Universiteit Leiden
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# Opleiding Informatica

Applying a Mixed-integer evolutionary strategy

for the configuration and parameterization of a CMA-ES

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BACHELOR THESIS

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## Abstract

This paper will focus on finding improvements in a Covariance Matrix Adaption Evolution Strategy (CMA-ES). We will make use of a framework that allows us to combine the many variants of the CMA-ES to find the best combination. The configuration of the framework consist of a mix of discrete options and parameter values. To find a optimization for both the framework and the parameters we will make use of a Mixed Integer Evolution Strategy(MIES). We made an attempt to find improvements over previous results. However while the combination of MIES and the CMA-ES were better then common variants, previous experiments that did not involve parameter optimizations were mostly better with the same amount of resources.

# Contents

# Chapter 1

# Introduction

Genetic Algorithms (GA) and Evolutionary Strategies (ES) have been proven useful in solving a wide variety of optimization problems. This paper will focus on the Covariance Matrix Adaption Evolutionary Strategy(CMA-ES). Recent study [1] has shown that optimizing the configuration of a CMA-ES using a GA is a useful way of improving it. The configurations found by the GA have been proven better than any literature described configuration for the CMA-ES. Our goal is to explore if even more improvement can be found. In the previous study only eleven configuration options were altered during the process. We add the ability to alter the remaining continuous and integer parameters, in order to explore the performance of tuned configurations.

## 1.1 Applications

We will make use of the original framework from [1].To find the best configuration and parameterization for the CMA-ES, we will use a Mixed Integer Evolution Strategy(MIES) [2]. The MIES is an ES that can be used to find an approximately optimal solution for a problem whose solution consists of discrete, integer and continuous values. The configuration for the CMA-ES consists of eleven discrete values, one integer value and fourteen continuous values.

## 1.2 Thesis Overview

The thesis consists of the problem description in Chapter 2. Next we will explain the techniques used in solving the problem in the Chapter 3 . The experiments and results will be shown in the Chapter 4 and the results will be discussed in Chapter 5. Finally, in Chapter 6 we present some ideas for future work.

# Chapter 2

# Problem

Our goal is to find the best algorithm for any given optimization problem. Specifically to explore if improvements can be found over previous results by van Rijn et al [1]. We do this by comparing our MIES-based results to those of the original research. For testing the optimization, we use the common Black Box Optimization Benchmark (BBOB) suite [3].

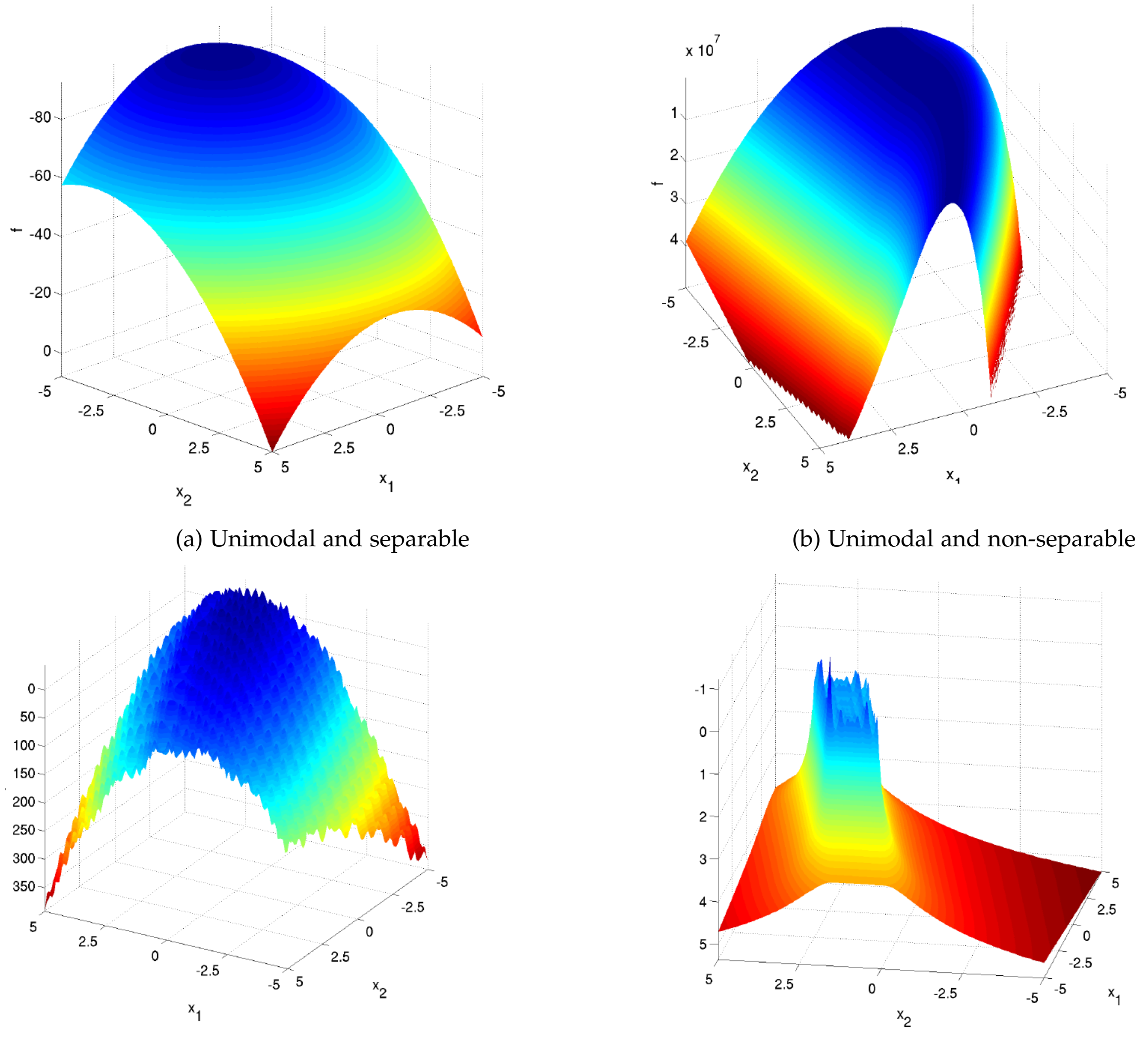
## 2.1 BBOB

The BBOB suite is a standardized collection of 24 test functions that are representative for real-world optimization problems. This allows us to compare optimization algorithms by comparing their performance results on the test problems in the BBOB suite.

The 24 functions are split into four subgroups:3 unimodal and separable,7 unimodal and non-separable, 2 multimodal and separable and 12 multimodal and non-separable. Examples of these subgroups can be found in Figure 2.1.

Unimodal means that only a single global optimal point in the function exists. Multimodal means that there are multiple local optima.

A function is called separable if the global optimal point composed of all optimal points from each dimension. In other words, the function can be optimized independently for each dimension. Non-separable means that the global optimal point cannot be optimized individually for each dimension.




(a) Unimodal and separable

(b) Unimodal and non-separable

(c) Multimodal and seperable

(d) Multimodal and non-seperable

Figure 2.1: Examples of the four function subgroups in the BBOB suite.

# Chapter 3

# Evolution Strategies

There are many forms of Evolution strategies. In this paper we will be using two different ones. The CMA-ES for solving the benchmark functions and the MIES for determining the configuration of the CMA-ES.

## 3.1 Standard Evolution strategy

An evolution strategy is based on biological evolution. Initially, individual are created as possible solutions for an optimization problem. These initial individuals will be the first generation. Each individual is evaluated using a fitness function.By recombining the best individual and applying mutation a new generation is created. The idea is that each generation will produce better results then its predecessor. This process will repeat itself until a fitness objective is met or the resources run out.

