1 problem.

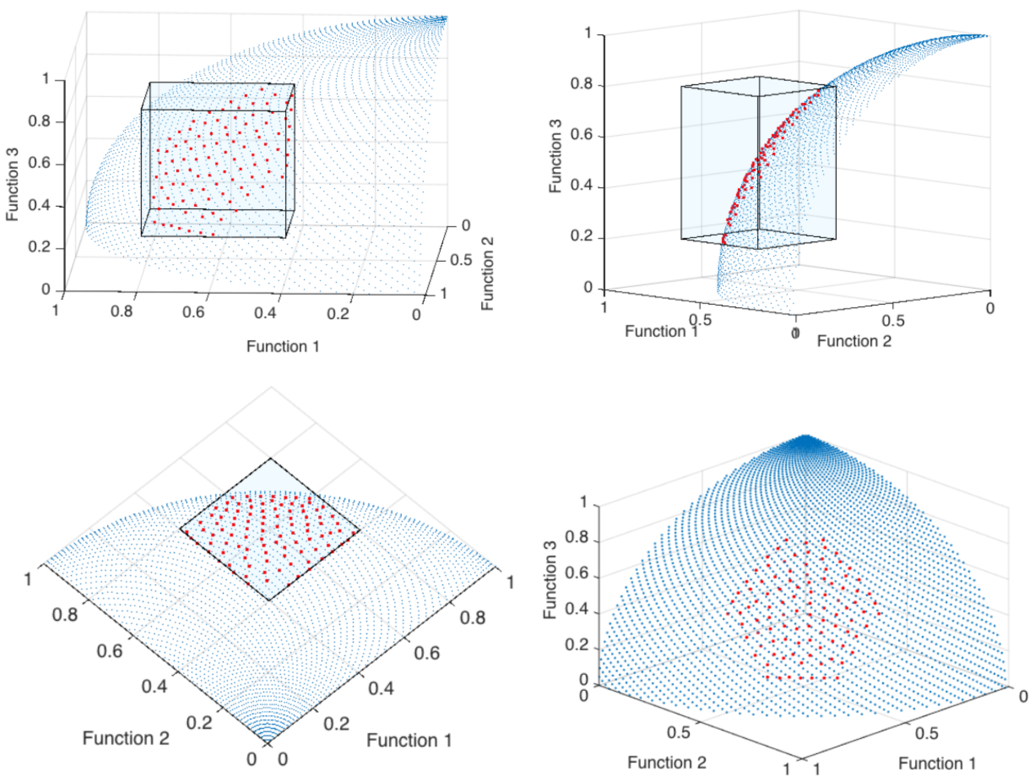

FIGURE 5.7: Representative PF approximations of T-SMS-EMOA in a cubic target region on three objective DTLZ2 problem.

5.4 Knapsack Problems

Knapsack Problems have been studied first by Dantzig in the late 1950′s (Dantzig, 1957). The problem is a general, understandable, and one of the most representative discrete optimization problem. At the same time, it is difficult to solve (NP-hard). In the study, we use the Multi-objective 0/1 Knapsack Problems from Zitzler and Thiele (Zitzler and Thiele, 1999) as discrete test problems. Formally, the multi-objective 0/1 knapsack problem can be formulated as the following maximization problem:

$$
\begin{aligned}
&maxmize \quad f(x) = (f_1(x), f_2(x), \cdots, f_n(x)) \\
&subject\ to \quad \sum_{j=1}^{m} w_{ij} x_j \leq c_i, \qquad i = 1, \cdots, n, \\
&\qquad\qquad\quad x_j \in \{0, 1\}, \qquad j = 1, \cdots, m, \\
&where \quad f_i(x) = \sum_{j=1}^{m} p_{ij} x_j, \qquad i = 1, \cdots, n,
\end{aligned}
$$

In this formulation, m is the number of items and n is the number of knapsacks, w_{ij} is the weight of item j according to knapsack i, p_{ij} is the profit of item j according to knapsack i and c_i is the capacity of knapsack i. The Multi-objective 0/1 Knapsack Problem is to find Pareto optimal vectors x = $(x_1, x_2, \cdots, x_m)$ and x_j = 1 when item j is selected and $x_j = 0$ otherwise.

Figure 5.8 shows solutions obtained in a random single run when the number of knapsack is 2 and the number of items is 250. The results of SMS-EMOA, R2-EMOA and NSGA-II are PF approximation without converting the target region into constraints. The target region for T-SMS-EMOA, T-R2-EMOA and T-NSGA-II is highlighted by the gray box. The lower bound is (9000,9000), the upper bound is (9800,9800).

Statistical results of the median of hypervolume are presented in Table 5.6. No additional constraints of the target region are converted in the description of the problem for the results of original MOEAs in the table. In our experiments, two and three objectives are taken into consideration, in combination with 250 and 500 items. The test data sets are from (Zitzler and Thiele, 1999). The target region of knapsack-250-2 is from (9000,9000) to (9800,9800), of knapsack-250-3 is from (8500,8500,8500) to (10000,10000,10000), of knapsack-500-2 is from (18000,18000) to (20000,20000), of knapsack-500-3 is from (17000,17000,17000) to (19000,19000,19000).

TABLE 5.6: The median of hypervolume on Knapsack problems with respect to different target regions

Algorithms Problems	T-SMS -EMOA	T-R2 -EMOA	T-NSG A-II	SMS- EMOA	R2- EMOA	NSGA -II
Knapsack -250-2	0.2117	0.2160	0.2230	0.2102	0.2099	0.1821
Knapsack -250-3	0.0295	0.0296	0.0311	0.0204	0.0293	0.0085
Knapsack -500-2	0.3273	0.3318	0.3230	0.3225	0.3272	0.2857
Knapsack -500-3	0.1936	0.1855	0.1713	0.1699	0.1747	0.0718

