Universiteit Leiden

Opleiding Informatica & Economie

Examining different strategies

for the card game Sueca

Name: Vanessa Lopes Carreiro

Date: July 13, 2016

1st supervisor: Walter Kosters
2nd supervisor: Rudy van Vliet

BACHELOR THESIS

Leiden Institute of Advanced Computer Science (LIACS)
Leiden University
Niels Bohrweg 1
2333 CA Leiden
The Netherlands

Examining different strategies for the card game Sueca

Vanessa Lopes Carreiro

Abstract

Sueca is a point-trick card game with trumps popular in Portugal, Brazil and Angola. There has not been done any research into Sueca. In this thesis we will study the card game into detail and examine different playing strategies, one of them being basic Monte-Carlo Tree Search. The purpose is to see what strategies can be used to play the card game best. It turns out that the basic Monte-Carlo strategy plays best when both team members play that strategy.

Acknowledgements

I would like to thank my supervisor Walter Kosters for brainstorming with me about the research and support but also for the conversations about life. It was a pleasure working with him. I am also grateful for Rudy van Vliet for being my second reader and taking time to read this thesis and providing feedback.

Contents

Chapter 1

Introduction

In this thesis we study the game Sueca, a point-trick card game with trumps. It is very popular in Portugal, Brazil and Angola. The game is played by two teams, each team consisting of two members. The goal is to score as many points as possible and to win four games of Sueca.

To the best of our knowledge, there has not been done any research into Sueca, neither about similar card games. For this reason we first study the card game in more detail. This is to understand the game better before determining which of the different strategies used plays Sueca best.

The second chapter explains how the card game is played. The third chapter describes other similar card games in short. Chapter 4 will give some insight into factors which might influence the game. In the next chapter, Chapter 5, some strategies are explained, like the Always High strategy. Chapter 6 describes an important strategy, namely the basic Monte Carlo Tree Search. The results of the four different strategies are shown in Chapter 7. In Chapter 8 we will give our conclusions and provide ideas for future work.

This thesis was written as a Bachelor project at the Leiden Institute of Advanced Computer Science (LIACS) of Universiteit Leiden, and has been supervised by Walter Kosters and Rudy van Vliet.

Chapter 2

The game

In this section we introduce the game Sueca, a point-trick partner card game played in Portugal, Brazil and Angola. We will discuss the rules of Sueca, mostly based on [1,9].

2.1 The deck and players

Sueca is played with a 40-card deck. The 8s, 9s and 10s are removed from a standard 4-suit 52-card deck. The ranking of the cards, from high to low, is: Ace, Seven, King, Jack, Queen, Six, Five, Four, Three and Two. The Ace has a value of 11 points, the Seven is worth 10 points, the King 4 points, the Jack 3 and the Queen 2. The cards Six till Two are not worth any points, as can be seen in Table 2.1. These values result in a deck worth 120 points, each suit with 30 points.

Card	**Value**
Ace	11
Seven	10
King	4
Jack	3
Queen	2
Six	0
Five	0
Four	0
Three	0
Two	0

Table 2.1: Value in points of each card in the game Sueca.

Sueca is played with four players, which form two teams. The players in a team are sitting across each other. Players are not allowed to communicate, both verbally and non-verbally, with each other. This is considered as cheating, as the players then get more information about other players' cards than they should have.

2.2 The deal

The game is played counter-clockwise. First, a dealer is chosen randomly. The player to the dealer's right shuffles the deck and his partner cuts the deck. The dealer then gives each player ten cards, in one batch. He may start with the player at his left side or he starts with his own cards.

When the dealer starts with the player at his left, the last card dealt, which is the last card of the dealer, will determine the suit of the trump. When he starts with dealing his own cards, the first card dealt will determine the trump. Then the dealer continues with the player on his left side. The card which determined the trump stays visible to every player for at least one trick.

2.3 The play

The first player to lead the first trick is the player to the dealers right. Other players have to follow suit when they can. Otherwise they are allowed to play any other card. The trick is won by the highest trump card. If there is not any trump card played, then the highest card of the lead suit wins. The points made during the trick will be added to the points of the winner's team. The winner leads the next trick and is allowed to play any card in his hand. There are in total ten tricks to be played.

2.4 Scoring

The goal of Sueca is to gain as many points as possible as a team. Each game has of a total of 120 points, which means a team has to score more than 60 points to win the game. The first team to score four games wins the "rubber".

During a game it is possible to score 'more than one game'. When a team scores more than 90 points it gains two games instead of one. When scoring all tricks, all 120 points and all zero points cards, the team wins the rubber immediately. Taking 120 points but losing a trick (worth zero points) is not sufficient to win the rubber, so the team only wins two games instead of four.

When both teams score 60 points, which is a draw, the game is skipped. Neither of the teams score a game. In this thesis we refer to the players as North, West, South and East, where PN and PS are a team and PW and PE are a team.

Chapter 3

Similar card games

Sueca is a point trick-taking card game and there are several similar card games which also belong to this category. The closest relatives are Einwerfen, a German card game, and Briscola, an Italian card game. But these are not very popular worldwide, so we will focus on two more known and somewhat similar card games: Klaverjas and Bridge.

3.1 Klaverjas

Klaverjas [7], popular in the Netherlands, is a game for four players played with a 32-card deck, using less cards than Sueca does. The cards Two till Six are left out. The trump suit is normally randomly determined by a card which is left out. Before starting the game there is a bidding. If a player accepts the deal, his team has to win the game. Just as Sueca, players have to follow suit. However, unlike Sueca, players are obligated to play a trump card when they cannot follow suit. The trump suit has a different value ranking than the other suits. Moreover, it is possible to score additional points during a trick, the so-called ”roem” [7]. The scoring is thus very different from Sueca. There are several variations of the game regarding the bidding and the way it is obligated to trump.

3.2 Bridge

Bridge [7] is another point trick-taking card game, played with four players with the regular 52-card deck. It is a very popular game played all over the world. Like Klaverjas, the game starts with a bidding part but this time much more complicated. Two players make a deal to win at least a number of tricks during the game given a certain trump, or with no trump at all. The team member of the player has to spread his hand open on the table and is played by the player himself. Every player has to follow suit and when that is not possible each player is allowed to play any card he wants, which is the same for Sueca. The scoring on the other hand is very different as it is determined whether the team has achieved the deal or not. There has been a lot of research for the bidding part of Bridge and there are several tactics for the playing part [8].

Chapter 4

How to win Sueca

First, we take a closer look at the card game itself. Generally, the different cards are categorized by their value. Bad cards are considered the ones worth zero points, being Two, Three, Four, Five and Six. Regular cards are worth some points, but their worth is not significantly big. These cards are the Queen, Jack and King. Lastly, high cards are the most valuable cards: the Seven and the Ace.

The card game has imperfect information, given the deck is randomly shuffled. This makes it harder to solve Sueca and to predict whether a team will win the game or not. There are several factors that can influence the outcome of the game: what player leads the first trick, which suit the trump is, your hand and your teammate's hand. In this chapter we examine whether just playing randomly can provide insight into some simple situations.

4.1 Leading the first trick

We are curious to see if leading the first trick will give you a head start during the game. When you lead the first trick, you decide what suit needs to be followed by the other players, which may influence the following tricks.

When starting with a high card, chances are the leader wins that trick and is allowed to start the next trick as well. This may result in an advantage for the leader. To test this, we played 1 000 000 games with only one deck. We did this multiple times starting with a different leader, as can be seen in Table 4.1. We also varied the trump suit to be sure the trump suit did not result in an advantage. In Table 4.2 the number of cards each team has for each suit is shown.

To make the average scores in Table 4.1 more clear, they are from one perspective: the team of players *N* & *S*. It is clear from the results that leading the first trick does not have an advantage, as all the scores are the same whoever starts. It also does not matter how many trump cards a team has, for having an advantage when starting as first player. So maybe it gives a slight advantage in one particular game, but overall it does not.

Another interesting result from Table 4.1 is the importance a trump suit can bring to a game. We used one single random deck and the scores vary between 36 and 65 points, all depending on what suit the trump suit is and how many a team has.

	Leader			
Trump suit	**P***N*	**P***W*	**P***S*	**P***E*
0	65	65	65	65
1	36	36	36	36
2	36	36	36	36
3	50	50	50	50

Table 4.1: Different average scores from the perspective of team 0 & 2, played 1 000 000 times with one random deck.

	Team players	
Suit	**P***N* **& P***S*	**P***W* **& P***E*
0	8	2
1	3	7
2	4	6
3	5	5

Table 4.2: Number of cards per suit per team at the used deck at Table 4.1.