## 3.2 CMA-ES

What makes a CMA-ES different from other ES's is that it uses a covariance matrix to adapt the distribution of mutations during the search process. This covariance matrix is updated each generation. For a more detailed description of the CMA-ES, please refer to [4]. A 2-dimensional example is shown in figure3.1
The framework we use for the CMA-ES [4] has eleven independent modules that each have two or three options resulting in 4608 different module combinations. Some of these modules have associated continuous parameters. Some parameters are not dependent on a module but are simply parameters of the basic CMA-ES. These parameters are the integer parameter population size $\mu$ and the continuous parameters $\alpha_{mu}$, $c_\sigma$, damps, $c_c$, $c_1$ and $c_{mu}$.

The eleven modules and their respective parameters are as follows:

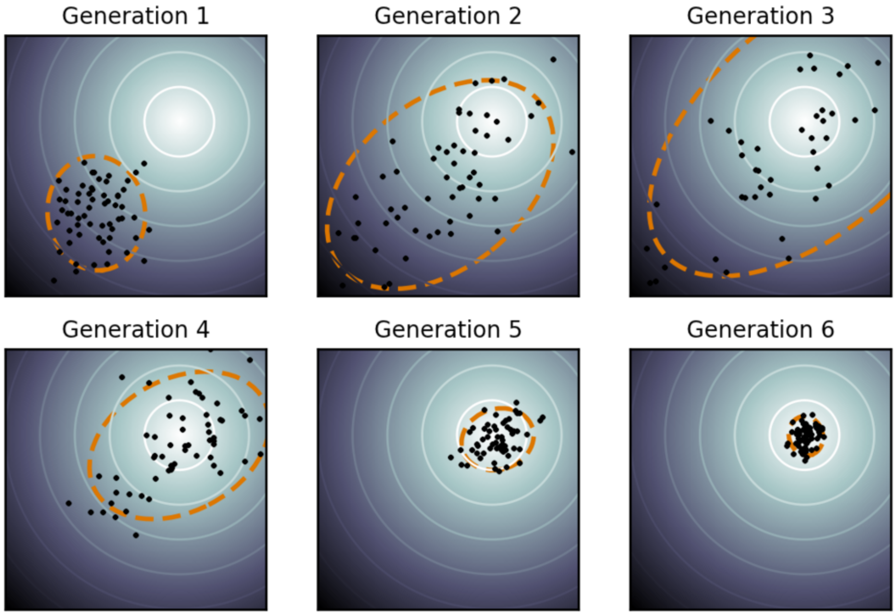



Figure 3.1: The black dots represent the individuals generated by the ES. The white rings mean equal fitness in the search space. The white circle is the global optimum, and the orange dotted line shows the sampling distribution described by the covariance matrix.

1. Active update

   Active update uses not only the best individuals in the population, but also the least favored individuals in order to adapt the covariance matrix. Without active update, only the best individuals will be used. The Active Update is introduced by Jastrebski el al. [5].

2. Elitism

   Elitism allows individuals from previous generations to survive and reproduce if no better solutions are found. If turned off the algorithm is forced to choose an new set of parents from the current generation. If this option is enabled $(\mu + \lambda)$ is used. If this option is disabled $(\mu, \lambda)$ is used.

3. Mirrored Sampling

   Mirrored sampling is used to evenly spread the sampling of the search space. First sampling half of the intended number of mutation vectors, and then using those and their mirror images to create the new candidates. [6].

4. Orthogonal Sampling

   Another sampling options is orthogonal sampling [7]. The desired number of samples is first drawn from the usual distribution. Then the Gram-Schmidt process is used to orthonormalize the set of vectors. Although orthogonal sampling was introduced in combination with Mirrored Sampling, both can be used independently in this framework.

5. Sequential Selection

   Normally the entire new generation is evaluated before selecting the new best individual. However Sequential Selection stops the evaluation of further offspring once an improvement has been found. [6].

6. Threshold Convergence
   Threshold Convergence can be used to prevent the the ES to get stuck on a local optimum. [8]. Threshold convergence forces the ES to stay in the exploratory phase for a longer time. It does this by requiring the mutation vectors to be of greater or equal length than a certain threshold. The decrease over time of this threshold is governed by associated continuous parameters 'initial-threshold' and 'decay-factor'.

7. Two-Point Step-Size Adaptation (TPA)
   TPA determines if the step-size of the CMA-ES should increase or decrease [9]. It does this by creating two additional candidates after the selection step. They are sampled along the direction of the weighted average mutation vector, one shorter and one longer. The step size is increased if the candidate with longer mutation step is more successful, otherwise the step size is decreased. It makes use of the following continuous parameters:'tpa-factor', '$\beta_{tpa}$, $c_\alpha$ and $\alpha$.

8. Pairwise Selection
   In order to prevent mirrored-sampling pairs from (partially) cancelling each other out when updating the covariance matrix or step size, Pairwise Selection as introduced by Auger, et al. [10] can be used. It selects the best offspring from each pair, and then regular selection is applied to that set. Although Pairwise Selection was introduced together with Mirrored Sampling and Orthogonal Sampling, all can be used individually.

9. Recombination Weights
   In the CMA-ES, recombination is performed with the following weight vector: $w_i = log(\mu + \frac{1}{2}) - \frac{log(i)}{\Sigma_j w_j}$ for $i, \cdots, \mu$. Alternative weights are the arithmetic mean $w_i = \frac{1}{\mu}$.

10. Quasi-Gaussian Sampling
    Instead of drawing samples from a Gaussian distribution sampling they can also be drawn from a Quasi-random uniform sequence, which are then transformed to a Gaussian distribution. This was proposed in [11]. Available Quasi-random sequences are the 'Sobol' and 'Halton' sequences.

11. Increasing Population Size (IPOP/BIPOP)
    With no improvement in recent generations one can restart an ES with increasing generation size(IPOP) [12]. Alternatively bi-population(BIPOP) [13] can be used. BIPOP varies between increasing and decreasing population size. Both IPOP and BIPOP make use of the continuous parameter 'population-increase-factor'.

## 3.3 MIES

Mixed Integer Evolution Strategies(MIES) are designed for optimizing problems where the objective function uses a mix of continuous, integer and discrete values. As a regular ES assumes that the candidate consists of only of continuous values, typical mutation would not work for discrete and integer values. MIES has a separate mutation process for each set of values.

Table I: *A table showing all continuous and integer variables and the associated module. λ is the only integer value, all the other values are continuous. μ is shown as a percentage of λ*

| Integer/Continuous variable | default values | Associated module |
|---|---|---|
| $\lambda$ | $4 + (3 \cdot \log(n))$ | |
| $\mu$ | 0.5 | |
| $\alpha_{mu}$ | 2 | |
| $c_\sigma$ | $\frac{\mu_{eff}+2}{\mu_{eff}+n+5}$ | |
| damps | $1 + 2 \cdot \sqrt{\frac{\mu_{eff}-1}{n+1}} - 1 + c_\sigma$ | |
| $c_c$ | $\frac{(4+\mu_{eff}/n)}{(n+4+2\cdot\mu_{eff}/n)}$ | |
| $c_1$ | $\frac{2}{(n+1.3)^2+\mu_{eff}}$ | |
| $c_{mu}$ | $min(1 - c_1, \alpha_{mu} \cdot \frac{\mu_{eff}-2+1/\mu_{eff}}{(n+2)^2+\alpha_{mu}\cdot\mu_{eff}/2})$ | |
| initial-threshold | 0.2 | Threshold Convergence |
| decay-factor | 0.995 | Threshold Convergence |
| tpa-factor | 0.5 | TPA |
| $\beta_{tpa}$ | 0 | TPA |
| $c_\alpha$ | 0.3 | TPA |
| $\alpha$ | 0.5 | TPA |
| 'population-increase-factor' | 2 | IPOP/BIPOP. |

The candidate solution representation is split into three subsets. Let $S$ denote the entire representation of a candidate solution. Then $D^n$, $I^m$ and $R^k$ are the sets of discrete, integer and continuous values from S respectively. Thus an individual in the MIES can be represented by $S = D^n I^m R^k$. In the remainder of this section we will elaborate on the mutations of each of these three sets of values.