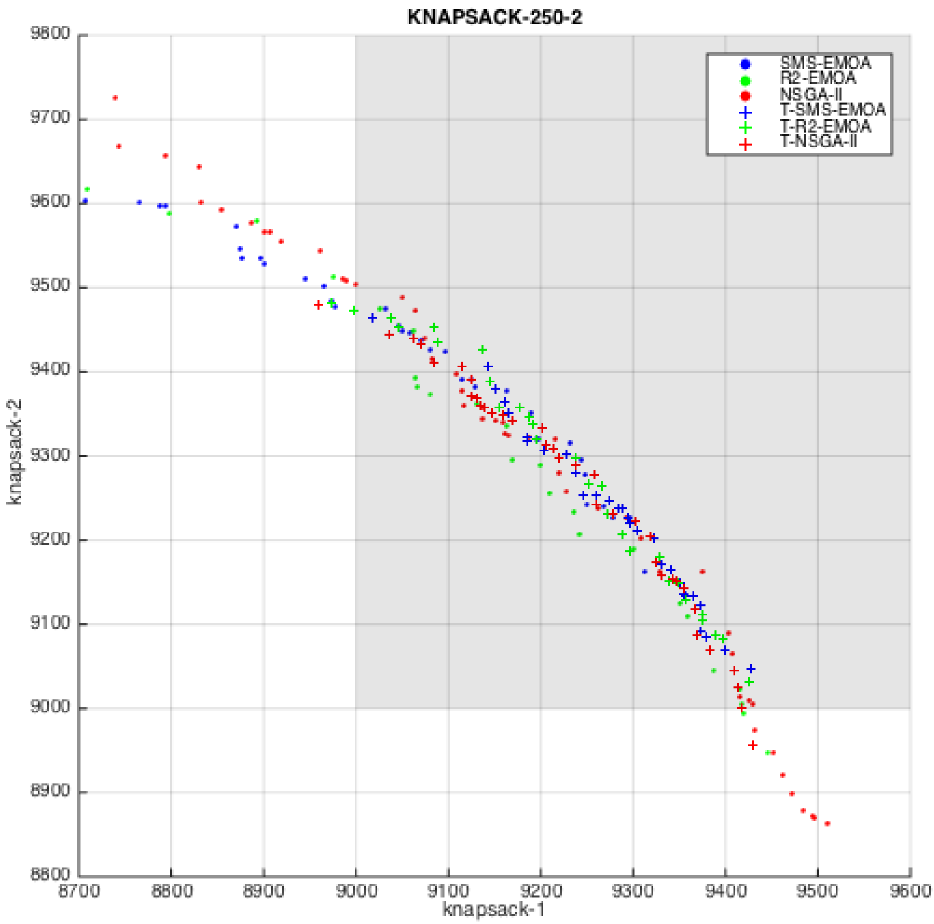

FIGURE 5.8: Representative PF approximations on knapsack-250-2 problem.

Chapter 6

Enhanced Algorithms and Experiments

6.1 Algorithm Improvement

As mentioned in Chapter 4 on page 7, the second ranking criterion (Hypervolume, R2 indicator or crowding distance) in new algorithms only works for solutions in the target region, which means if the intersection of the target region and true PF is empty, the second ranking criterion becomes useless. Under this condition, well-distributed solutions can not be obtained because solutions are guided only by Non-dominated sorting and the Chebyshev distance. In addition, if multiple target regions are specified, sometimes the obtained solutions only converge to one target region. Inspired by some ideas from R-NSGA-II (Deb and Sundar, 2006), two methods are adopted to overcome these limitations and strengthen the proposed algorithms.

6.1.1 Separate Population to Different Targets

The first method can attract the population to different targets and it is used in the calculation of both the second ranking criterion (Hypervolume, R2 indicator or crowding distance) and the third ranking criterion (the Chebyshev distance). The aim of this method is to support multiple targets.

Taking R2 indicator as an example, after coordinate transformation, for all target regions, R2 indicator values of all solutions on the worst ranked front are calculated and the solutions are sorted in descending order of R2 indicator. Thereafter, R2 indicator values are replaced by *R2 indicator ranks*: solutions with the largest R2 indicator values for all target regions are assigned the same largest *R2 indicator rank*, solutions having next-to-largest R2 indicator values for all target regions are assigned the same next-to-largest *R2 indicator rank*, and so on, until the number of solutions which have been assigned the *R2 indicator rank* for each target region reaches its proportion in population. If the uniform distribution of solutions for all target regions is expected, the proportion of each target region should be divided equally between all target regions. For example, when the number of target regions is two, the proportion for each target region should be $1/2$ and the number of solutions being assigned the *R2 indicator rank* should reach half of the size of the worst ranked front. Lastly, for solutions haven't been assigned the *R2 indicator rank*, their R2 indicator values should be mapped to values smaller than the least *R2 indicator rank*. One way to do this is to calculate their R2 indicator values in the combined region of all target regions and normalized them to values lower than all *R2 indicator ranks*. By this way, solutions with larger R2 indicator values in each target region are emphasized more.

The *Chebyshev rank* is used in place of the Chebyshev distance when the method is added to work with the third ranking criterion. First, the Chebyshev distances of each solution to the union of all targets are calculated and the solutions are sorted in ascending order of distance. The *Chebyshev rank* of the solutions closest to all targets is assigned to be the same smallest rank of zero, the *Chebyshev rank* of the solutions having next-to-smallest Chebyshev distance is assigned to be the same next-to-smallest rank of one, and so on, until the number of solutions which have been assigned the *Chebyshev rank* for each target reaches its proportion in population. Figure 6.1 shows an example of assigning *Chebyshev ranks* to solutions on the current worst ranked front. In the example, red Chebyshev ranks are assigned by sorting solutions in ascending order of the Chebyshev distance to Target Region 1 and blue Chebyshev ranks are assigned by sorting solutions in ascending order of the Chebyshev distance to Target Region 2, solutions with lower ranks are encouraged to remain in the population. The worst solution will be chosen from two solutions having the Chebyshev rank 3 randomly.

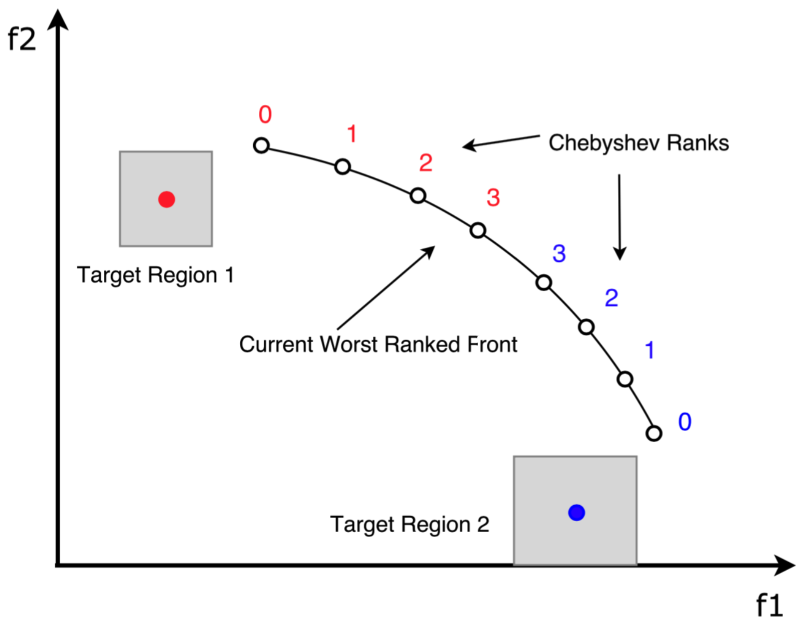

FIGURE 6.1: An example of assigning Chebyshev ranks to solutions on the current worst ranked front.

6.1.2 Improve Diversity

Under the circumstance that the second ranking criterion (Hypervolume, R2 indicator or crowding distance) doesn't work, the diversity is lost and the solution set would converge to one point when the number of evaluation is high enough. To solve the problem, a parameter ϵ is introduced in basic algorithms and works with the third ranking criterion (the Chebyshev distance) to improve the diversity. First, the solution with the shortest Chebyshev distance to the target is picked out. Thereafter, all other solutions having a Chebyshev distance less than the sum of the current shortest Chebyshev distance and ϵ are assigned a relatively large distance to discourage them to remain in the population. Then, another solution (not already considered earlier) is picked and the above procedure is performed again. This way, only one solution within a ϵ-neighborhood is emphasized and the diversity of the solution set is improved.