4.2 Trump

One of the factors which can determine the outcome of a game is the suit of the trump. It can of course be very useful to have a lot of trump cards in your hand.

4.2.1 Trump suit

The suit of the trump cards is determined by the first or the last card of the dealer, depending how he deals the cards. This card stays open on the table for at least one trick. To see if this gives the team of the dealer an advantage we played 1 000 000 games varying the dealer where the suit of the first card is the trump suit. We set the leader, the player to start the game, on player *N*. As seen in table 4.3, the team of the dealer does have an advantage of three points. All the scores seen in Table 4.3 are from the perspective of team *N* and *S*.

	P*N*	**P***W*	**P***S*	**P***E*
Score	63	63	63	63

Table 4.3: Average score of 1 000 000 games where the suit of the first card of the dealer the trump is.

4.2.2 Number of trump cards per player

We played 1 000 000 random games per number of trump cards varying the number of trump cards in one player's hand. The leader is in all cases PN as it does not give an advantage as discussed in 4.1. In Table 4.4 the average score by the team of that player is shown. The fewer trump cards a player has, the fewer points his team will achieve on average during a game. This means a trump card is valuable to have in your hand. Since the main purpose of a trump card is to beat the follow suit card in a trick, it makes sense a trump card is valuable. When we count every difference between the average scores and divide it by ten we get the average value of one trump card. When applying this formula we can conclude that each trump card is worth seven points.

It is also interesting to notice that when you have five trump cards or less the opposite team has a chance to win 120 points.

Number of trump cards	**Score**		
individually	**Lowest**	**Average**	**Highest**
10	120	120	120
9	51	110	120
8	36	102	120
7	18	93	120
6	6	85	120
5	0	77	120
4	0	69	120
3	0	61	120
2	0	56	120
1	0	52	120
0	0	49	120

Table 4.4: Number of points played with 1 000 000 games with different number of trump cards.

Ten trump cards

One way to win a game is when one player has all the trumps in his hand. Since no other player has a card higher than a trump card, the team of the player with all the trump cards wins the game.

Nine trump cards

When a player has nine trump cards and only one regular card, he will still win the game with an average score of 110 points as seen in Table 4.4. As this is an average score we will discuss different scenarios to see the fluctuations in points with different cards.

Having nine trumps in your hand, there are a few general possibilities for what kind of cards those can be. Either you have high trump cards or less high cards, like missing the Ace trump card. And either you have a high, regular or low alternative card, like an Ace or a Five .

The trump card the player does not have in his or her hand is called T. If $T = Ace$ and the opposite team has T, it is certain the player's team will lose one trick. It is also possible to lose two tricks, as the winning player with the T card may win a trick with a regular non trump suit card. The team loses at least 11 and maximally 39 points, as seen in Table 4.5. However, it is very unlikely to lose 39 points as the player can play much better. It is possible to win 120 points when the team member of the player has T.

If $T = Two$, the team might lose zero or one trick. Losing a trick might happen when a player of the opposite team has a higher card when the player plays his regular card. If the player is unlucky, his team might lose points, as seen in Table 4.5.

In the other cases, the team of the player can lose zero, one or two tricks. Although, with playing smart it is possible to minimize the loss. When playing a trump at the first trick, as leader, the player removes the tenth trump card. The following eight tricks the player can play his trump cards in a random order, as no one has a trump card to take over the lead. The last trick the player has to play his non-trump card. It now depends on the cards of the other players to determine who wins the last trick.

	$T =$	
Score	**Two**	**Ace**
Highest	120	120
Average	103	113
Lowest	58	81

Table 4.5: Different scores when played 1 000 000 random games where T is the card player N does not have.

Eight trump cards

As can be seen in Table 4.4, the average score gained by the team is 102 points. This means it is still possible to win Sueca with two games. The difference between having nine trump cards and eight trump cards is eight points, which is significant. This is explained because the player has now two regular cards which makes the outcome of the game more unpredictable. The opposite team might win maximally four tricks, in the way described when having nine trump cards.

4.3 Your and your teammate's hand

An extremely important factor to consider is that Sueca is played in a team. Whether you will win or not, also depends on how your teammate plays the game.

4.3.1 Number of trump cards per team

It is interesting to see what value a trump card has for one player, but as already said Sueca is played in teams. When one player has five trump cards in his hand, his partner could have the remaining five trump cards, leaving the opposite team with no trump cards. To find out what impact the number of trump cards has as a team, we played 1 000 000 random games per number of trump cards having different number of trump cards as a team.

The first thing to notice is the difference between having ten trump cards individually and as a team. When one player has all ten trump cards it is certain his team wins the game with all available points. However, as a team that does not apply. The average score is 99 points, as can be seen in Table 4.6. This is explained by the fact that each player has to follow suit. When, for example, each team player has five trump cards it means they also have five regular cards. Consequently this means it is possible they both have the same regular suit in their hands, for example Hearts. When a player leads the trick with a card of the suit Hearts, both players are obligated to play their card of Hearts instead of their trump card. In this way their trump cards are less valuable.

It is even possible to lose the game when having ten trump cards as a team. By playing at the first trick a non-trump suit card the opposite team could win this trick with the highest value cards, like Ace and Seven. This can happen for all three non-trump suits. If that happens then it is likely that the team with the trump cards can only use their trump cards on zero point cards. The only points they will score are the points of the trump suit, worth 30 points.

The decline in points when having fewer trump cards as a team is similar to the decline when having a number of trump cards as a player. The value of a trump card is in this case 8 points. The main difference is that the score starts at 99 points instead of 120 and it ends at 20 instead of 49 points. It is also interesting to see that the opposite team is able to win "the rubber" when having only three trump cards.

4.3.2 Number of points per team

Another factor that might be important to consider besides the number of trump cards in the hand of the team, is the number of points of the cards. It is useful to have a lot of Ace cards in your hand, especially the trump one. When having high cards in your hand the chance of winning a trick is greater than having zero points cards.

We varied the number of total points in the hands of a team, varying from 0 to 120 points. We played 1 000 000 random games per number of points varying the number of points as a team, as can be seen in Table 6.3. Each interval of the number of points include the first and last points of that interval. The trump suit in each game is determined by a random card. The average score is roughly equal to the number of points the team has in their hands at the start of the game. Therefore, we can conclude that the number of points in the hand of a team is an important predictor of the outcome of the game.

It is also interesting that a team with 110 to 120 points in their hand, will at their worse still score more than 30 points. Vice versa, a team having 0 to 10 points will of course not be able to score 120 points. Both cases might be explained by the trump cards. In the first case the team is likely to have many trump cards worth points and their value is certain for the team. In the second case, the team cannot win many tricks without trump cards.

The scores in Table 6.3 are the results from the experiments. Theoretically the maximal score when having between 0 and 10 points is 99 points, which is when a player has eight trump cards. The player then misses the Ace and Seven of the trump suit, which is 21 points in total.

Number of trump cards as a team	**Score**		
	Min	**Average**	**Max**
10	30	99	120
9	8	92	120
8	2	85	120
7	0	78	120
6	0	69	120
5	0	60	120
4	0	50	120
3	0	42	120
2	0	34	117
1	0	27	110
0	0	20	88

Table 4.6: Average number of points played with 1 000 000 games with different number of trump cards as a team.

Number of points as a team	**Score**		
	Min	**Average**	**Max**
110 - 120	32	106	120
100 - 110	18	101	120
90 - 100	0	92	120
80 - 90	0	83	120
70 - 80	0	74	120
60 - 70	0	64	120
50 - 60	0	55	120
40 - 50	0	45	120
30 - 40	0	36	120
20 - 30	0	27	115
10 - 20	0	18	100
0 - 10	0	13	83

Table 4.7: Different scores with different total points in the hands of a team, played 1 000 000 random games.

Chapter 5

Strategies

In this chapter we will introduce different strategies used to play Sueca. All the strategies are implemented in C++ and run on a computer with 800 GB SSD and 16 GB RAM memory.

5.1 Random

The most basic way a player can play is random. Of course, each player has to play the game fair. For this reason, before choosing a random card we first look at which suit is to be followed. The cards with the follow suit are set aside. From these cards one random card is chosen and played. When the player does not have the follow suit, a random card is chosen to be played from all his cards in his hand. When the player is to lead the trick there is no suit to follow so all his cards may be played.

When all four players play randomly the outcome will be 60 points for each team, as expected. In Figure 5.1 the distribution when playing a million games is shown. The standard deviation tells us how close the scores achieved are to the mean [2]. The standard deviation is 24.00 meaning that 68% lies between 36 and 84 points. With two standard deviations, which consists 13% of the time, the team loses or wins two game points.