### 3.3.1 Nomenclature

In the next sections the following definitions are used:

$N(a, b)$ denotes a normally distributed random number with expectation $a$ and standard deviation $b$.

$U(a, b)$ is a uniformly distributed number between $a$ and $b$.

$T[a, b](x)$ is the transformation function, as proposed in [2] that keeps the value $x$ between upper and lower bound $a$ and $b$. The algorithm is shown in Algorithm 1.

**Algorithm 1** Transformation function

$y = (x - a)/(b - a)$
**if** $\lfloor y \rfloor \mod 2 = 0$ **then**
  $y = |y - \lfloor y \rfloor|$
**else**
  $y = 1 - |y - \lfloor y \rfloor|$
**end ifreturn** y

### 3.3.2 Discrete values

Each discrete parameter in $D^n$ has its own step-size value. The step-size is represented as $P_1 \cdots P_k$ with $P_i$ the step-size of $D_i$. As shown in algorithm 2 in step 3-5 all step-sizes are updated. Then if $U(0,1)$ is lower then the step-size, the discrete value is randomly chanced to a different possible value in step 6-7.

**Algorithm 2** Discrete values

1: $N_c = N(0,1)$
2: Initialize global and local learning curve $\tau = \frac{1}{\sqrt{2k}}$ ;$\tau' = \frac{1}{\sqrt{2\sqrt{k}}}$
3: **for all** $i \in \{1, \cdots n\}$ **do**
4: $\quad P_i = \frac{1}{1+\frac{1-P_i}{P_i} \cdot exp(-\tau \cdot N_c - \tau' \cdot N(0,1))}$
5: $\quad P_i = T[\frac{1}{3n}, 0.5]P_i$
6: $\quad$ **if** $U(0,1) < P_i$ **then**
7: $\quad\quad$ choose random from set of possible discrete values without the current value of the discrete
8: $\quad$ **end if**
9: **end for**

### 3.3.3 Integer Values

A individual step-size is kept for each integer parameter. The step-size is represented as $\varsigma_1 \cdots \varsigma_m$ with $\varsigma_i$ the step-size of $I_i$.

Algorithm 3 shows the mutation procedure for integer values. First the step-size is updated in step 4. Then variables are created in order to create $G_1$ and $G_2$. Which are then added and subtracted from the value of the initial integer respectively.

**Algorithm 3** Integer values

1: $N_c = N(0,1)$
2: Initialize global and local learning curve $\tau = \frac{1}{\sqrt{2k}}$ ;$\tau' = \frac{1}{\sqrt{2\sqrt{k}}}$
3: **for all** $i \in \{0, \cdots, m\}$ **do**
4: $\quad \varsigma_i = max(1, \varsigma_i exp(\tau N_c + \tau' N(0,1))$
5: $\quad u_1 = U(0,1); u_2 = U(0,1)$
6: $\quad \psi = 1 - (\varsigma_i/m)(1 + \sqrt{1 + (\frac{\varsigma_i}{m})})^{-1}$
7: $\quad G_1 = \lfloor \frac{ln(1-u_1)}{ln(1-\psi)} \rfloor$
8: $\quad G_2 = \lfloor \frac{ln(1-u_2)}{ln(1-\psi)} \rfloor$
9: $\quad I_i = I_i + G_1 - G_2$
10: **end for**

### 3.3.4 Continuous Values

The mutation of the continuous values is similar to integer mutation. A individual step-size is kept for each continuous parameter. As shown in algorithm 4, the step-size is updated with the learning curve and added to the previous continuous value. The value of the parameter is then kept in-bounds by the transformation function T. The step-size is represented as $\sigma_1 \cdots \sigma_k$ with $\sigma_i$ the step-size of $R_i$.

**Algorithm 4** Continuous values

$N_c = N(0, 1)$
Initialize global and local learning curve $\tau = \frac{1}{\sqrt{2k}}$ ;$\tau' = \frac{1}{\sqrt{2\sqrt{k}}}$
**for all** $i \in \{0, \cdots, k\}$ **do**
    $\sigma_i = \sigma_i \cdot exp(\tau N_c + \tau' N(0, 1))$
    $R_i = Ri + \sigma i$
    $R_i = T(max_i, min_i)R_i$
**end for**

# Chapter 4

# Experiments

The experiments reported in this thesis are based on the ideas discussed in [1]. The key difference between our study and [1] is that the MIES changes all the parameters. As it is well known that tuned algorithms perform better. These experiments test the possibility of tuning the CMA-ES while optimizing its structure at the same time.

The experiments are run on all 24 Black Box Optimization Problems. The evolutionary strategy was executed om 2,3 and 5 dimensional problems. All test are run 10 times to get a solid estimate of how good this approach works.

## 4.1 Settings

The previous experiments of [1] use a generation size of 12 and a budget of 20 generations for the Genetic Algorithm (GA) that optimizes the configuration. Although the search-space is larger due to the inclusion of the CMA-ES parameters, we choose to give the MIES the same budget of 20 generations. This is done both to facilitate a more direct comparison with previous results, and due to time-constraints.

MIES sets the configuration of the CMA-ES. In order for the MIES to properly work, all integer and continuous values of the CMA-ES need to have an upper and lower bound. For the single integer value $\lambda$ (population size), we use a minimum of 2 and a maximum of 200. It is argued that with a budget of $1000 \cdot D$(where D denotes the dimensionality of the problem), a larger population size would probably lead to too few generation.
For continuous values we define the range as $[0.7\sigma_i, 1.3\sigma_i]$ where $\sigma_i$ are the default values from literature as mentioned in Chapter 3. A standard range of [0,1] was used if a default value was 0.

## 4.2 Conditional mutation

Roughly half of the continues variables are uniquely used by a discrete option. If they are mutated while the discrete option is not enabled, we will get no feedback on this continuous value and potentially lose information. Therefore if a discrete option is disabled all continuous values that depend on the option will remain unaltered. All discrete options with the corresponding continuous values can be found in Table I.

## 4.3 FCE and ERT

In order to make the results measurable we make use of the same criteria as in [1]. Fixed Cost Error (FCE) and Expected Run Time (ERT) will be used. FCE is the lowest distance to the global optimum after using fixed computation cost. The target fitness is set at $10^{-8}$. If a run reached the target within the given resources it has been successful, then the FCE will be $10^{-8}$. The ERT gives more information but can only be calculated if at least one of the runs reached the target fitness. If none of the runs reach the target fitness, the ERT is considered invalid and only FCE can be used.

ERT is calculated as follows:

$$ERT(f_{target}) = RT_S \frac{1 - p_s}{p_s} RT_{US}$$

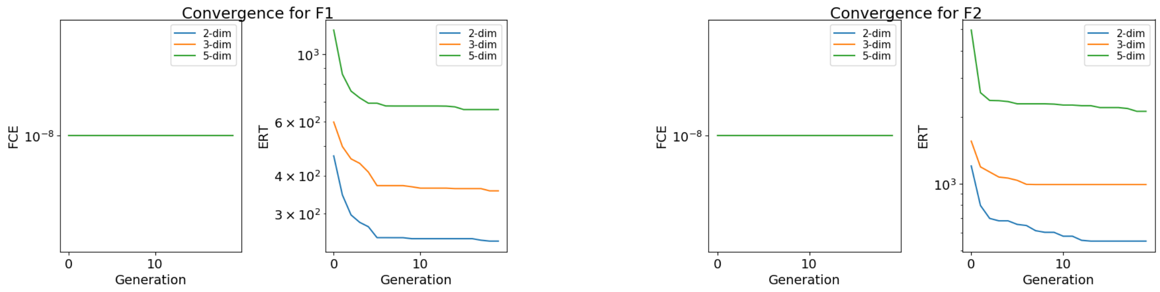
With $RT_S$ the mean of $r_s$, and $r_s$ rated with the number of function evaluations needed to reach $f_{target}$. Furthermore, let $RT_{US}$ be the mean of $r_{us}$ with $r_{us}$ runs that did not reach $f_{target}$ and are considered unsuccessful. These are rated with the full run time. The ratio of successful runs to all runs yields the value $p_s$.
In words, the ERT is the time you expect the CMA-ES to take in order to reach the taget value $f_{targer}$.