Figure 6.2 illustrates how the parameter ϵ takes effect when working in the calculation of the Chebyshev distance together. The Chebyshev distance between two vectors is the greatest of their differences along any coordinate dimension. Therefore, in the graph, the Chebyshev distances of solutions on the current font are distances to the center point along f_2. Apparently, point a is the point with the shortest Chebyshev distance to the center point, point b has a Chebyshev distance less than the sum of a's Chebyshev distance and ϵ, thus, it will be assigned an relatively large distance. So will point d and e. Point b, d and e are discouraged to remain in the population.

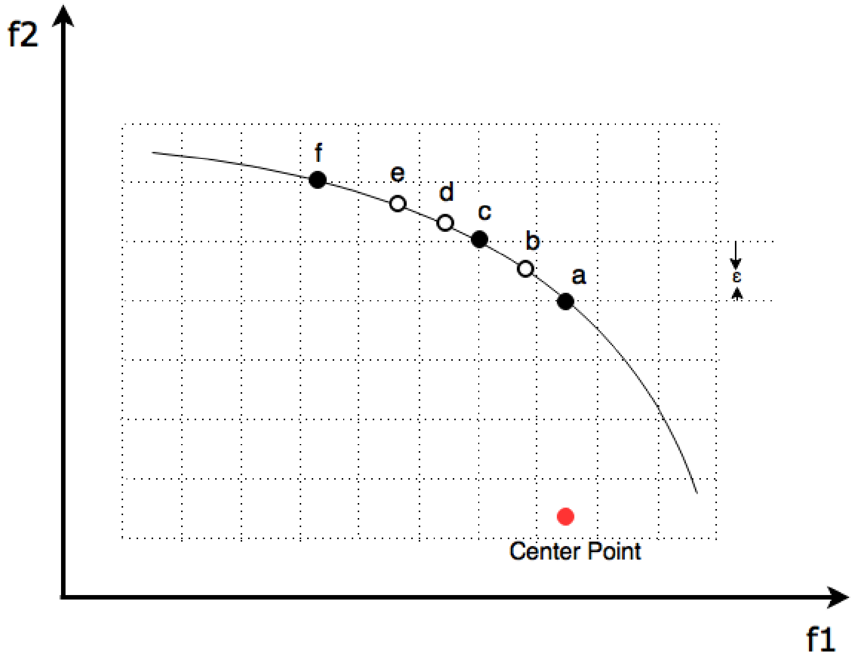

FIGURE 6.2: Illustration of ϵ parameter.

6.1.3 Structure of Enhanced Algorithms

Involving above two methods in basic algorithms, Algorithm 4, 5 and 6 show the structures of enhanced algorithms.

Algorithm 4 T-SMS-EMOA Enhanced version

$P_0 \leftarrow$ init( ) $/*$ Initialise random population $*/$
$t \leftarrow 0$
repeat
  $q_{t+1} \leftarrow$ generate(P_t) $/*$ generate offspring by variation $*/$
  $P_t = P_t \cup \{q_{t+1}\}$
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  for all targets:
    $\forall x \in R_v$: compute $D_{Ch}(x)/*$ Chebyshev distance to current target $*/$
    $R_v^{'} \leftarrow R_v/*$ sort in ascending order of $D_{Ch} */$
    for $x \in R_v^{'}$: $/ * x$ is not labelled, start from the first in $R_v^{'} */$
      Label x
      $\forall x^{'} \in R_v^{'} \setminus X \quad / * X$: the set of points have been labelled $*/$
        if $D_{Ch}(x^{'}) < D_{Ch}(x) + \epsilon$
          Label $x^{'}$
          $D_{Ch}(x^{'}) \leftarrow$ relatively large value
    $R_v \leftarrow R_v^{'}/*$ sort in ascending order of $D_{Ch} */$
    for $x \in R_v$: $/*$ only for assigned number of solutions $*/$
      $D_{Ch}(x) = D_{Ch}\_rank(x)/*$ start from the smallest D_{Ch}_rank: 0 $*/$
  for $x (\in R_v)$ without being assigned a D_{Ch}_rank:
    $D_{Ch}(x) = Normalized\_D_{Ch}(x)/ * Normalized\_D_{Ch}(x)$>largest $D_{Ch}\_rank */$
  $R_{v1} \cup R_{v2} \leftarrow R_v/*$solutions not in the target region $\rightarrow R_{v1}$; solutions in the target region $\rightarrow R_{v2} */$
  $\forall x \in R_{v1} : HC(x) = 0$
  $R_{v2} \leftarrow$ Coordinate Transformation(R_{v2})
  $\forall x \in R_{v2} : HC(x) = HV(R_{v2}) - HV(R_{v2} \backslash x)$
  for all targets:
    $R_v^{'} \leftarrow R_v/*$ sort in descending order of $HC */$
    for $x \in R_v^{'}$: $/*$ only for assigned number of solutions $*/$
      $HC(x) = HC\_rank(x)/*$ start from the largest $HC\_rank : |R_v^{'}| */$
  for $x (\in R_v^{'})$ without being assigned a HC_rank:
    $HC(x) = Normalized\_HC(x)/ * Normalized\_HC(x)$ <smallest $HC\_rank */$
  if unique $argmin\{HC(x) : x \in R_v\}$ exists
    $x^* = argmin\{HC(x) : x \in R_v\}$
  else
    $x^* = argmax\{D_{Ch}(x) : x \in R_v\}/*$ in case of tie, choose randomly $*/$
  $P_{t+1} = P \backslash \{x^*\}$
  $t \leftarrow t + 1$
until termination condition fulfilled

Algorithm 5 T-R2-EMOA Enhanced version

$P_0 \leftarrow$ init( ) /$*$ Initialise random population $*$/
$t \leftarrow 0$
repeat
  $q_{t+1} \leftarrow$ generate(P_t) /$*$ generate offspring by variation $*$/
  $P_t = P_t \cup \{q_{t+1}\}$
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  for all targets:
    $\forall x \in R_v$: compute $D_{Ch}(x)$/$*$ Chebyshev distance to current target $*$/
    $R_v^{'} \leftarrow R_v$/$*$ sort in ascending order of $D_{Ch} *$ /
    for $x \in R_v^{'}$: $/ * x$ is not labelled, start from the first in $R_v^{'} *$ /
      Label x
      $\forall x^{'} \in R_v^{'} \setminus X \quad / * X$: the set of points have been labelled $*$/
        if $D_{Ch}(x^{'}) < D_{Ch}(x) + \epsilon$
          Label $x^{'}$
          $D_{Ch}(x^{'}) \leftarrow$ relatively large value
    $R_v \leftarrow R_v^{'}$/$*$ sort in ascending order of $D_{Ch} *$ /
    for $x \in R_v$: /$*$ only for assigned number of solutions $*$/
      $D_{Ch}(x) = D_{Ch}\_rank(x)$/$*$ start from the smallest D_{Ch}_rank: 0 $*$/
  for $x (\in R_v)$ without being assigned a D_{Ch}_rank:
    $D_{Ch}(x) = Normalized\_D_{Ch}(x) / * Normalized\_D_{Ch}(x)$>largest $D_{Ch}\_rank *$ /
  $R_{v1} \cup R_{v2} \leftarrow R_v$/$*$solutions not in the target region $\rightarrow R_{v1}$; solutions in the target region $\rightarrow R_{v2} *$ /
  $\forall x \in R_{v1} : r(x) = 0$
  $R_{v2} \leftarrow$ Coordinate Transformation(R_{v2})
  $\forall x \in R_{v2} : r(x) = R2(P \backslash \{x\}; \Lambda; i) / * i$: ideal point$*$/
  for all targets:
    $R_v^{'} \leftarrow R_v$/$*$ sort in descending order of $R2 *$ /
    for $x \in R_v^{'}$: /$*$ only for assigned number of solutions $*$/
      $r(x) = r\_rank(x)$/$*$ start from the largest $r\_rank : |R_v^{'}| *$ /
  for $x (\in R_v^{'})$ without being assigned a r_rank:
    $r(x) = Normalized\_r(x) / * Normalized\_r(x)$ <smallest $r\_rank *$ /
  if unique $argmin\{r(x) : x \in R_v\}$ exists
    $x^* = argmin\{r(x) : x \in R_v\}$
  else
    $x^* = argmax\{D_{Ch}(x) : x \in R_v\}$/$*$in case of tie, choose randomly$*$/
  $P_{t+1} = P \backslash \{x^*\}$
  $t \leftarrow t + 1$
until termination condition fulfilled