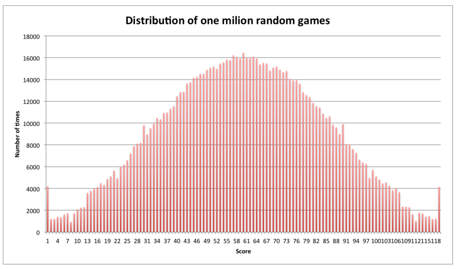

Figure 5.1: Distribution of a million random games.

5.2 Obligated to trump

A strategy a player can use is to play a trump card whenever that is possible. During a trick the player will play a trump card when he does not have the suit to be followed. He will do this when either his or the opposite team is winning the trick. If the player does not have a card with the suit to be followed nor a trump card, he will play a random card in his hand.

This strategy, obligating to play a trump card, is a rule at the card game Klaverjas. By simulating this rule as a strategy we can examine the effect of the rule on the outcome of the game.

When the player leads the trick, there is no other card to trump. To see the effect of starting a trick with a trump card or a random card, we played both variants. As can be seen in Table 5.1, the strategy where a player starts a trick with a random card gets a score of 61.4 points. When a team plays this way the team scores 62.3 points. This is higher than leading the trick with a trump card which results in 60.1 points, the same as the random strategy. This way the player spoils his trump cards by playing them when leading a trick and therefore obligating the other players to play a trump card as well. During the following tricks the player has fewer trump cards to use, when it might be necessary. We have chosen to use the strategy where the player leads a trick with a random card.

	Leading random	**Leading trump card**
Players	**Score**	**Score**
One player	61.4	60.1
Team	62.3	60.2

Table 5.1: Average score when one player or a team play 1 000 000 games with two versions of the Obligated trump strategy.

5.3 Always play the highest card

A player may also play at every turn his highest card in his hand. With this strategy the player will play his highest card whether it is a trump card or not. If the player has multiple cards of the same value, a random card from these is chosen. In this way the player may win a trick and give his team as many points as possible but it also may happen that he gives the opposite team unnecessary points.

A variant of this strategy is that a player only uses this strategy when he is the third player to play during a trick. This variant is based on the game of Bridge. This tactic is called Third Hand High [6]. The idea is to play a high card, as there is another tactic that the second seat will play a low card, referred to as ducking [4]. This may lead to a higher score during that trick.

Both strategies are tested by playing 1 000 000 games. The variant where the player only plays his highest card as a third player is significantly worse than the other. When just one player always uses this strategy his team scores an average of 63.7 points. When one team plays this way they score 66.1 points, as seen in Table 5.2.

	Always	**As third player**
Players	**Score**	**Score**
One player	63.7	60.0
Team	66.1	60.1

Table 5.2: Average score when one player or a team play 1 000 000 games with the always high card strategy.

Chapter 6

Monte-Carlo strategy

Another strategy used is Monte Carlo. This is an effective and frequently used strategy to predict the outcomes of many games [3]. We will study different implementations of this strategy.

6.1 Applying Monte-Carlo Tree Search

First, we will explain the basic method of Monte-Carlo Tree Search. Then we will look into different implementations of Monte-Carlo to decide which implementation to use in further analysis of the game Sueca.

6.1.1 Method

Monte-Carlo Tree Search (MCTS) is a strategy based on constructing a search tree, commonly used in games [3]. There are several variants. The basic variant of MCTS predicts the outcome of the game for every possible move by playing the game randomly a certain times from that move and further. In this way a search tree is built and each possible move is given a value based on the outcome of the randomly played games. Each possible move has a value, the best move is chosen to be played by the MC player. Another variant of MCTS is the Upper Confidence Bounds (UCB). UCB chooses the node to be expanded by looking at the value of the node and the number of times the node and the parent of that node have been visited during the game. In this way the most valuable nodes will be visited most times [5].

The way we use MCTS at the game Sueca will be the basic variant without UCB. For example, if player N is playing a MCTS strategy, when it is his turn MCTS predicts all possible cards he is allowed to play. Before MCTS plays the playout, it first shuffles the cards of the other three players, since player N does not know which cards each player has in their hands. During shuffling the cards MCTS also takes into account which suit each player certainly does not have: when one player does not follow the suit which is obligated to be followed, it means the player does not have that suit. Then MCTS plays the game randomly from that move till the end of the game and registers the outcome, from the perspective of player N. This happens multiple times for each possible move to search for the best card to play. When the MC player has only one possible card to play, he plays that card without shuffling the cards from the other three players and without the playouts, since there is no other choice than to play that card.

6.1.2 Different Monte-Carlo Strategies

There are several ways to implement the Monte-Carlo strategy how to select the card to play. We have implemented three different variants: Normal, Average and Extra.

Monte-Carlo Normal

Monte-Carlo Normal remembers the card which got the highest score during the playouts. That card is chosen to play by the MC player. As all the players, including the MC player, play random during the payouts the highest score may be a lucky "shot". This is also seen in the standard deviation which is 23.75 points. It is just a little less than the standard deviation of random, which is 24.00 points. However the average score of MCTS Normal is higher than random, namely 64 points as seen in Figure 6.1.

Monte-Carlo Average

The Monte-Carlo Average variant sums all the scores of the playouts from one card to calculate the mean. The card with the highest mean is chosen to play. The standard deviation is 23.45 which is less than MC Normal, but the difference is very small. A big difference between the distributions of MC Normal and Average is the peak of the mountain. The peak at MC Average starts around 70 points and declines around a score of 90. This can also be seen at the average which is 69 points, seen in Figure 6.2.

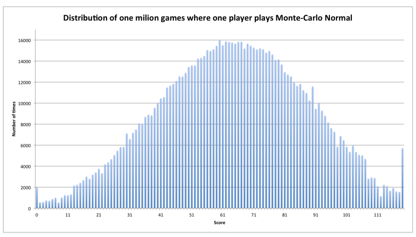

Figure 6.1: Distribution of a million games where one player plays MC normal strategy.

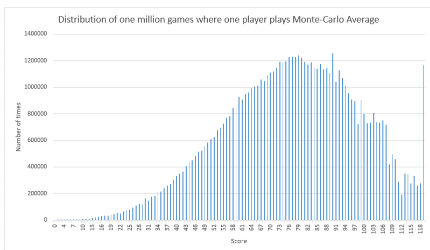

Figure 6.2: Distribution of a million games where one player plays MC Average strategy.

Monte-Carlo Extra

The Monte-Carlo Extra variant is an extension of the MC Average strategy. After the cards of the three other players have been shuffled it does not play only one game with those cards, but multiple games with the same cards. Because during a playout each card is chosen randomly, the playout only represents a very small part of the possible outcomes. This way it might be possible that MCTS can shuffle the cards less often to achieve the same result. To see what the optimal number of extra playouts is we played 1 000 000 games with a number of shuffles of ten. As can be seen in Table 6.1, with just two playouts per shuffle for every possible move the score increases with almost two points. When we play five extra playouts the difference in points is smaller and it takes a long time.

Number of playouts	MC Extra	
per shuffled cards	Score	Time (m)
1	69.97	24
2	71.50	43
5	72.36	93

Table 6.1: Score per number of extra playouts Monte-Carlo Extra when played 1 000 000 games with 10 normal playouts.

When we make a distribution of 1 000 000 games where one player plays the Monte-Carlo Extra strategy with two extra playouts we have a mean of 72 and a standard deviation 23.32 which is slightly smaller than the standard deviation of Monte-Carlo Average.

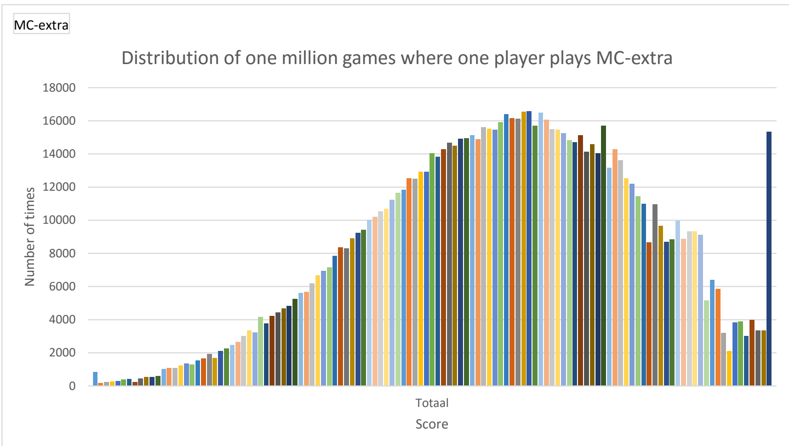

Figure 6.3: Distribution of a million games where one player plays MC Extra strategy.