As the goal of an optimizer is to find the best possible (i.e. minimal) fitness value in the least amount of time, we want to minimize both FCE and ERT measures. Comparisons between two configurations are done on FCE only if neither have valid ERT values, otherwise the only valid/better ERT wins.

## 4.4 Results

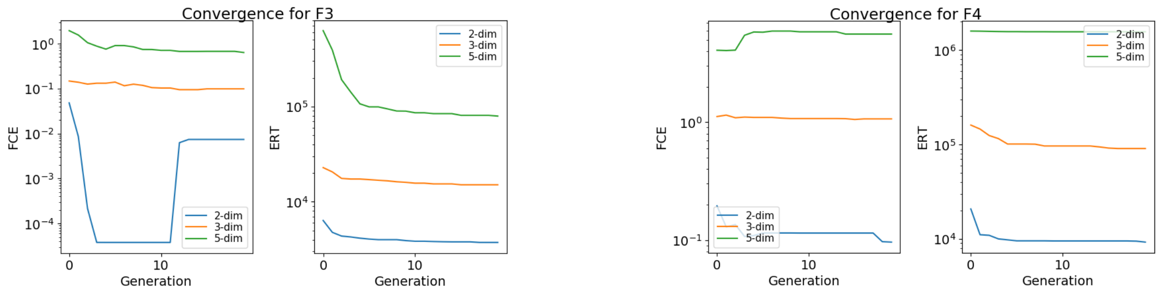
As you can see in the graphs of F1,F3 and F6 for example, the FCE line is horizontal with a value of 1e-8. This is because that is the target value of the fitness function and therefore the minimum(best) result. If the target value has been found in the first generation, the FCE will most likely stay at that value.

The FCE can still rise during the process( for example, F3, F5 and F16). This is because in that generation only higher values were found.




(a) F1 (b) F2

Figure 4.1: plots of F1 - F2




(a) F3 (b) F4

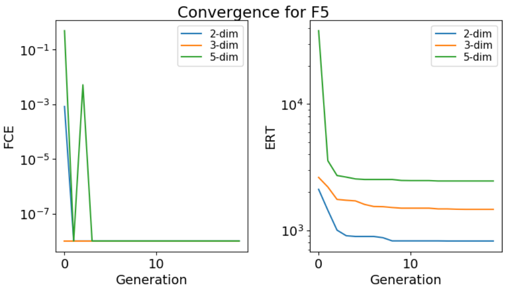
Figure 4.2: plots of F3 - F4

However, the ERT (Expected Run Time) will not increase because, only when improvements are found the ERT can be updated.
While most improvements of ERT are found in later generation, it is very possible for the best solution to be found somewhere in the first few generation. For example F10 3-dim and 5-dim show no improvement after generation 2, and F10 2-dim has reached its best result in generation 1.
However it can be discussed that better results will be found after the initial 20 generations.

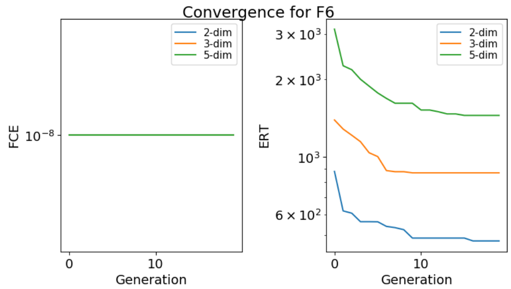
If we look at F6 2-dim, we can see that there are multiple moments where improvements were found. And we could argue that given more generations, even better results will be found.

It is often that a higher dim results in a higher ERT. In almost all cases ERT of 5-dim is higher then 3-dim which in turn is higher then 2-dim. However this is not always the case. F10 and F18 3-dim has a higher ERT then 5-dim. It would make sense that a higher dimensionality would cost more run time.

All the data has been processed into graphs and tables. The graph shows the average improvements over 10 runs. And the tables show the best results found. In 4.1 - 4.12 the results of 20 generations with F1- F24 in 2, 3 and 5 dimensions are shown.
All best results are shown in Table I and Table II. The results are compared to previous results in Table III and Table IV to the results from [1].