Algorithm 6 T-NSGA-II Enhanced version

$P_0 \leftarrow$ init( ) /$*$ Initialise random population $*$/
$t \leftarrow 0$
repeat
  $Q_t \leftarrow$ generate(P_t) /$*$ generate offsprings by variation $*$/
  $P_t = P_t \cup Q_t$
  for all targets:
    $\forall x \in P_t$: compute $D_{Ch}(x)$/$*$ Chebyshev distance to current target $*$/
    $P_t^{'} \leftarrow P_t$/$*$ sort in ascending order of $D_{Ch} * /$
    for $x \in P_t^{'}$: $/ * x$ is not labelled, start from the first in $P_t^{'} * /$
      Label x
      $\forall x^{'} \in P_t^{'} \setminus X \quad / * X$: the set of points have been labelled $*/$
        if $D_{Ch}(x^{'}) < D_{Ch}(x) + \epsilon$
          Label $x^{'}$
          $D_{Ch}(x^{'}) \leftarrow$ relatively large value
    $P_t \leftarrow P_t^{'}$/$*$ sort in ascending order of $D_{Ch} * /$
    for $x \in P_t$: /$*$ only for assigned number of solutions $*$/
      $D_{Ch}(x) = D_{Ch}\_rank(x)$/$*$ start from the smallest D_{Ch}_rank: 0 $*$/
  for $x(\in P_t)$ without being assigned a D_{Ch}_rank:
    $D_{Ch}(x) = Normalized\_D_{Ch}(x) / * Normalized\_D_{Ch}(x)$>largest $D_{Ch}\_rank * /$
  $\{R_1, ..., R_v\} \leftarrow$ fast-nondominated-sort(P_t)
  for i = rank 1,...,v do
    $R_{i1} \cup R_{i2} \leftarrow R_i$/$*$solutions not in the target region $\rightarrow R_{i1}$; solutions in the target region $\rightarrow R_{i2} * /$
    $\forall x \in R_{i1} : D_c(x) = 0 / * D_c$: crowding distance$*$/
    $R_{i2} \leftarrow$ Coordinate Transformation(R_{i2})
    $\forall x \in R_{i2}$: compute $D_c(x)$
    for all targets:
      $R_i^{'} \leftarrow R_i$/$*$ sort in descending order of $D_c * /$
      for $x \in R_i^{'}$: /$*$ only for assigned number of solutions $*$/
        $D_c(x) = D_c\_rank(x)$/$*$ start from the largest $D_c\_rank : |R_i^{'}| * /$
    for $x(\in R_i^{'})$ without being assigned a D_c_rank:
      $D_c(x) = Normalized\_D_c(x) / * Normalized\_D_c(x)$ <smallest $D_c\_rank * /$
  $P_{t+1} \leftarrow$ half the size of P_t based on rank, D_c and then D_{Ch}
  $t \leftarrow t + 1$
until termination condition fulfilled

6.2 Experiments on Improved Algorithms

6.2.1 Multiple Target Regions

If multiple target regions of interest can be found simultaneously, the DM can make a more effective selection towards finding ultimate preferred solution(s). The enhanced algorithms in Section 6.1 can guide the search toward multiple target regions. Three pair of spherical target regions are used on three objective DTLZ1 problems separately to demonstrate differences of results between T-SMS-EMOA (Figure 6.3), T-R2-EMOA (Figure 6.4) and T-NSGA-II (Figure 6.5). The center point and radius of two target regions are shown above each graph. The first pair of target regions have same radius and both center points are on the PF which is a three-dimensional, triangular hyperplane. The second pair of target regions have same radius, but one center point is on the PF, the other is not. The third pair of target regions have different radius, but both center points are on the PF. Experimental results over consecutive 30 runs show that all three algorithms can obtain uniform solutions in two target regions, no outliers exist. When the assigned population size is 100, each target region obtains 50 solutions for all 30 runs. While when the number of runs increases to 50, the solution of 49 and 51 also appears once.