6.1.3 Number of Monte-Carlo playouts

An important variable at the strategy Monte-Carlo is the number of playouts per possible card. This determines how many possible games are played and thus how reliable the chosen card is. To see what the impact is of the number of playouts we played 1 000 000 games with different number of playouts for both MC Normal, Average and Extra. As can be seen in Table 6.2, the score at MC Normal has only a slight increase till 50 playouts and then it decreases. With only one playout it achieves a score of 63 points which is three points more than random. Because MCTS takes the highest score obtained and the playouts are random, the chance to get a score of 120 or the highest possible is greater with more playouts. But in reality the chance is very small that the game happens just like in the playout to obtain a score that high.

The Monte-Carlo Average strategy score keeps increasing because it searches many possibilities. The increase from one to ten playouts is the largest from a score of 63 to 66 points. A disadvantage is that it takes a lot of time to execute MCTS with a large number of playouts. With 500 playouts it takes 22 hours. The time it takes to execute the program is a bit higher for MCTS Average than Normal, because it has to calculate the average score per card.

The number of playouts of Monte-Carlo Extra is actually twice that of MC Normal and Average. This is because MC Extra plays two games for each playout with the same shuffled cards. This can also be seen in the time it takes to execute the program. Where MC Normal and Average take more or less 250 minutes for 100 playouts, MC Extra takes roughly 400 minutes. But the score MC Extra achieves is significantly higher than the other ones: almost 73 points.

	MC Normal		**MC Average**		**MC Extra**	
Number of shuffles	**Score**	**Time (m)**	**Score**	**Time (m)**	**Score**	**Time (m)**
1	63.35	3	63.32	3	67.84	5
10	64.22	25	66.67	25	71.50	42
50	64.48	122	68.45	128	72.72	207
100	64.47	244	68.93	256	72.95	413
250	64.36	605	69.35	644	-	-
500	64.30	1275	69.54	1358	-	-

Table 6.2: Average score when varying the number of shuffles when played 1 000 000 games.

Score per points per team

To have more insight into the average score achieved by Monte-Carlo, we experimented with the number of points a team has in their hand. Just like in Chapter 4, we split the number of points as can be seen in Table 6.3 and played 1 000 000 games. Each game the team has a number of points as a team. This means that the points between the two players may be divided differently each game. We hypothesized that a game can not be won or played very well when a team has little points. And vice versa, a game having almost all possible points could easily win.

In Table 6.3 we see that in all cases MC Normal is better than Random, MC Average is better than MC Normal and MC Extra is better than MC Average. In most of the cases there is a difference of three points between the scores of each strategy. It is interesting to see that the difference between Random and MC Extra is smaller when the number of points is high, 110-120 points, and low, 0-10 points. Here the difference is just eight points where the difference when the team has 60-70 points is 14 points. This difference may be explained because the team has little opportunity to win a trick when having very few points even when you play optimally. Therefore, MCTS cannot play much better than Random.

Number of points in hand as team	**Random Score**	**MC Normal Score**	**MC Average Score**	**MC Extra Score**
110 - 120	106	107	110	114
100 - 110	101	103	106	110
90 - 100	92	95	99	103
80 - 90	83	87	91	95
70 - 80	74	78	83	87
60 - 70	64	69	74	78
50 - 60	55	60	65	68
40 - 50	45	50	55	58
30 - 40	36	41	45	48
20 - 30	27	31	34	37
10 - 20	18	22	24	27
0 - 10	13	16	18	20

Table 6.3: Different scores with different total points in the hands of a team, played 1 000 000 random games.

6.2 Shuffling the cards

Before each Monte-Carlo playout MCTS shuffles the cards of the other three players. It does this by randomly distributing the cards to the three players. We have chosen for this approach to keep the shuffle random and fair. Each card has an equal chance to get to a player, even if a player does not have the suit of that card in his hand. For that reason, MCTS checks after shuffling the cards whether it is a correct possibility. This means that it checks if each player does not have a suit in his hands which he has previously during the game proved not to have. If the shuffled cards are not a correct possibility MCTS shuffles again.

In Figure 6.4 an example is shown of a situation when shuffling the cards. The player who plays the Monte-

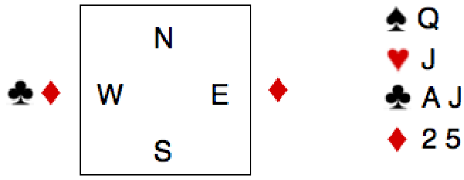

Figure 6.4: Example of a situation when shuffling the cards.

Carlo strategy is N and his team mate is S. Beside each player the suits are listed which that player does not have in his hand. At the right side of Figure 6.4 the cards to be shuffled are stated. This is without the cards of player N, as MCTS knows exactly which cards he has.

There are in total six cards to be shuffled, so we know that each player gets two cards. It can also happen that one or two players have one card more than the others. This depends on what seat player N belongs to. If player N is the leader all three other players have yet to play a card and thus have an equal number of cards in their hand. But when player N is second, the leader has already played a card and has one card less than the other two players.

Both players W and E do not have the suit Diamonds and there are two Diamond cards to be shuffled, so these belong to player S. Player W also does not have suit Clubs which means that both Clubs cards belong to player E. The last two cards belong thus to player W.

As the shuffle method is completely random and fair, this is not the way MCTS distributes the cards. It may happen that the same wrong result occurs multiple times and this may take a long time. For this reason we decided to set a limit to the number of shuffles MCTS does. When this limit is reached, MCTS takes the true value, referred to as the ”truth”.

Recall that the shuffle for the Monte-Carlo playout is done in the context of a game played from a certain random distribution of the 40 cards. The "truth" is simply the restriction of this distribution to the set of the unplayed cards which are not in the hand of the Monte-Carlo player himself. Thus, this is what MCTS resorts to, if it cannot find a correct random distribution of the cards in time.

6.2.1 Number of shuffles

We experimented with several maximal numbers of shuffles to see how many times MCTS takes the truth and how much time it takes. We used 100 Monte-Carlo playouts. The results can be seen in Table 6.4. MCTS takes the truth 8.4% of the total number of Monte-Carlo playouts when the maximal number of shuffles is 10. When increasing the maximal number of shuffles, the percentage of truth taken decreases in the way which can be seen in Figure 6.5. The decrease is big at first, up to 60 shuffles, and becomes smaller from then on. The difference between maximal 10 shuffles and maximal 500 shuffles is just 39 minutes. This can be explained because with maximal 10 shuffles 91.6% succeed in less than 10 shuffles. This means that only the remaining 8.4% need more shuffles than 10 and thus need more time.

Number of max shuffles	Times truth (%)	Time (m)
10	8.4	223
20	4.4	230
30	2.9	234
40	2.1	239
50	1.7	242
60	1.4	244
70	1.2	245
80	1.0	244
90	0.9	245
100	0.8	247
250	0.3	257
500	0.1	262

Table 6.4: Number of times Monte-Carlo takes the truth for the playouts when played 1 000 000 games.

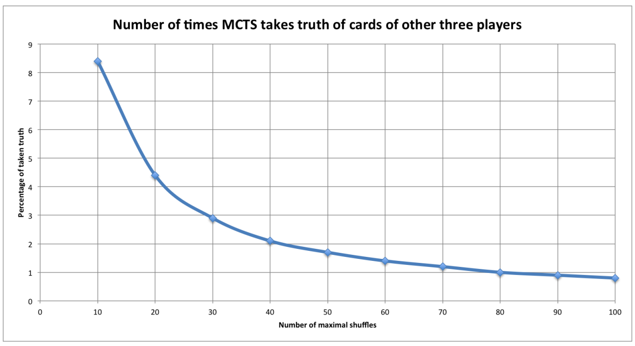

Figure 6.5: Graph of the number of times MCTS takes the truth for the playouts when played 1 000 000 games.

Rounds

To get more insight when MCTS takes the truth for the cards of the other three players, we show in what rounds of the game this happens most often. As can be seen in Figure 6.6, from the 1% in which the truth is taken in Monte-Carlo, it happens the most at rounds 5, 6 and 7. At these rounds MCTS has between 9 and 18 cards to shuffle; nine cards during round seven when the MC player is last to play with four cards in his hand and the other three players with three cards in their hands and 18 cards at round five when all three players have six cards left. This means that the number of combinations is between 1680 with nine cards and 17 153 136 with 18 cards.

Both the first and the last round have zero. At the first round there has not been played any cards yet which means there is no information available about players not having a suit. This is when the MC player is the leader. When the MC player is not the first leader it may happen that a player who already has played his card does not have the suit to be followed in his hand. The last round the MC player only has one card in his hand. So it has no use to shuffle the cards as there is no other choice than to play that last card in his hand.