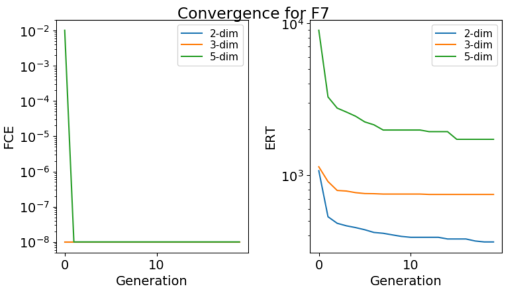



(a) F5

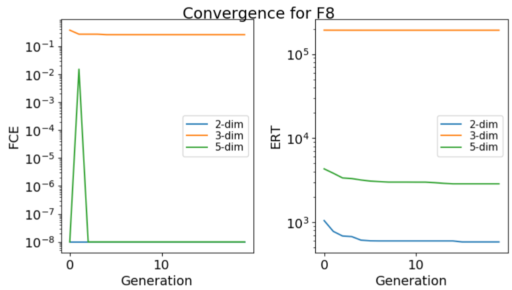



(b) F6

Figure 4.3: plots of F5 - F6

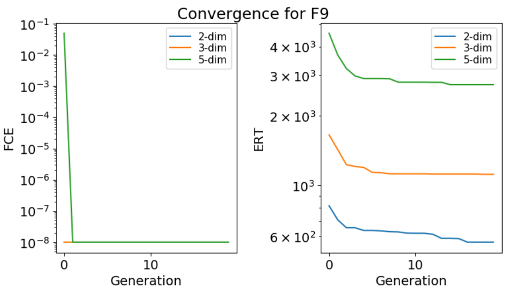



(a) F7

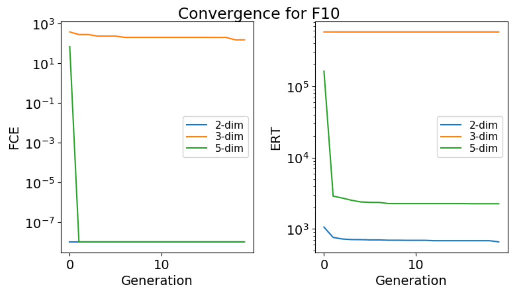



(b) F8

Figure 4.4: plots of F7 - F8

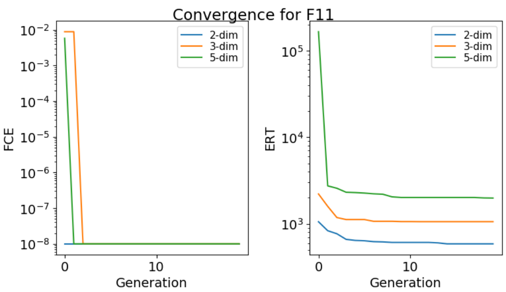



(a) F9

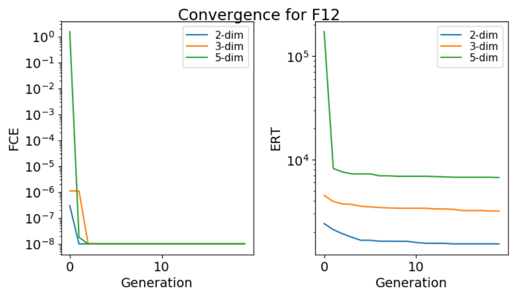



(b) F10

Figure 4.5: plots of F9 - F10

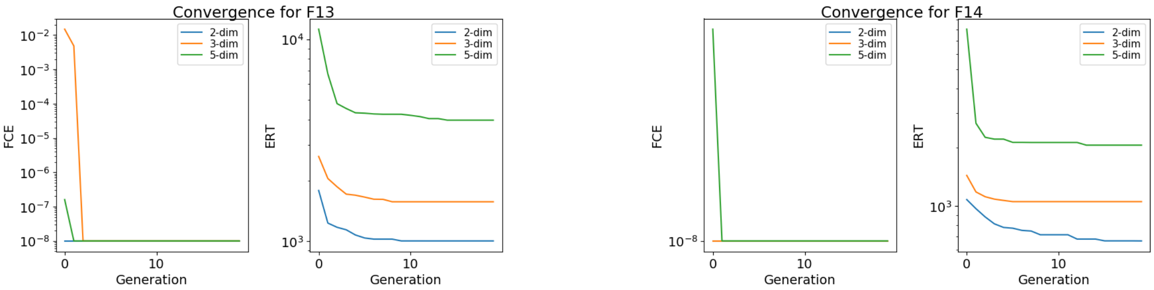



(a) F11



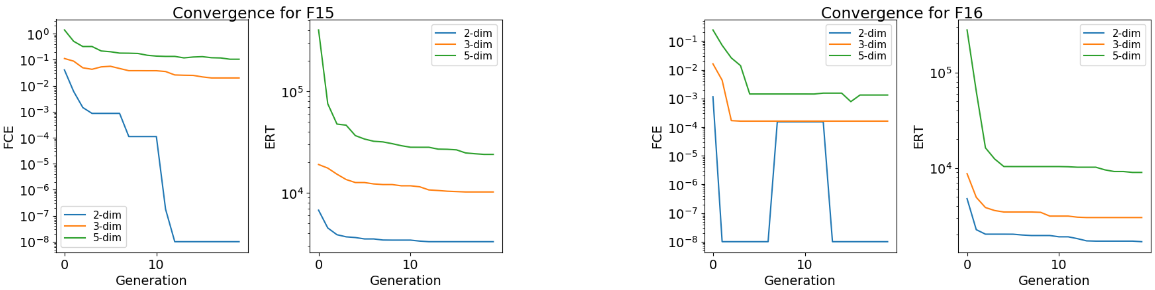

(b) F12

Figure 4.6: plots of F11 - F12



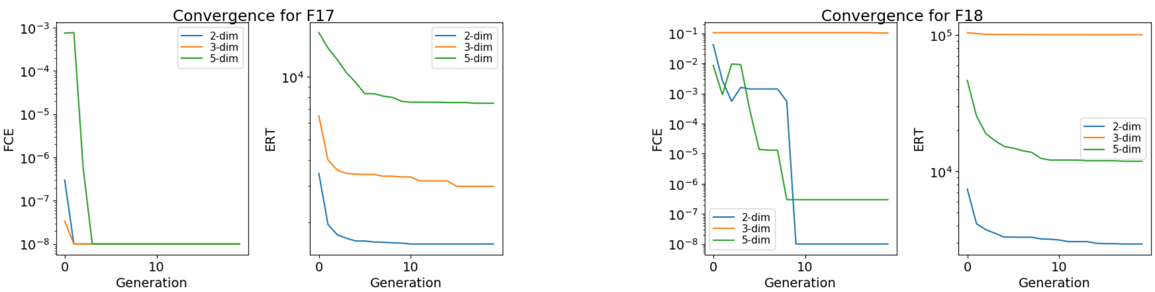

(a) F13 (b) F14

Figure 4.7: plots of F13 - F14



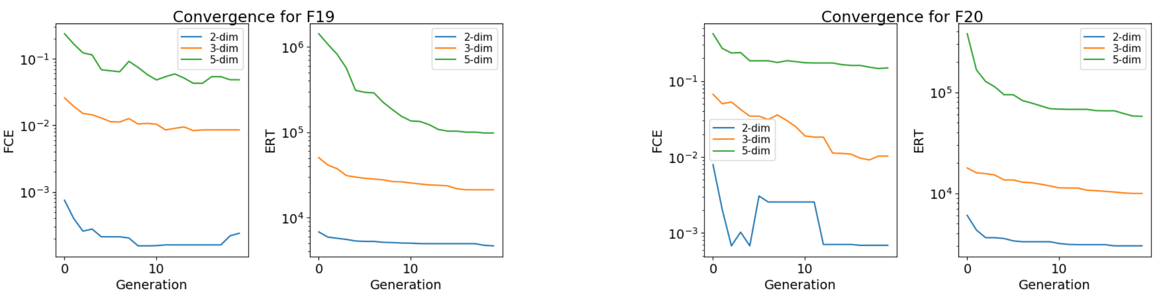

(a) F15 (b) F16

Figure 4.8: plots of F15 - F16



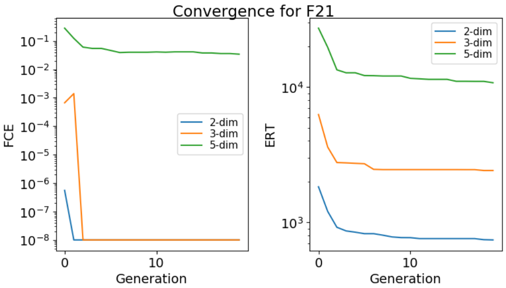

(a) F17 (b) F18

Figure 4.9: plots of F17 - F18

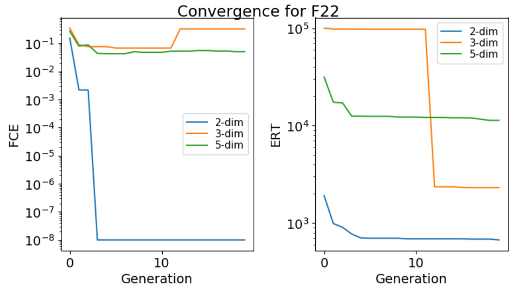



(a) F19 (b) F20

Figure 4.10: plots of F19 - F20

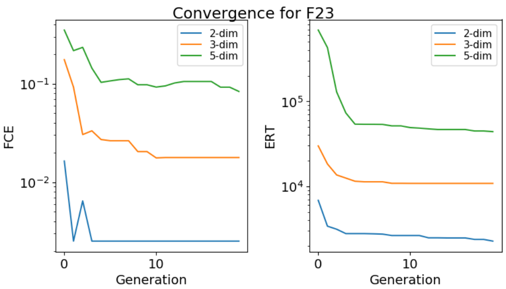



(a) F21

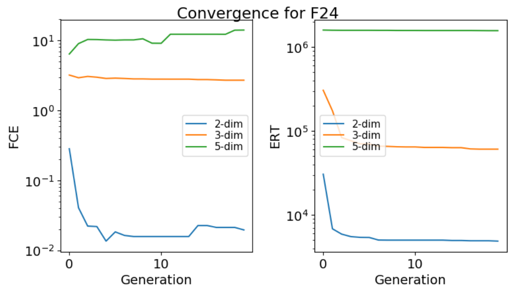



(b) F22

Figure 4.11: plots of F21 - F22




(a) F23




(b) F24

Figure 4.12: plots of F23 - F24

# Chapter 5

# Conclusion

Looking at table III and table IV in the CV/MIES column you can clearly see that the MIES mostly performs better then the Common Variant. If an improvement has been found the value is higher then 1. If the value is lower, the MIES failed to find better results. When comparing the MIES against the GA, results aren't as clear. While some improvements are found (for example F18 dim 5 in table IV), most results of the GA are still better or somewhat the same. Of the total 72 results, 9 improvements were found, 9 results are somewhat the same ( between 0.95 and 1.05) and 54 are worse.

The reason that most results are worse is most likely due to the fact that there are too many parameters and discrete values to calculate within the given calculation time. While the GA only needed to optimize 11 discrete values, the MIES had to optimize 15 additional parameters. This causes the MIES to create results that are mostly worse then the GA.