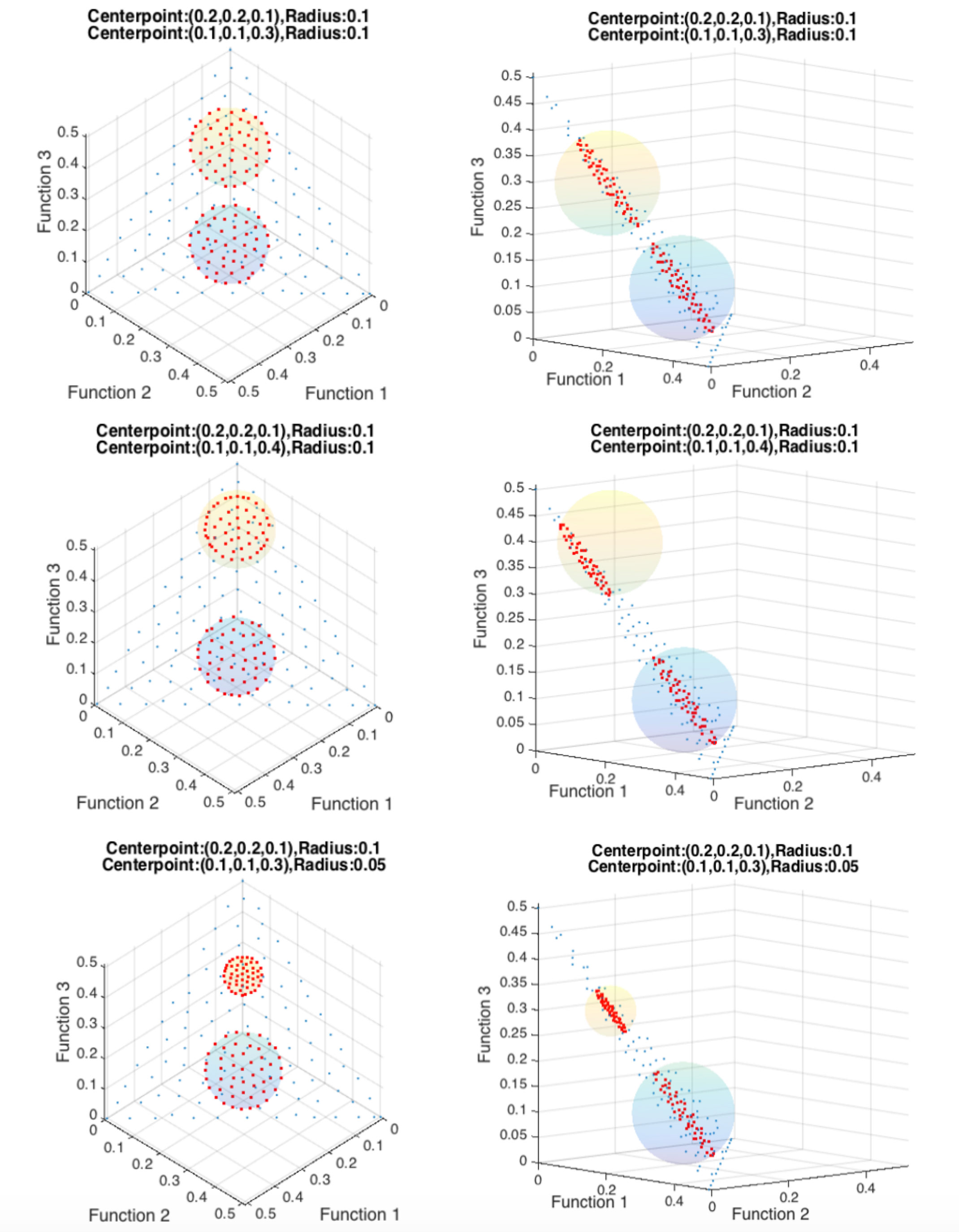

FIGURE 6.3: Representative PF approximations of T-SMS-EMOA in two spherical target regions on three objective DTLZ1.

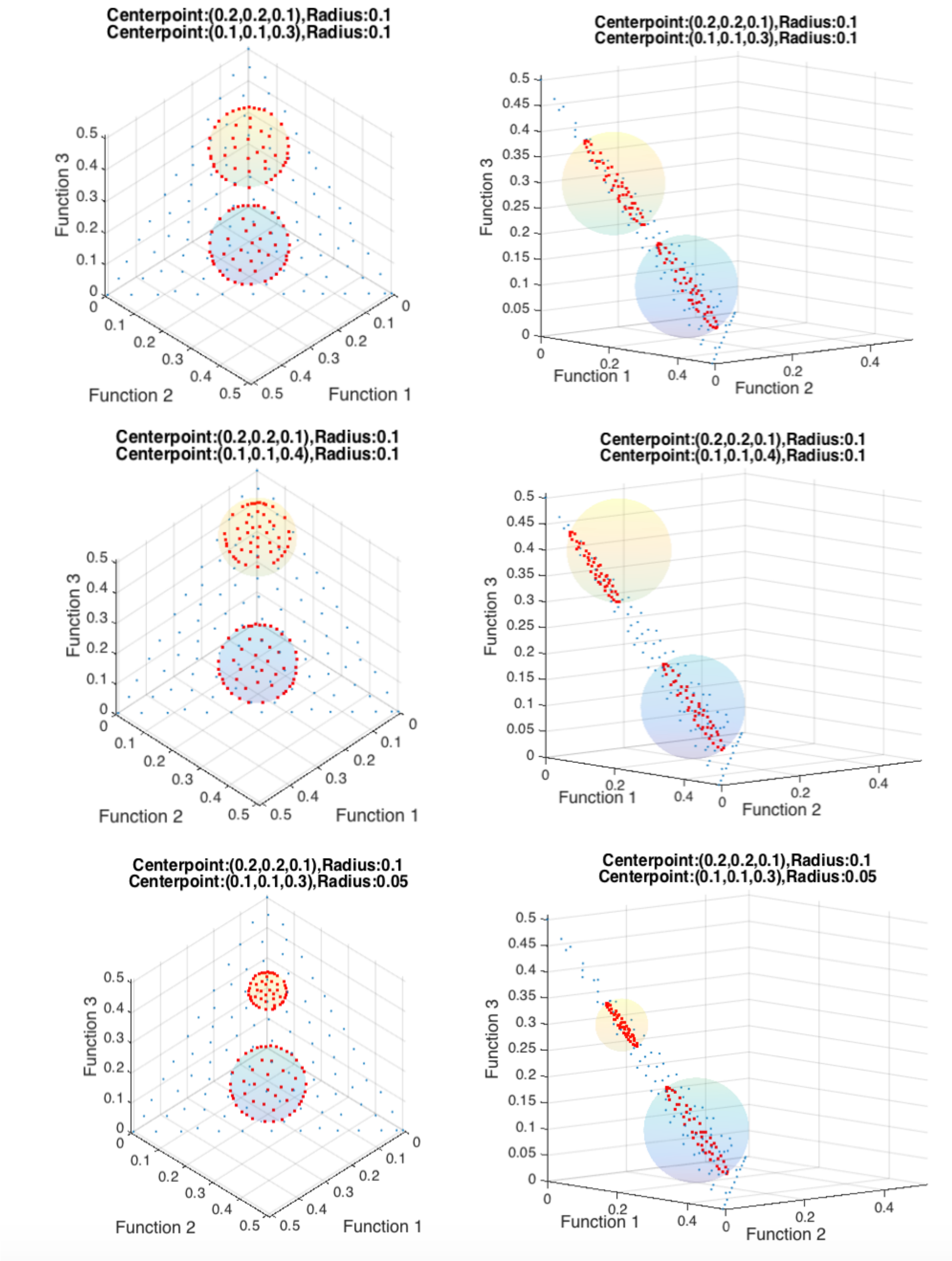

FIGURE 6.4: Representative PF approximations of T-R2-EMOA in two spherical target regions on three objective DTLZ1.