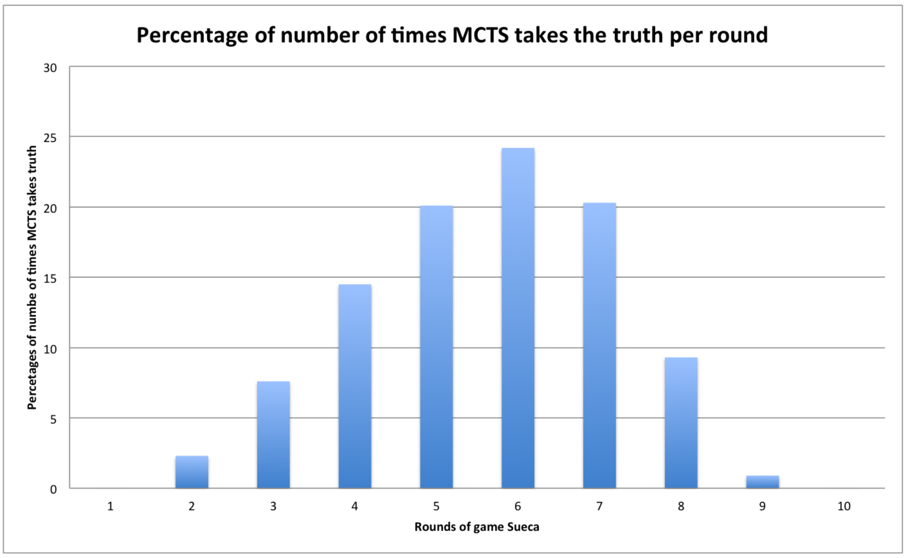

Figure 6.6: Graph of the number of times MCTS takes the truth with max 100 shuffles for the playouts per round when played 1 000 000 games.

Number of prohibited suits

Another valuable piece of information is to know how many suits the other three players do not have. The number of missing suits of the Monte-Carlo player does not matter because his cards are not being shuffled. In Figure 6.7 can be seen that more than 70% of the times there are two or three suits other players do not have in their hands. It also occurs relatively often with four missing suits. The remaining scenarios occur with just one, five and six missing suits.

The scenarios with a number of missing suits may relate to several different situations. For example, with two prohibited suits it can be that one player misses two suits or that two players each miss one suit. Moreover, these suits can be different from each other or the same. In Figure 6.7 we do not have any information in what round it happens. So we also do not know how many cards are to be shuffled and thus how many possible combinations there are in those situations. When there is just one correct combination it is possible that it takes many attempts to reach that combination.

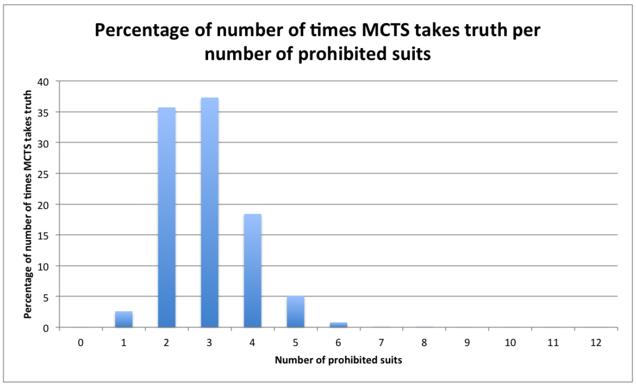

Figure 6.7: Graph of the number of times MCTS takes the truth for the playouts per missing suit when played 1 000 000 games.

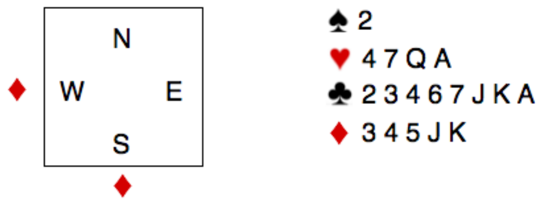

Figure 6.8: Example of a situation when shuffling the cards with a high number of shuffles (7500).

To get more insight in what scenarios it happens most that MCTS takes the truth, we set the maximal number of shuffles very high. We noticed that with maximal 7500 shuffles only one type of scenario occur: two players who do not have one and the same suit. For example, two players do not have the suit Hearts in their hands. Of course there might be other suits that players do not have. In Figure 6.8 an example is seen of such a situation. Both player W and S do not have the suit Diamonds. There are in total 18 cards to be shuffled where five of them are Diamonds. This means each player has to receive six cards. As there is only one player that is allowed to have Diamond cards, all of the five Diamond cards have to go to player E. The remaining cards can go to any player.

New way to shuffle

As we now know what kind of scenarios occur most of the times when shuffling, we can extend the way to shuffle the cards of the other three players. Before MCTS shuffles the cards, we first check whether there are two players that both do not have one type of suit. We also check whether there are cards with that suit to be shuffled. If that is the case, it gives the cards of that suit to the player that does have that suit in his hand. When applying this to the example of Figure 6.8 it gives the Diamond cards to player *E*. After that MCTS shuffles the remaining cards in a random and fair way, just like before. Otherwise, the shuffle is done in the usual way.

When we apply this way of shuffling when playing 1 000 000 games using 100 playouts, we see in Table 6.5 that the percentage that MCTS takes is lower than before. With just maximal ten shuffles the percentage is already 30% less than before. When the maximal number of shuffles is very high, 500, the percentage is only 0.02%. What also is interesting to see is that the execution time is also smaller. This could be because there are fewer shuffles necessary when the cards are taken aside when two players do not have one suit. This might save time.

	Regular way		**New way**	
Number of max shuffles	**Times truth (%)**	**Time (m)**	**Times truth (%)**	**Time (m)**
10	8.4	223	6.0	222
20	4.4	230	2.8	228
30	2.9	234	1.7	231
40	2.1	239	1.1	232
50	1.7	242	0.8	234
60	1.4	244	0.6	236
70	1.2	245	0.5	238
80	1.0	244	0.4	237
90	0.9	245	0.4	238
100	0.8	247	0.3	237
250	0.3	257	0.07	241
500	0.1	262	0.02	243

Table 6.5: Number of times Monte-Carlo takes the truth for the playouts when played 1 000 000 games with the new shuffle.

Chapter 7

Results

We played every combination with each strategy to see which strategy plays Sueca best: Random, Obligated Trump, Always High and Monte-Carlo Extra.

We played 1 000 000 games for each combination of the strategies. All 1 000 000 decks are randomly shuffled and used for each experiment. This is to limit the chance of coincidence. For the strategy Obligated Trump we used that a player does not have to start a trick with a trump card. When a player plays the strategy Always High he will do that every trick and not just at third seat. We decided to use the Monte-Carlo Extra strategy with 50 shuffles and playing two playouts per shuffle per possible card. This means that MCTS plays in total 100 playouts per possible card. We have set the maximal number of shuffles at 100, which takes the truth approximately 0.3% of the time.

For each result we calculated the mean and the standard deviation. The strategy Random is shown with 'R', strategy Monte-Carlo is 'MC', 'T' is strategy Obligated Trump and Always High is 'A'.

7.1 Results per team

To present the results in a clear way, we divided the tables per team. Per team we briefly discuss the highlights of the results. The games won by the team is shown in bold.

Random strategy

Team Random & Random

We see in Table 7.1 that the team loses all games, except against his equivalent team. It is interesting to see that the team does not achieve a score less than 30 points, which means that the team does not lose two game points.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
R	R	R	R	**60.1**	59.9	24.3
R	R	R	MC	46.8	73.2	23.3
R	R	R	T	58.7	61.3	24.3
R	R	R	A	57.7	62.3	24.3
R	R	MC	R	47.3	72.7	23.3
R	R	MC	MC	36.1	83.9	21.5
R	R	MC	T	46.5	73.5	23.2
R	R	MC	A	45.3	74.7	23.1
R	R	T	R	58.8	61.2	24.3
R	R	T	MC	46.0	74.0	23.2
R	R	T	T	57.8	62.2	24.3
R	R	T	A	56.4	63.6	24.1
R	R	A	R	57.7	62.3	24.3
R	R	A	MC	44.8	75.2	23.1
R	R	A	T	56.4	63.6	24.1
R	R	A	A	54.9	65.1	24.3

Table 7.1: Average score of different strategies when played 1 000 000 games.