# Chapter 6

# Future research

There are some possible improvements in this research. Because there are so many parameters that influence the result, what could be done is running 20 generations to find the best discrete value combination for the CMA-ES configuration and using another 20 generations for parameter optimization.

Right now all continuous values have been set at a predetermined setting and given a limit 30% up and down of that value. It would be possible to find better results in the long run if the limits are allowed to optimize in a wider range

The discrete options IPOP/BIPOP both use the same parameter 'increase-population-size'. This means that if IPOP is being used, the parameter for BIBOP will change. This could give undesirable results. a suggestion would be to split the parameter 'increase population size' into two different parameters.
There are still experiments from 10 and 20 dimensionality that can be tested and compared.

Table I: *All best found by the MIES F1 -F12. discrete values represent the same option as in the previous experminents from [1].The first number in the next colom is the population size $\lambda$. The discrete values from left to right are $\mu$ , $\alpha_{mu}$ $c_{\sigma}$ , damps , $c_c$, $c_1$ , $c_{mu}$ ,'initial-threshold' , 'decay-factor', 'tpa-factor , 'beta-tpa','c-alpha', 'alpha' and population-increase-factor.*

| F-ID | N | GA-discrete | parameters | | | | | | | | | | | | | | | FCE | ERT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F1 | 2 | 00011010000 | 18 | 0.068 | 2.43 | 0.1 | 1.285 | 0.13 | 0.003 | 0.654 | 0.2 | 0.955 | 0.383 | 0.462 | 0.214 | 0.546 | 2.133 | 1e-08 | 157.72 |
| F1 | 3 | 00110010112 | 11 | 0.062 | 1.854 | 0.077 | 0.838 | 0.153 | 0.002 | 0.664 | 0.227 | 0.906 | 0.408 | 0.053 | 0.254 | 0.585 | 2.564 | 1e-08 | 251.91 |
| F1 | 5 | 01011010112 | 11 | 0.059 | 1.748 | 0.097 | 1.181 | 0.129 | 0.002 | 0.255 | 0.2 | 0.955 | 0.415 | 0.192 | 0.221 | 0.441 | 2.254 | 1e-08 | 453.84 |
| F2 | 2 | 00010011002 | 8 | 0.101 | 2.22 | 0.101 | 1.373 | 0.153 | 0.002 | 0.597 | 0.2 | 0.955 | 0.392 | 0.621 | 0.375 | 0.383 | 2.209 | 1e-08 | 426.44 |
| F2 | 3 | 10010011001 | 9 | 0.092 | 1.96 | 0.098 | 1.113 | 0.096 | 0.003 | 0.454 | 0.151 | 0.885 | 0.426 | 0.298 | 0.366 | 0.422 | 1.515 | 1e-08 | 878.13 |
| F2 | 5 | 01000010012 | 21 | 0.072 | 1.797 | 0.068 | 1.299 | 0.152 | 0.002 | 0.131 | 0.2 | 0.955 | 0.527 | 0.473 | 0.371 | 0.379 | 1.81 | 1e-08 | 168.00 |
| F3 | 2 | 01011010112 | 52 | 0.076 | 2.211 | 0.104 | 0.782 | 0.142 | 0.002 | 0.958 | 0.2 | 0.955 | 0.559 | 0.239 | 0.215 | 0.434 | 2.381 | 1e-08 | 3213.44 |
| F3 | 3 | 10111011021 | 111 | 0.098 | 1.731 | 0.091 | 1.396 | 0.121 | 0.002 | 0.857 | 0.2 | 0.955 | 0.386 | 0.388 | 0.314 | 0.463 | 1.817 | 0.0679 | 12877.64 |
| F3 | 5 | 00000010001 | 146 | 0.091 | 2.588 | 0.072 | 1.402 | 0.137 | 0.003 | 0.324 | 0.2 | 0.955 | 0.401 | 0.126 | 0.278 | 0.462 | 1.935 | 0.308 | 47132.73 |
| F4 | 2 | 11110000011 | 18 | 0.108 | 1.56 | 0.087 | 1.154 | 0.174 | 0.002 | 0.141 | 0.2 | 0.955 | 0.374 | 0.545 | 0.28 | 0.469 | 1.713 | 0.0509 | 7054.41 |
| F4 | 3 | 00101000022 | 13 | 0.061 | 2.176 | 0.066 | 1.226 | 0.124 | 0.002 | 0.127 | 0.253 | 0.936 | 0.601 | 0.246 | 0.328 | 0.394 | 2.211 | 0.887 | 62078.69 |
| F4 | 5 | 11100000102 | 45 | 0.101 | 1.936 | 0.08 | 0.823 | 0.108 | 0.003 | 0.283 | 0.2 | 0.955 | 0.441 | 0.687 | 0.35 | 0.571 | 2.587 | 3.48 | 1553072.00 |
| F5 | 2 | 10101100022 | 25 | 0.085 | 1.523 | 0.103 | 1.228 | 0.126 | 0.003 | 0.893 | 0.223 | 0.818 | 0.5 | 0 | 0.3 | 0.5 | 2.196 | 1e-08 | 505.25 |
| F5 | 3 | 10111000020 | 24 | 0.078 | 2.284 | 0.121 | 1.35 | 0.155 | 0.002 | 0.329 | 0.141 | 0.755 | 0.605 | 0.024 | 0.339 | 0.566 | 1.756 | 1e-08 | 809.13 |
| F5 | 5 | 01101000001 | 14 | 0.083 | 1.705 | 0.116 | 1.416 | 0.108 | 0.002 | 0.106 | 0.254 | 1.052 | 0.5 | 0 | 0.3 | 0.5 | 2.31 | 1e-08 | 1772.66 |
| F6 | 2 | 11010001112 | 7 | 0.093 | 2.59 | 0.093 | 1.184 | 0.169 | 0.003 | 0.965 | 0.2 | 0.955 | 0.471 | 0.32 | 0.281 | 0.36 | 2.154 | 1e-08 | 334.72 |
| F6 | 3 | 11110010010 | 13 | 0.102 | 2.486 | 0.114 | 1.066 | 0.145 | 0.002 | 0.218 | 0.2 | 0.955 | 0.405 | 0.298 | 0.373 | 0.446 | 1.888 | 1e-08 | 566.69 |
| F6 | 5 | 00110001120 | 6 | 0.087 | 2.037 | 0.072 | 0.812 | 0.119 | 0.002 | 0.06 | 0.2 | 0.955 | 0.466 | 0.018 | 0.302 | 0.571 | 2 | 1e-08 | 1099.38 |
| F7 | 2 | 11001010012 | 14 | 0.086 | 2.392 | 0.118 | 1.009 | 0.096 | 0.002 | 0.184 | 0.172 | 0.787 | 0.62 | 0.809 | 0.221 | 0.448 | 2.318 | 1e-08 | 268.63 |
| F7 | 3 | 00000011121 | 12 | 0.08 | 2.061 | 0.077 | 1.393 | 0.125 | 0.003 | 0.695 | 0.2 | 0.955 | 0.471 | 0.361 | 0.217 | 0.472 | 2.367 | 1e-08 | 564.66 |
| F7 | 5 | 10000010021 | 94 | 0.072 | 1.455 | 0.091 | 1.301 | 0.173 | 0.002 | 0.675 | 0.195 | 0.754 | 0.384 | 0.947 | 0.279 | 0.431 | 2.25 | 1e-08 | 1270.25 |
| F8 | 2 | 01001000100 | 5 | 0.101 | 1.983 | 0.093 | 0.939 | 0.101 | 0.002 | 0.44 | 0.229 | 0.727 | 0.423 | 0.347 | 0.211 | 0.52 | 1.933 | 1e-08 | 506.09 |
| F8 | 3 | 00000111002 | 2 | 0.096 | 1.538 | 0.092 | 1.197 | 0.137 | 0.002 | 0.059 | 0.153 | 0.902 | 0.603 | 0.746 | 0.332 | 0.615 | 1.67 | 1e-08 | 1041.81 |
| F8 | 5 | 11000111111 | 12 | 0.075 | 2.067 | 0.078 | 1.323 | 0.118 | 0.002 | 0.01 | 0.234 | 0.737 | 0.482 | 0.591 | 0.383 | 0.456 | 2.579 | 1e-08 | 2406.09 |
| F9 | 2 | 01001011011 | 9 | 0.099 | 2.049 | 0.113 | 1.072 | 0.118 | 0.002 | 0.21 | 0.219 | 0.899 | 0.648 | 0.023 | 0.222 | 0.496 | 1.903 | 1e-08 | 398.00 |
| F9 | 3 | 11010010011 | 16 | 0.081 | 1.632 | 0.098 | 0.998 | 0.132 | 0.003 | 0.514 | 0.2 | 0.955 | 0.575 | 0.058 | 0.289 | 0.465 | 2.096 | 1e-08 | 959.75 |
| F9 | 5 | 00110010011 | 20 | 0.074 | 1.592 | 0.113 | 1.416 | 0.162 | 0.002 | 0.442 | 0.2 | 0.955 | 0.501 | 0.02 | 0.31 | 0.552 | 1.492 | 1e-08 | 2307.69 |
| F10 | 2 | 11000000110 | 4 | 0.101 | 2.491 | 0.088 | 0.844 | 0.117 | 0.003 | 0.07 | 0.183 | 0.779 | 0.471 | 0.222 | 0.365 | 0.529 | 2.207 | 1e-08 | 502.38 |
| F10 | 3 | 10110011101 | 16 | 0.082 | 2.006 | 0.113 | 1.2 | 0.162 | 0.002 | 0.523 | 0.2 | 0.955 | 0.488 | 0.562 | 0.297 | 0.399 | 2.59 | 1e-08 | 956.50 |
| F10 | 5 | 00110010002 | 20 | 0.093 | 1.452 | 0.119 | 0.937 | 0.154 | 0.002 | 0.165 | 0.228 | 0.789 | 0.507 | 0.571 | 0.251 | 0.432 | 1.902 | 1e-08 | 1915.88 |
| F11 | 2 | 10011010020 | 24 | 0.108 | 1.734 | 0.093 | 0.927 | 0.149 | 0.002 | 0.81 | 0.218 | 0.979 | 0.645 | 0.561 | 0.287 | 0.457 | 1.958 | 1e-08 | 448.65 |
| F11 | 3 | 00111010011 | 13 | 0.058 | 2.557 | 0.103 | 1.132 | 0.163 | 0.003 | 0.478 | 0.2 | 0.955 | 0.428 | 0.081 | 0.365 | 0.51 | 1.442 | 1e-08 | 864.66 |
| F11 | 5 | 00001010012 | 22 | 0.077 | 2.312 | 0.112 | 1.033 | 0.161 | 0.003 | 0.308 | 0.164 | 0.798 | 0.649 | 0.199 | 0.317 | 0.429 | 1.532 | 1e-08 | 1272.06 |
| F12 | 2 | 11001011122 | 23 | 0.099 | 1.525 | 0.096 | 0.908 | 0.125 | 0.003 | 0.336 | 0.247 | 0.867 | 0.522 | 0.729 | 0.219 | 0.476 | 1.68 | 1e-08 | 1126.00 |
| F12 | 3 | 01111010102 | 22 | 0.099 | 2.52 | 0.116 | 1.119 | 0.122 | 0.002 | 0.341 | 0.143 | 1.164 | 0.63 | 0.069 | 0.222 | 0.37 | 2.042 | 1e-08 | 2672.03 |
| F12 | 5 | 01010010022 | 12 | 0.085 | 1.462 | 0.076 | 0.765 | 0.161 | 0.002 | 0.272 | 0.142 | 0.795 | 0.471 | 0.114 | 0.356 | 0.419 | 2.049 | 1e-08 | 6082.81 |