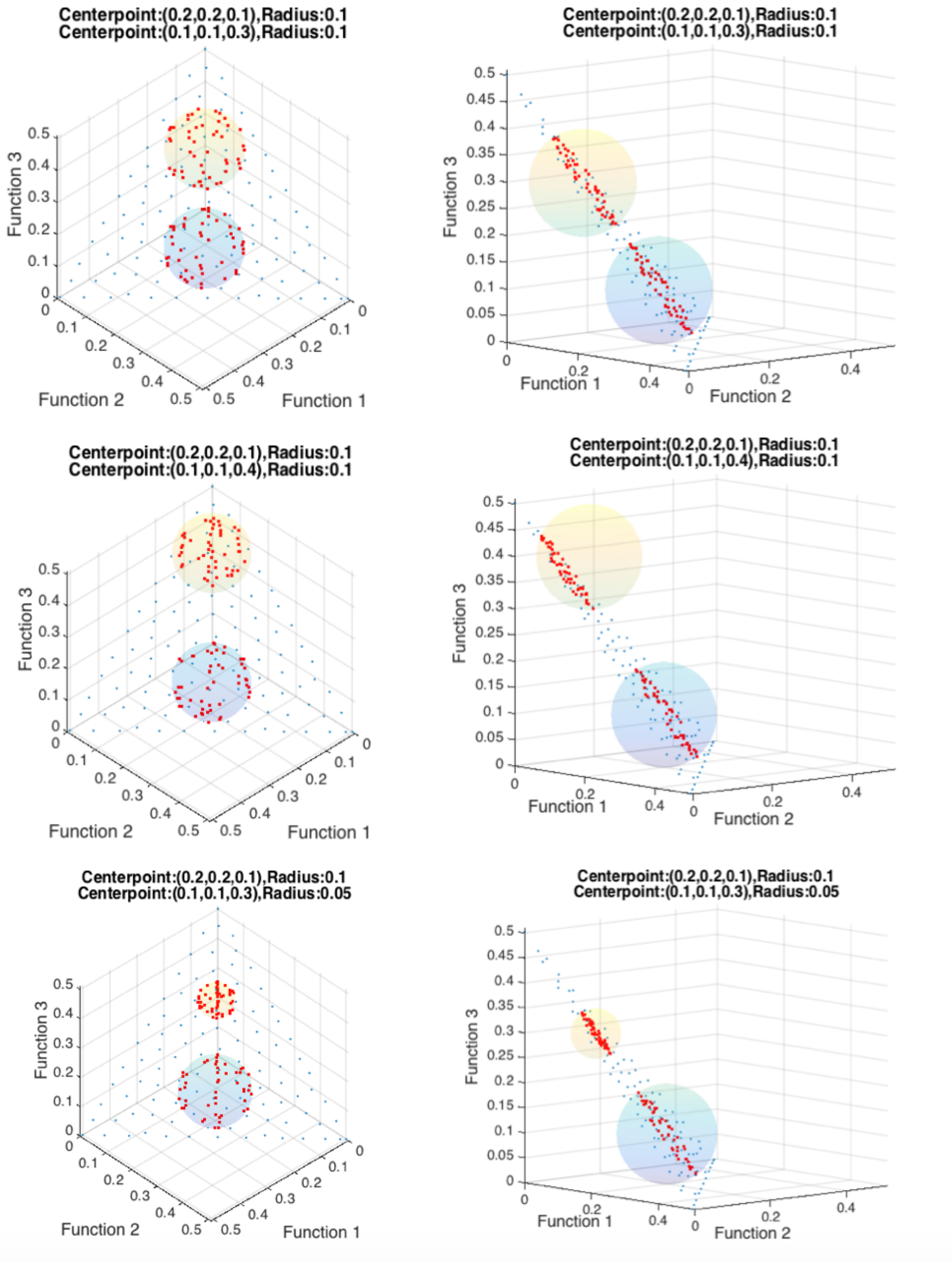

FIGURE 6.5: Representative PF approximations of T-NSGA-II in two spherical target regions on three objective DTLZ1.

In the above experiments for T-SMS-EMOA (Figure 6.3), T-R2-EMOA (Figure 6.4) and T-NSGA-II (Figure 6.5), we specify that solutions are equally distributed in multiple target regions. For the population size of 100, this means that there are 50 solutions in each target region. It is also possible that we hope different proportion of solutions for each target region. For example, we find that the real intersection areas of the third pair of target regions and the PF are obviously different. Therefore, $1/4th$ of population size is specified as the number of obtained solutions in the small intersection area and $3/4th$ of population size is specified as the number of obtained solutions in the larger intersection area. Figure 6.6 shows the difference between equally distributed solutions (left graph) and solutions with specified proportion for each target region (right graph) of T-SMS-EMOA.

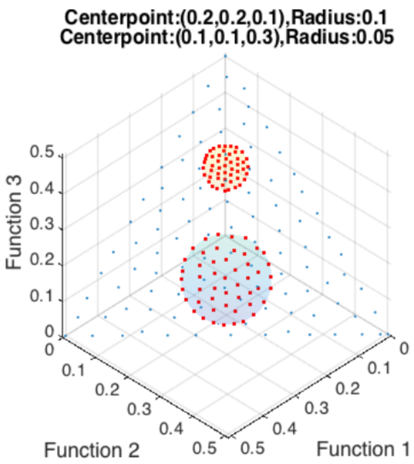

FIGURE 6.6: Representative PF approximations of T-SMS-EMOA with different solution distribution in two spherical target regions on three objective DTLZ1.

When there is no intersection between the target region and the PF, the enhanced algorithms can still obtain solutions close to the target region. Figure 6.7 shows PF approximations for different target regions which don't intersect with the PF.

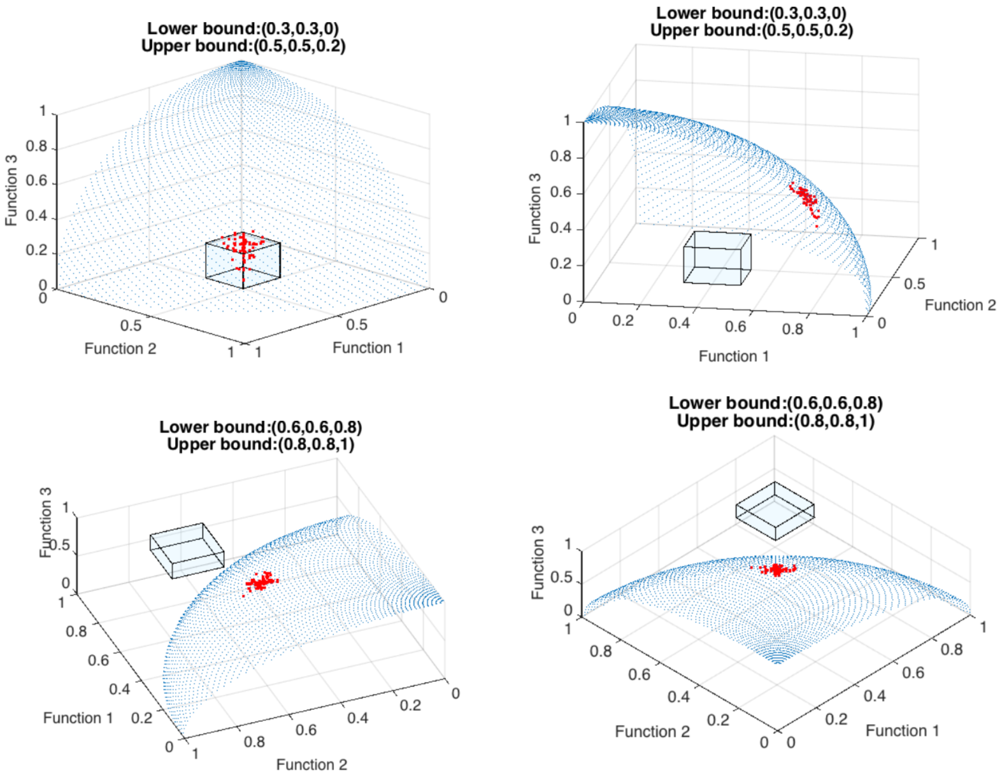

FIGURE 6.7: Representative PF approximations of T-SMS-EMOA on three objective DTLZ2; ϵ=0.001.