Team Random & Monte-Carlo

We see in Table 7.2 that the team Random & Monte-Carlo with a mean of 65.1 plays significantly better than Random & Random with an average of 52.0 points. The team wins most of the games.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
R	MC	R	R	**72.9**	47.1	23.4
R	MC	R	MC	59.6	60.4	23.8
R	MC	R	T	**71.2**	48.8	23.6
R	MC	R	A	**71.6**	48.4	23.7
R	MC	MC	R	**60.2**	59.8	23.8
R	MC	MC	MC	48.6	71.4	23.2
R	MC	MC	T	59.0	61.0	23.7
R	MC	MC	A	59.1	60.9	23.9
R	MC	T	R	**71.2**	48.8	23.7
R	MC	T	MC	58.6	61.4	23.8
R	MC	T	T	**70.0**	50.0	23.8
R	MC	T	A	**69.8**	50.2	23.7
R	MC	A	R	**71.6**	48.4	23.7
R	MC	A	MC	58.4	61.6	23.9
R	MC	A	T	**69.6**	50.4	23.8
R	MC	A	A	**69.9**	50.1	24.4

Table 7.2: Average score of different strategies when played 1 000 000 games.

Team Random & Obligated Trump

In Table 7.3 we see that this team loses almost all games, but with a higher score than team Random & Random.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
R	T	R	R	**61.4**	58.6	24.3
R	T	R	MC	48.6	71.4	23.6
R	T	R	T	**60.1**	59.9	24.3
R	T	R	A	59.2	60.8	24.2
R	T	MC	R	49.0	71.0	23.6
R	T	MC	MC	38.2	81.8	22.0
R	T	MC	T	48.2	71.8	23.5
R	T	MC	A	47.4	72.6	23.4
R	T	T	R	60.0	60.0	24.3
R	T	T	MC	47.8	72.2	23.5
R	T	T	T	59.1	60.9	24.4
R	T	T	A	58.0	62.0	24.1
R	T	A	R	59.3	60.7	24.3
R	T	A	MC	46.8	73.2	23.4
R	T	A	T	58.0	62.0	24.1
R	T	A	A	56.8	63.2	24.3

Table 7.3: Average score of different strategies when played 1 000 000 games.

Team Random & Always High

Team Random & Always High is also not able to win multiple games, except against a team Random & Random, as can be seen in Table 7.4.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
R	A	R	R	**62.5**	57.5	24.2
R	A	R	MC	48.2	71.8	23.7
R	A	R	T	**60.8**	59.2	24.2
R	A	R	A	**60.1**	59.9	26.0
R	A	MC	R	48.6	71.4	23.6
R	A	MC	MC	37.0	83.0	22.0
R	A	MC	T	47.6	72.4	23.3
R	A	MC	A	46.7	73.3	24.8
R	A	T	R	**60.8**	59.2	24.2
R	A	T	MC	47.2	72.8	23.4
R	A	T	T	59.5	60.5	24.2
R	A	T	A	58.4	61.6	25.7
R	A	A	R	60.0	60.0	26.0
R	A	A	MC	46.3	73.7	24.7
R	A	A	T	58.4	61.6	25.7
R	A	A	A	57.2	62.8	27.5

Table 7.4: Average score of different strategies when played 1 000 000 games.

Monte-Carlo strategy

Team Monte-Carlo & Random

As can be seen in Table 7.5, this team wins half of the games and the other half the team loses.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
MC	R	R	R	**72.7**	47.3	23.3
MC	R	R	MC	59.0	61.0	23.7
MC	R	R	T	**70.8**	49.2	23.7
MC	R	R	A	**71.7**	48.3	23.9
MC	R	MC	R	59.7	60.3	23.7
MC	R	MC	MC	47.8	72.2	23.0
MC	R	MC	T	58.5	61.5	23.8
MC	R	MC	A	58.8	61.2	24.0
MC	R	T	R	**70.9**	49.1	23.6
MC	R	T	MC	57.9	62.1	23.7
MC	R	T	T	**69.8**	50.2	23.8
MC	R	T	A	**69.7**	50.3	23.9
MC	R	A	R	**72.0**	48.0	23.8
MC	R	A	MC	58.3	61.7	23.9
MC	R	A	T	**69.9**	50.1	23.9
MC	R	A	A	**70.5**	49.5	24.8

Table 7.5: Average score of different strategies when played 1 000 000 games.

Team Monte-Carlo & Monte-Carlo

A team both playing Monte-Carlo wins all the games with the highest score of 83.6 and lowest score of 69.1 points, as can be seen in Table 7.6.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
MC	MC	R	R	**83.6**	36.4	21.5
MC	MC	R	MC	**70.4**	49.6	23.3
MC	MC	R	T	**81.4**	38.6	22.1
MC	MC	R	A	**83.2**	36.8	22.3
MC	MC	MC	R	**71.1**	48.9	23.3
MC	MC	MC	MC	59.1	60.9	23.8
MC	MC	MC	T	**69.6**	50.4	23.5
MC	MC	MC	A	**70.8**	49.2	23.8
MC	MC	T	R	**81.6**	38.4	22.1
MC	MC	T	MC	**69.1**	50.9	23.5
MC	MC	T	T	**80.2**	39.8	22.4
MC	MC	T	A	**80.8**	39.2	22.6
MC	MC	A	R	**83.3**	36.7	22.2
MC	MC	A	MC	**70.1**	49.9	23.8
MC	MC	A	T	**80.7**	39.3	22.6
MC	MC	A	A	**82.4**	37.6	23.7

Table 7.6: Average score of different strategies when played 1 000 000 games.

Team Monte-Carlo & Obligated Trump

In Table 7.7 is shown that this team wins most of the games, but also loses some games. The scores are lower than the team MC & MC.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
MC	T	R	R	**73.5**	46.5	23.1
MC	T	R	MC	**60.3**	59.7	23.8
MC	T	R	T	**71.6**	48.4	23.6
MC	T	R	A	**72.6**	47.4	23.6
MC	T	MC	R	**60.9**	59.1	23.7
MC	T	MC	MC	49.3	70.7	23.2
MC	T	MC	T	59.6	60.4	23.9
MC	T	MC	A	**60.1**	59.9	23.9
MC	T	T	R	**71.8**	48.2	23.5
MC	T	T	MC	59.1	60.9	23.8
MC	T	T	T	**70.6**	49.4	23.7
MC	T	T	A	**70.6**	49.4	23.7
MC	T	A	R	**72.8**	47.2	23.5
MC	T	A	MC	59.7	60.3	23.9
MC	T	A	T	**70.8**	49.2	23.7
MC	T	A	A	**71.6**	48.4	24.4

Table 7.7: Average score of different strategies when played 1 000 000 games.

Team Monte-Carlo & Always High

The team MC & Always High achieves roughly the same results as the team MC & Obligated Trump, as can be seen in Table 7.8.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
MC	A	R	R	**74.7**	45.3	23.0
MC	A	R	MC	**60.4**	59.6	23.8
MC	A	R	T	**72.5**	47.5	23.4
MC	A	R	A	**73.8**	46.2	24.8
MC	A	MC	R	**60.9**	59.1	23.7
MC	A	MC	MC	48.7	71.3	23.2
MC	A	MC	T	59.4	60.6	23.8
MC	A	MC	A	**60.2**	59.8	25.1
MC	A	T	R	**72.6**	47.4	23.3
MC	A	T	MC	59.1	60.9	23.8
MC	A	T	T	**71.0**	49.0	23.6
MC	A	T	A	**71.4**	48.6	24.8
MC	A	A	R	**74.0**	46.0	24.7
MC	A	A	MC	59.9	60.1	25.1
MC	A	A	T	**71.5**	48.5	24.9
MC	A	A	A	**72.5**	47.5	26.9

Table 7.8: Average score of different strategies when played 1 000 000 games.

Obligated Trump strategy

Team Obligated Trump & Random

These results, as can be seen in Table 7.9, are very similar as the results from the team Random & Obligated Trump.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
T	R	R	R	**61.4**	58.6	24.3
T	R	R	MC	48.5	71.5	23.6
T	R	R	T	60.0	60.0	24.4
T	R	R	A	59.2	60.8	24.3
T	R	MC	R	49.0	71.0	23.7
T	R	MC	MC	38.1	81.9	22.0
T	R	MC	T	48.2	71.8	23.6
T	R	MC	A	47.3	72.7	23.4
T	R	T	R	60.0	60.0	60.0
T	R	T	MC	47.7	72.3	23.5
T	R	T	T	60.0	60.0	60.0
T	R	T	A	60.0	60.0	60.0
T	R	A	R	59.3	60.7	24.2
T	R	A	MC	46.8	73.2	23.4
T	R	A	T	58.0	62.0	24.1
T	R	A	A	56.8	63.2	24.3

Table 7.9: Average score of different strategies when played 1 000 000 games.