TABLE II: *All best found by the MIES F13 -F24. discrete values represent the same option as in the previous experminents from [1].The first number in the next colom is the population size $\lambda$. The discrete values from left to right are $\mu$ , $\alpha_{mu}$ $c_{\sigma}$ , damps , $c_c$, $c_1$ , $c_{mu}$ ,'initial-threshold' , 'decay-factor', 'tpa-factor , 'beta-tpa','c-alpha', 'alpha' and population-increase-factor.*

| F-ID | N | GA-discrete | parameters | | | | | | | | | | | | | | | FCE | ERT |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| F13 | 2 | 10010010112 | 21 | 0.062 | 2.237 | 0.082 | 1.298 | 0.141 | 0.002 | 0.539 | 0.2 | 0.955 | 0.556 | 0.09 | 0.256 | 0.355 | 2.286 | 1e-08 | 699.25 |
| F13 | 3 | 01000010101 | 8 | 0.104 | 2.359 | 0.07 | 0.921 | 0.138 | 0.001 | 0.239 | 0.2 | 0.955 | 0.501 | 0.25 | 0.218 | 0.447 | 2.437 | 1e-08 | 1400.91 |
| F13 | 5 | 10110010021 | 30 | 0.091 | 1.925 | 0.118 | 1.279 | 0.118 | 0.001 | 0.295 | 0.2 | 0.955 | 0.508 | 0.286 | 0.339 | 0.426 | 1.994 | 1e-08 | 3371.44 |
| F14 | 2 | 00110011000 | 9 | 0.089 | 2.302 | 0.096 | 1.321 | 0.112 | 0.002 | 0.522 | 0.199 | 1.034 | 0.487 | 0.713 | 0.339 | 0.636 | 1.675 | 1e-08 | 571.188 |
| F14 | 3 | 00111000000 | 12 | 0.064 | 2.013 | 0.101 | 0.814 | 0.129 | 0.003 | 0.257 | 0.2 | 0.955 | 0.5 | 0 | 0.3 | 0.5 | 2.261 | 1e-08 | 893.75 |
| F14 | 5 | 01101111020 | 14 | 0.1 | 2.068 | 0.113 | 1.335 | 0.167 | 0.002 | 0.013 | 0.229 | 1.117 | 0.386 | 0.143 | 0.211 | 0.353 | 2 | 1e-08 | 1745.81 |
| F15 | 2 | 10010010022 | 17 | 0.071 | 1.858 | 0.103 | 1.185 | 0.146 | 0.003 | 0.602 | 0.186 | 0.926 | 0.634 | 0.53 | 0.32 | 0.481 | 1.821 | 1e-08 | 2813.91 |
| F15 | 3 | 11101110012 | 71 | 0.087 | 2.301 | 0.098 | 1.154 | 0.162 | 0.003 | 0.043 | 0.202 | 0.871 | 0.366 | 0.828 | 0.357 | 0.495 | 1.982 | 1e-08 | 8473.56 |
| F15 | 5 | 00100010101 | 154 | 0.093 | 2.125 | 0.077 | 0.818 | 0.097 | 0.002 | 0.188 | 0.2 | 0.955 | 0.411 | 0.998 | 0.224 | 0.535 | 2.05 | 0.0938 | 21929.10 |
| F16 | 2 | 11110010001 | 10 | 0.087 | 1.491 | 0.111 | 1.111 | 0.169 | 0.002 | 0.473 | 0.141 | 1.217 | 0.499 | 0.118 | 0.326 | 0.635 | 1.477 | 1e-08 | 1171.25 |
| F16 | 3 | 00100010001 | 53 | 0.078 | 1.56 | 0.11 | 1.263 | 0.118 | 0.001 | 0.288 | 0.2 | 0.955 | 0.551 | 0.937 | 0.369 | 0.536 | 1.475 | 1e-08 | 2091.88 |
| F16 | 5 | 00111010002 | 111 | 0.096 | 1.781 | 0.079 | 1.231 | 0.161 | 0.003 | 0.603 | 0.2 | 0.955 | 0.553 | 0.854 | 0.335 | 0.424 | 1.499 | 1e-08 | 6479.91 |
| F17 | 2 | 00110011122 | 28 | 0.083 | 1.63 | 0.09 | 1.303 | 0.144 | 0.002 | 0.821 | 0.247 | 1.192 | 0.484 | 0.38 | 0.215 | 0.635 | 2.387 | 1e-08 | 1397.16 |
| F17 | 3 | 00110110122 | 44 | 0.07 | 2.479 | 0.075 | 0.966 | 0.094 | 0.002 | 0.019 | 0.167 | 0.825 | 0.565 | 0.473 | 0.363 | 0.502 | 2.177 | 1e-08 | 2346.06 |
| F17 | 5 | 00100011002 | 87 | 0.069 | 1.646 | 0.091 | 0.849 | 0.153 | 0.002 | 0.354 | 0.21 | 0.785 | 0.4 | 0.446 | 0.336 | 0.504 | 1.822 | 1e-08 | 5595.91 |
| F18 | 2 | 11000011001 | 7 | 0.105 | 2.207 | 0.102 | 0.793 | 0.153 | 0.001 | 0.388 | 0.2 | 0.955 | 0.534 | 0.125 | 0.35 | 0.571 | 1.572 | 1e-08 | 1723.22 |
| F18 | 3 | 11010011112 | 13 | 0.101 | 2.007 | 0.067 | 0.964 | 0.136 | 0.003 | 0.314 | 0.2 | 0.955 | 0.43 | 0.144 | 0.383 | 0.403 | 2.433 | 1.02 | 3904.69 |
| F18 | 5 | 10010010022 | 13 | 0.091 | 2.487 | 0.08 | 1.233 | 0.156 | 0.002 | 0.246 | 0.2 | 0.955 | 0.541 | 0.028 | 0.23 | 0.42 | 2.292 | 0.000000449 | 9190.69 |