6.2.2 Single Target Point

The enhanced algorithms are not only capable of obtaining solutions in the target region, but also they may belong to reference point-based approaches. When the lower bound and the upper bound of the target region specified in the algorithms are the same, the target region shrinks to a target point. In this section, only results of T-SMS-EMOA are presented, T-R2-EMOA and T-NSGA-II can obtain similar results. Figure 6.8 shows PF approximations of T-SMS-EMOA for different single target point: point around the PF, point near the border, point in the feasible area and point in the infeasible area.

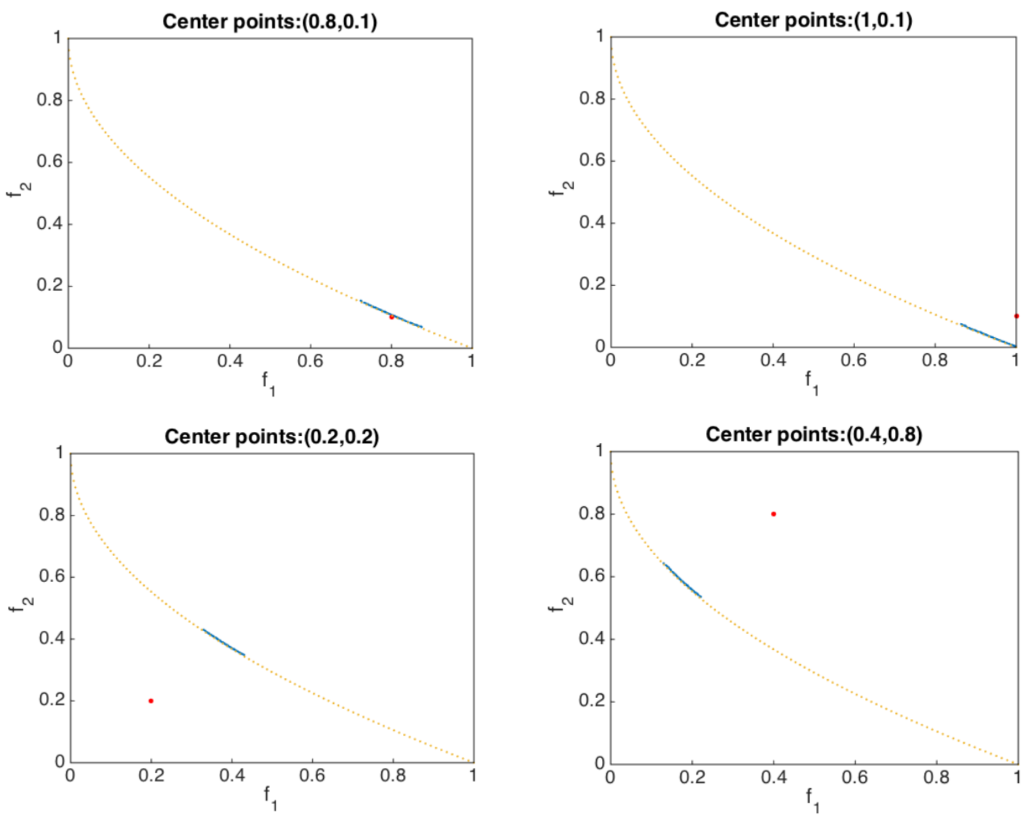

FIGURE 6.8: Representative PF approximations of T-SMS-EMOA on ZDT1; ϵ=0.0001.

For three objective problem, the parameter ϵ plays an essential role in balancing convergence and diversity of the solutions near the target point. Figure 6.9 shows PF approximations of T-SMS-EMOA for one target point when the values of parameter ϵ are different. The black point is the target point and red points are obtained solutions; blue points indicate the PF. It is observed that when the parameter ϵ is smaller, obtained solutions are denser and more concentrated.

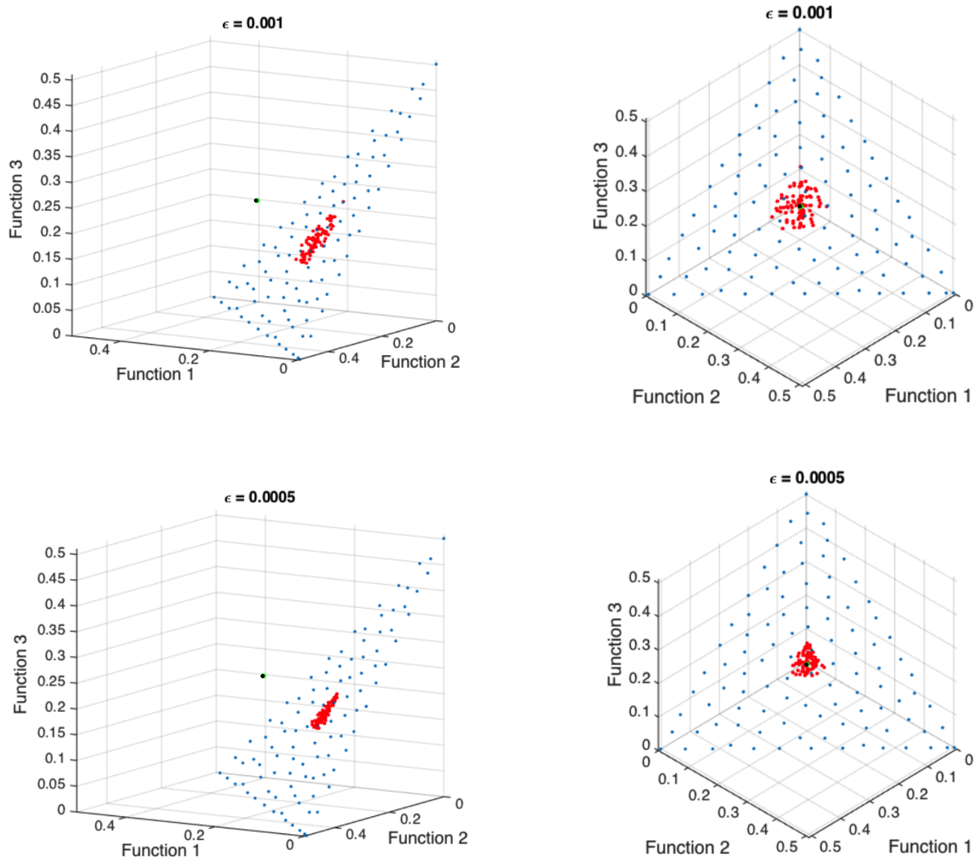

FIGURE 6.9: Representative PF approximations of T-SMS-EMOA on three objective DTLZ1 problem for one target point: (0.25, 0.25, 0.25).

6.2.3 Multiple Target Points

The enhanced algorithms can also work on multiple target points. Increasing the number of evaluation to 20000, Figure 6.10 shows PF approximations of T-SMS-EMOA for two target points on ZDT1 problem when the values of parameter ϵ are different. The red points are the target points and blue points are obtained solutions; purple points indicate the PF.