Team Obligated Trump & Monte-Carlo

In Table 7.10 we see that the team wins most of the games.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
T	MC	R	R	**73.6**	46.4	23.2
T	MC	R	MC	**60.7**	59.3	23.8
T	MC	R	T	**71.9**	48.1	23.5
T	MC	R	A	**72.5**	47.5	23.4
T	MC	MC	R	**61.3**	58.7	23.8
T	MC	MC	MC	49.9	70.1	23.4
T	MC	MC	T	**60.2**	59.8	23.8
T	MC	MC	A	**60.5**	59.5	23.9
T	MC	T	R	**71.9**	48.1	23.5
T	MC	T	MC	59.6	60.4	23.9
T	MC	T	T	**70.8**	49.2	23.7
T	MC	T	A	**70.8**	49.2	23.5
T	MC	A	R	**72.4**	47.6	23.4
T	MC	A	MC	59.7	60.3	23.8
T	MC	A	T	**70.6**	49.4	23.6
T	MC	A	A	**71.0**	49.0	24.1

Table 7.10: Average score of different strategies when played 1 000 000 games.

Team Obligated Trump & Obligated Trump

In Table 7.11 we see that the team does not play very well.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
T	T	R	R	**62.3**	57.7	24.2
T	T	R	MC	49.7	70.3	23.7
T	T	R	T	**60.9**	59.1	24.4
T	T	R	A	**60.5**	59.5	24.2
T	T	MC	R	50.1	69.9	23.8
T	T	MC	MC	39.4	80.6	22.4
T	T	MC	T	49.3	70.7	23.8
T	T	MC	A	48.8	71.2	23.6
T	T	T	R	**61.0**	59.0	24.4
T	T	T	MC	48.8	71.2	23.7
T	T	T	T	60.0	60.0	24.5
T	T	T	A	59.2	60.8	24.2
T	T	A	R	**60.5**	59.5	24.2
T	T	A	MC	48.3	71.7	23.6
T	T	A	T	59.2	60.8	24.2
T	T	A	A	58.4	61.6	24.3

Table 7.11: Average score of different strategies when played 1 000 000 games.

Team Obligated Trump & Always High

In Table 7.12 we see that this team plays better than a team playing Obligated Trump.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
T	A	R	R	**63.6**	56.4	24.0
T	A	R	MC	50.0	70.0	23.7
T	A	R	T	**62.0**	58.0	24.1
T	A	R	A	**61.6**	58.4	25.6
T	A	MC	R	50.4	69.6	23.7
T	A	MC	MC	39.1	80.9	22.4
T	A	MC	T	49.3	70.7	23.5
T	A	MC	A	48.9	71.1	24.8
T	A	T	R	**62.0**	58.0	24.1
T	A	T	MC	49.1	70.9	23.5
T	A	T	T	**60.8**	59.2	24.2
T	A	T	A	60.0	60.0	25.4
T	A	A	R	**61.6**	58.4	25.6
T	A	A	MC	48.6	71.4	24.8
T	A	A	T	60.0	60.0	25.4
T	A	A	A	59.2	60.8	27.0

Table 7.12: Average score of different strategies when played 1 000 000 games.

Always High strategy

Team Always High & Random

We see in Table 7.13 that the scores are slightly higher than the team playing Random & Always High.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
A	R	R	R	**63.7**	56.3	23.8
A	R	R	MC	49.1	70.9	23.1
A	R	R	T	**61.8**	58.2	23.9
A	R	R	A	**62.0**	58.0	25.7
A	R	MC	R	49.8	70.2	23.2
A	R	MC	MC	37.9	82.1	21.3
A	R	MC	T	48.7	71.3	23.0
A	R	MC	A	48.6	71.4	24.4
A	R	T	R	**61.9**	58.1	23.9
A	R	T	MC	48.0	72.0	22.9
A	R	T	T	**60.5**	59.5	23.9
A	R	T	A	**60.2**	59.8	25.4
A	R	A	R	**62.0**	58.0	25.7
A	R	A	MC	47.9	72.1	24.3
A	R	A	T	**60.2**	59.8	25.4
A	R	A	A	59.8	60.2	27.5

Table 7.13: Average score of different strategies when played 1 000 000 games.

Team Always High & Monte-Carlo

The team wins all games, except against a team where both teammates play Monte-Carlo, as can be seen in Table 7.14.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
A	MC	R	R	**75.9**	44.1	22.8
A	MC	R	MC	**61.6**	58.4	23.5
A	MC	R	T	**73.8**	46.2	23.1
A	MC	R	A	**75.2**	44.8	24.6
A	MC	MC	R	**62.3**	57.7	23.5
A	MC	MC	MC	49.8	70.2	23.1
A	MC	MC	T	**60.9**	59.1	23.5
A	MC	MC	A	**61.8**	58.2	25.0
A	MC	T	R	**73.7**	46.3	23.2
A	MC	T	MC	**60.1**	59.9	23.6
A	MC	T	T	**72.2**	47.8	23.3
A	MC	T	A	**72.8**	47.2	24.6
A	MC	A	R	**75.1**	44.9	24.5
A	MC	A	MC	**60.9**	59.1	24.9
A	MC	A	T	**72.6**	47.4	24.6
A	MC	A	A	**73.7**	46.3	26.7

Table 7.14: Average score of different strategies when played 1 000 000 games.

Team Always High & Obligated Trump

Most of the games are won, but not with a very high score, as can be seen in Table 7.15.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
A	T	R	R	**64.7**	55.3	23.7
A	T	R	MC	50.8	69.2	23.2
A	T	R	T	**63.0**	57.0	23.7
A	T	R	A	**63.4**	56.6	25.3
A	T	MC	R	51.5	68.5	23.3
A	T	MC	MC	40.1	79.9	21.8
A	T	MC	T	50.5	69.5	23.2
A	T	MC	A	50.7	69.3	24.5
A	T	T	R	**63.0**	57.0	23.7
A	T	T	MC	49.7	70.3	23.2
A	T	T	T	**61.8**	58.2	23.9
A	T	T	A	**61.7**	58.3	25.1
A	T	A	R	**63.4**	56.6	25.3
A	T	A	MC	50.0	70.0	24.4
A	T	A	T	**61.7**	58.3	25.2
A	T	A	A	**61.6**	58.4	27.0

Table 7.15: Average score of different strategies when played 1 000 000 games.

Team Always High & Always High

We see in Table 7.16 that this team either wins or loses but does not play a draw.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
A	A	R	R	**66.1**	53.9	23.7
A	A	R	MC	50.6	69.4	23.6
A	A	R	T	**64.0**	56.0	23.8
A	A	R	A	**64.5**	55.5	26.9
A	A	MC	R	51.1	68.9	23.7
A	A	MC	MC	38.9	81.1	22.3
A	A	MC	T	49.9	70.1	23.5
A	A	MC	A	50.2	69.8	26.0
A	A	T	R	**64.0**	56.0	23.8
A	A	T	MC	49.3	70.7	23.4
A	A	T	T	**62.4**	57.6	23.9
A	A	T	A	**62.4**	57.6	26.6
A	A	A	R	**64.5**	55.5	26.9
A	A	A	MC	49.6	70.4	25.9
A	A	A	T	**62.4**	57.6	26.6
A	A	A	A	**62.2**	57.8	29.8

Table 7.16: Average score of different strategies when played 1 000 000 games.

7.2 Overview results per strategy

7.2.1 Random

We notice that when both team members play Random, the team loses all games. This means that every other strategy is better than Random. When one team member plays Random and the other plays Obligated Trump or Always High, the team only wins against a team playing both Random. When a team plays Random and Monte-Carlo they win or play draw, except when playing against a team playing both Monte-Carlo.

7.2.2 Monte-Carlo

A team that plays both Monte-Carlo strategy wins all games against each combination of the four strategies. The only draw they play is against a team that also plays Monte-Carlo, which make the teams equivalent. On average the team achieves a score of 76.1 points. It is better to have a player playing Always High strategy in a MC team, than a player playing Obligated Trump strategy. The average score with Obligated Trump is 65.9 and the average score with Always High is 66.4.

7.2.3 Obligated Trump

A team playing Obligated Trump and Random will at most play draw and sometimes lose the game. This combination is thus not very good. The same goes for a team where both team members play Obligated Trump with the exception when playing against a team Random.

7.2.4 Always High

We see that a team playing both Always High scores overall better than a team where both play Obligated Trump. The team Always High manage to win some games: against teams playing Random and Obligated Trump. We notice that an equivalent team, playing Random and Always High, does not result in a draw. However, the standard deviation is 25.7, which is higher than most of the other ones.

7.3 Observations

In this section we discuss several observations we made from the results. We reason what might be an explanation from these observations.

7.3.1 Same strategy, different player

When we take four combinations where in each combination a different player plays one strategy and the other three players play all Random, we see that the scores stay the same. In Table 7.17, we see the Monte-Carlo strategy played by a different player. The team with the Monte-Carlo player has an average score of more or less 73 points. This means that it does not matter in what order the strategies are played. This is consistent with the results from Chapter 4.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
MC	R	R	R	73.0	47.0	23.3
R	MC	R	R	72.9	47.1	23.3
R	R	MC	R	47.3	72.7	23.3
R	R	R	MC	46.8	73.2	23.3

Table 7.17: Average score of different strategies when played 1 000 000 games.

7.3.2 Obligated Trump vs. Always High

To compare the strategies Obligated Trump and Always High we take several combinations where each team use one of these strategies. As seen in Table 7.18, the strategy Always High is better than Obligated Trump except when playing against Random & Monte-Carlo. We could argue that the Obligated Trump strategy is more damaging against Monte-Carlo than the Always High strategy. This might be explained because the players play randomly during the playouts of MC. When a player has two possible cards where one of them is a trump card, MCTS takes a fifty-fifty chance for each card. But in reality the trump card will be chosen by the player playing Obligated Trump strategy. This might result that the trick is won by the player playing the Obligated Trump strategy. On the other hand, when the player does not play the Obligated Trump strategy but plays the Always High strategy instead, the chance is smaller that the player wins the trick.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
R	A	R	T	60.8	59.2	24.2
R	R	R	T	58.7	61.3	24.3
R	R	R	A	57.7	62.3	24.3
R	MC	R	T	71.2	48.8	23.6
R	MC	R	A	71.6	48.4	23.7

Table 7.18: Average score of different strategies when played 1 000 000 games.

7.3.3 Equivalent teams

When a team uses the same strategies as their opponent, we see in Table 7.19 that the score is 60 points, which is a draw. The score is more or less 60 points with a player playing Random and his team member playing any other strategy: Random, Monte-Carlo, Obligated Trump and Always High. This was to be expected as the value of each strategy is outplayed by the same strategy of the other team.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
R	R	R	R	60.1	59.9	24.3
R	MC	R	MC	59.6	60.4	23.8
R	T	R	T	60.1	59.9	24.3
R	A	R	A	60.1	59.9	26.0

Table 7.19: Average score of different strategies when played 1 000 000 games.

However, the difference with a team member playing Monte-Carlo is bigger than the other three strategies. When the Random player is the leader the score is 59.6 and when the MC player is the leader the score is 59.0, as can be seen in Table 7.20. This might be explained because the MC does not have any information about what suits players do not have. When the leader is Random, the MC player is the third player during the first trick and thus has already information for the playouts.

Team N & S		Team E & W		Average score team N & S	Average score team E & W	Standard deviation
R	MC	R	MC	59.6	60.4	23.8
MC	R	R	MC	59.0	61.0	23.7

Table 7.20: Average score of different strategies when played 1 000 000 games.

7.3.4 Teams

When both team members play the same strategy, we can compare one strategy against another. As seen in Table 7.21, when both teams are equivalent the score achieved is around 60 points. This is already seen in Table 7.19. The only exception is when all players play the Always High strategy. The average score is then 62.2 points. We might explain this because of the leader. The leader plays his highest card, so chances are that he wins the trick, which means that the same player starts the next trick as well. The standard deviation of two teams playing Always High is higher than the other standard deviations. What might explain this is that all players give their highest card which means that the first trick probably possess many points. The number of points is then higher than the average points per trick. The following tricks will probably possess a low number of points. This results in a high standard deviation.

What we also see is that a team playing Monte-Carlo strategy plays best, especially against Random with almost 84 points. But two teams playing Monte-Carlo does not result in a draw. This explanation is already given at Section 7.3.3.

Another observation is that the Always High strategy is often better than Obligated Trump, when playing against each other and comparing the scores when playing against a team playing Random. However, against a team playing Monte-Carlo the strategy Obligated Trump is better than Always High.

Team N **&** S		**Team** E **&** W		**Average score team** N **&** S	**Average score team** E **&** W	**Standard deviation**
R	R	R	R	60.1	59.9	24.3
R	R	MC	MC	36.1	83.9	21.5
R	R	T	T	57.8	62.2	24.3
R	R	A	A	54.9	65.1	24.3
MC	MC	MC	MC	59.1	60.9	23.8
MC	MC	T	T	80.2	39.8	22.4
MC	MC	A	A	82.4	37.6	23.7
T	T	T	T	60.0	60.0	24.5
T	T	A	A	58.4	61.6	24.3
A	A	A	A	62.2	57.8	29.8

Table 7.21: Average score of different strategies when played 1 000 000 games.

Chapter 8

Conclusions and future work

8.1 Conclusions

In this study we have examined different strategies to play the card game Sueca. We also did research concerning important factors of the game and concluded that who starts the first trick does not have an advantage. The number of trump cards and the number of points a team has in their hands is very important and influences the possible outcome of the game.

We used four different strategies: Random, Obligated Trump, Always High and Monte-Carlo. It is clear from the results that the Monte-Carlo strategy plays better than the other strategies. A team where a player plays Monte-Carlo gets a higher score and thus wins the game. When both team members play Monte-Carlo, their score is the highest.

We can also conclude that it is likely that the outcome will be a draw when both teams play the same strategies. This make the teams equivalent and the value of each strategy is outplayed by the same strategy of the opponent.

8.2 Future work

There has not been done any research into Sueca thus there are several ways to continue this research. One is to explore the impact of the way the trump card is determined during the game. For our results we determined the suit of the trump cards randomly, but originally the suit of the trump is determined by the first or last card of the dealer. So the dealer has at least one trump card in his hand. We saw in Chapter 4 that this gives the dealer's team an advantage of three points. To see how the way the trump suit is determined affects the outcomes of the game we could run the experiments this way and compare the average scores with our present results.

The card which determines the suit of the trump cards is visible to all players. This means that the players have more information than we used in our experiments. This information could be used in the strategies. For example, for the Monte-Carlo strategy, when MCTS shuffles the cards it is known to what player that card belongs. This could result in a more accurate decision about which card to play. Perhaps, cards being played earlier in the game could also provide some more information about the location of the remaining cards. When a player plays a high value card when the trick is for his opponent, it may mean that he does not have any other cards of that suit.

We investigated what impact the number of trump cards and the number of points per team make. We saw that both have a great effect on the minimal, average and maximal score a team can achieve. To extend this, we could also combine those two to see the impact together. There may be a difference when a team has many points in their hands but no trump cards.

Of course, it is also possible to explore more strategies. As already said, we used a basic form of Monte-Carlo. A more complex variant of Monte-Carlo explores the potential options more thoroughly and the less valuable options less thoroughly. This way the evaluation of the cards will be more accurate. We could also develop more strategies, like a strategy where the player never plays a trump card when he does not have the follow suit.

Another way to improve the research is to combine several strategies, for example the strategies Always High and Obligated Trump. When the player does not have the follow suit and the trick is in favor of the teammate, it plays a high card and when it is in favor of the opponent it plays a trump card.

Bibliography

[1] Pagat rules of card games: Sueca. https://www.pagat.com/aceten/sueca.html. Accessed: 30-03-2016.

[2] J. Martin Bland and Douglas G. Altman. Statistics notes: measurement error. *British Medical Journal Publishing Group*, 313:744, (1996).

[3] Guillaume Chaslot, Sander Bakkes, Istvan Szita, and Pieter Spronck. Monte-Carlo Tree Search: A New Framework for Game AI. In *Proceedings of the Fourth Artificial Intelligence and Interactive Digital Entertainment Conference (AIIDE)*, pages 216–217, 2008.

[4] Ian Frank. *Search and Planning under Incomplete Information: a study using Bridge card play*. Springer Science & Business Media, 2012.

[5] Sylvain Gelly, Levente Kocsis, Marc Schoenauer, Michele Sebag, David Silver, Csaba Szepesvári, and Olivier Teytaud. The grand challenge of computer go: Monte-carlo tree search and extensions. *Communications of the ACM*, 55(3):106–113, 2012.

[6] Bridge Hands. http://www.bridgehands.com/T/Third_Hand_High.htm, 2016. [Online; accessed 13-06-2016].

[7] David Parlett. *The Penguin book of card games*. Penguin UK, 2008.

[8] Wikipedia. List of Play Techniques Bridge. https://en.wikipedia.org/wiki/List_of_play_techniques_(bridge), 2016. [Online; accessed 14-06-2016].

[9] Wikipedia. Sueca (card game). https://en.wikipedia.org/wiki/Sueca_(card_game), 2016. [Online; accessed 25-03-2016].