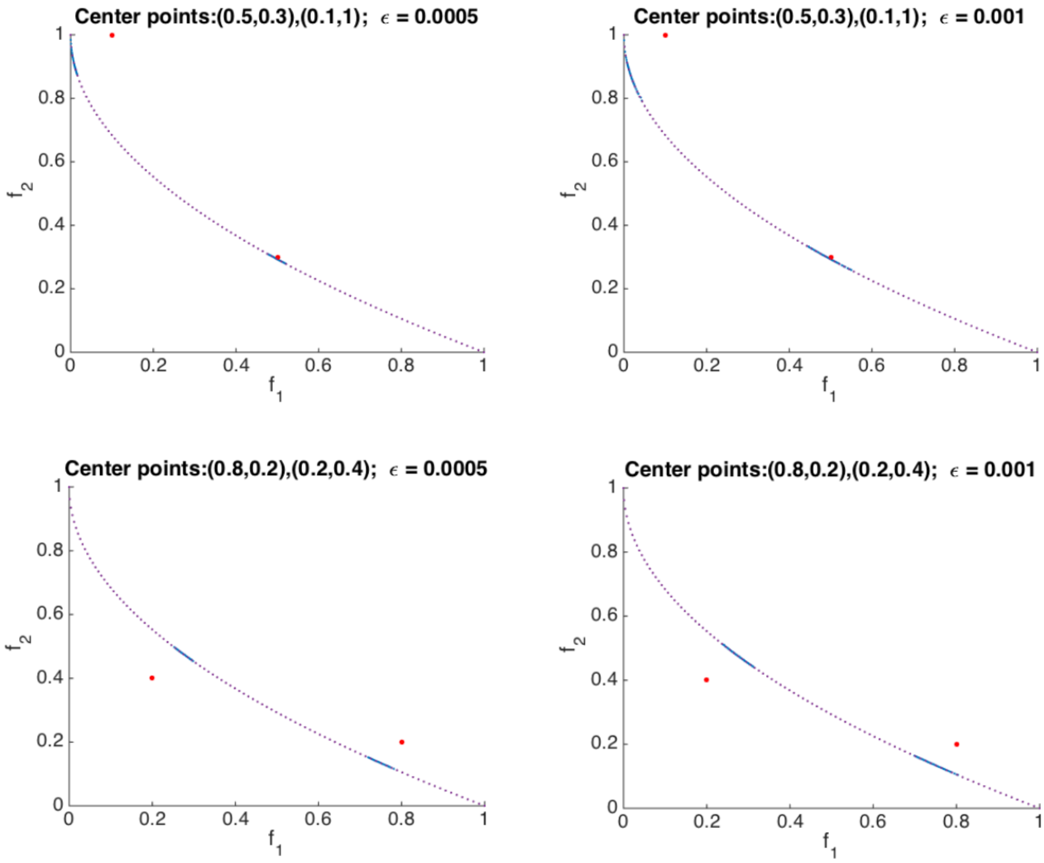

FIGURE 6.10: Representative PF approximations of T-SMS-EMOA on ZDT1 for two target points.

Increasing the number of evaluation to 50000, Figure 6.11 shows PF approximations of T-SMS-EMOA for two target points on three objective DTLZ1 problem when the parameter ϵ are different. The black points are the target points and red points are obtained solutions; blue points indicate PF.

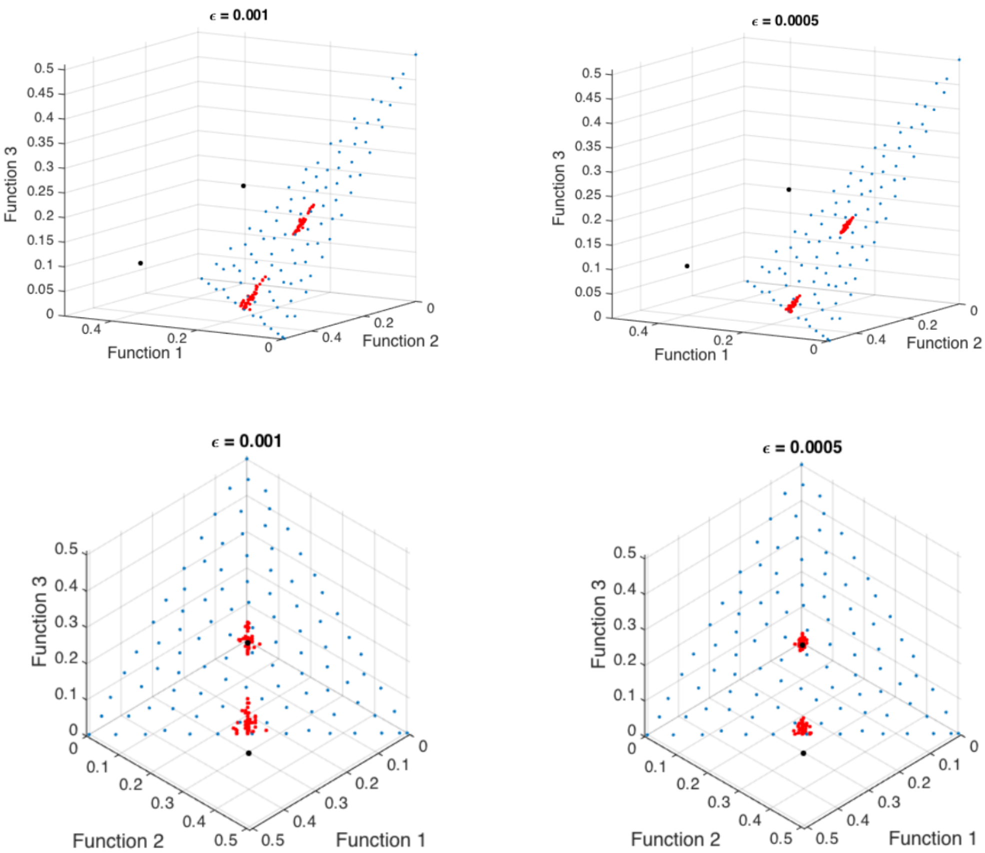

FIGURE 6.11: Representative PF approximations of T-SMS-EMOA on three objective DTLZ1 problem for two target points: (0.25, 0.25, 0.25) (0.4, 0.4, 0.1).

Chapter 7

Conclusion

In the thesis, a new target region based multi-objective evolutionary approach has been proposed. Three algorithms named T-SMS-EMOA, T-R2-EMOA and T-NSGA-II have been instantiated when combining the common algorithm framework with original SMS-EMOA, R2-EMOA and NSGA-II algorithm. These new algorithms have been applied to a number of continuous and combinational benchmark problems with two or three objectives. Experimental results show that the proposed algorithms can guide the search toward the preferred region on the Pareto optimal front. No outliers appear on a large number of evaluations. In addition, these basic algorithms have been improved. Enhanced algorithms are more powerful and do not only support multiple target regions but also target point(s). It is worth noting that different number of solutions can be allocated to different targets by assigning the proportion of population size for each target.

Also, the proposed algorithms presented similar performance with the original MOEAs on several tested problems by converting the target region into constraints in the problem description. The proposed algorithms save computational effort by guiding the search towards the preferred region without the calculation of the second ranking criterion in initial iterations. On the contrary, for original MOEAs, the increase in the number of constraints leads to the decrease of the search ability. Moreover, comparing with the original MOEAs, the proposed algorithms exhibit the trend of behaving better with the increase in the number of objectives. More importantly, when there is no intersection between targets and the PF, the proposed algorithms can still find Pareto optimal solutions close to the targets.

Very recently A new approach (Cheng et al., 2016) for multiple target region selection using sets of reference vectors. It will be interesting to compare to it in the future work.

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