| F19 | 2 | 11010010112 | 15 | 0.082 | 2.127 | 0.081 | 1.074 | 0.106 | 0.003 | 0.491 | 0.2 | 0.955 | 0.387 | 0.093 | 0.252 | 0.438 | 2.197 | 1e-08 | 3460.33 |
| F19 | 3 | 00101010022 | 119 | 0.066 | 2.222 | 0.084 | 1.314 | 0.167 | 0.002 | 0.808 | 0.2 | 0.955 | 0.577 | 0.656 | 0.317 | 0.411 | 2.207 | 0.0191 | 15316.14 |
| F19 | 5 | 10000011021 | 126 | 0.08 | 1.887 | 0.102 | 0.999 | 0.169 | 0.002 | 0.724 | 0.2 | 0.955 | 0.526 | 0.237 | 0.342 | 0.501 | 1.808 | 0.144 | 71535.75 |
| F20 | 2 | 00001010011 | 9 | 0.059 | 2.446 | 0.107 | 0.901 | 0.137 | 0.003 | 0.308 | 0.2 | 0.955 | 0.481 | 0.824 | 0.252 | 0.527 | 1.762 | 1e-08 | 2330.81 |
| F20 | 3 | 00101010022 | 9 | 0.068 | 1.626 | 0.103 | 1.416 | 0.1 | 0.003 | 0.32 | 0.2 | 0.955 | 0.644 | 0.262 | 0.33 | 0.407 | 2.415 | 0.000172 | 7668.806 |
| F20 | 5 | 11011110011 | 103 | 0.097 | 2.319 | 0.113 | 1.396 | 0.103 | 0.002 | 0.09 | 0.252 | 1.042 | 0.515 | 0.453 | 0.297 | 0.446 | 2.436 | 0.258 | 37833.27 |
| F21 | 2 | 11011110022 | 11 | 0.064 | 1.9 | 0.087 | 1.141 | 0.094 | 0.002 | 0.489 | 0.256 | 1.129 | 0.57 | 0.908 | 0.24 | 0.505 | 2.515 | 1e-08 | 590.19 |
| F21 | 3 | 11100110102 | 9 | 0.087 | 2.506 | 0.085 | 0.789 | 0.155 | 0.003 | 0.554 | 0.237 | 0.774 | 0.45 | 0.324 | 0.28 | 0.477 | 2.544 | 1e-08 | 1842.44 |
| F21 | 5 | 00100010122 | 12 | 0.086 | 2.115 | 0.103 | 1.214 | 0.104 | 0.002 | 0.367 | 0.2 | 0.955 | 0.599 | 0.337 | 0.321 | 0.479 | 1.887 | 0.102 | 7702.25 |
| F22 | 2 | 11000110112 | 18 | 0.103 | 2.572 | 0.101 | 0.886 | 0.166 | 0.002 | 0.551 | 0.193 | 0.95 | 0.506 | 0.443 | 0.259 | 0.431 | 1.948 | 1e-08 | 434.44 |
| F22 | 3 | 01010010111 | 26 | 0.065 | 2.5 | 0.096 | 1.298 | 0.095 | 0.002 | 0.614 | 0.165 | 1.04 | 0.447 | 0.541 | 0.211 | 0.573 | 1.762 | 3.22 | 1565.60 |
| F22 | 5 | 01001001022 | 12 | 0.069 | 1.567 | 0.086 | 0.909 | 0.155 | 0.002 | 0.176 | 0.2 | 0.955 | 0.5 | 0 | 0.3 | 0.5 | 2.352 | 0.0804 | 7979.37 |
| F23 | 2 | 00100010001 | 14 | 0.095 | 2.185 | 0.081 | 1.229 | 0.149 | 0.002 | 0.294 | 0.2 | 0.955 | 0.359 | 0.223 | 0.334 | 0.554 | 1.984 | 1e-08 | 1664.84 |
| F23 | 3 | 11011110022 | 19 | 0.084 | 1.414 | 0.086 | 0.924 | 0.096 | 0.003 | 0.022 | 0.219 | 1.03 | 0.639 | 0.136 | 0.389 | 0.514 | 2.47 | 0.00443 | 8568.60 |
| F23 | 5 | 10011000111 | 75 | 0.074 | 2.565 | 0.115 | 0.767 | 0.136 | 0.001 | 0.088 | 0.2 | 0.955 | 0.481 | 0.126 | 0.219 | 0.412 | 2.136 | 0.0622 | 22807.11 |
| F24 | 2 | 00010100111 | 17 | 0.085 | 1.895 | 0.111 | 1.173 | 0.151 | 0.002 | 0.54 | 0.224 | 0.679 | 0.382 | 0.185 | 0.223 | 0.399 | 2.214 | 0.0113 | 4016.00 |
| F24 | 3 | 00010100021 | 10 | 0.097 | 1.827 | 0.069 | 0.908 | 0.142 | 0.002 | 0.142 | 0.228 | 0.934 | 0.359 | 0.879 | 0.298 | 0.518 | 1.648 | 2.63 | 50193.29 |
| F24 | 5 | 11011011111 | 135 | 0.061 | 2.573 | 0.084 | 0.845 | 0.156 | 0.002 | 0.726 | 0.2 | 0.955 | 0.612 | 0.037 | 0.279 | 0.624 | 2.392 | 20 | 1557063.00 |

Table III: *F1 - F12 from the MIES Compared to the best from GA and best from Commen Variant, both from [1]. In the last two coloms, if the value is greater then* 1 *then there is an improvement. Note: All experiments from the MIES only had 10 runs while both other options had 30 tries.*

| F | dim | MIES ERT | MIES FCE | GA ERT | GA FCE | CV ERT | CV FCE | GA/MIES | CV/MIES |
|---|---|---|---|---|---|---|---|---|---|
| F1 | 2 | 157.72 | 1e-08 | 165.25 | 1e-08 | 294.75 | 1e-08 | **1.048** | **1.869** |
| F1 | 3 | 251.91 | 1e-08 | 235.38 | 1e-08 | 473.59 | 1e-08 | 0.934 | **1.880** |
| F1 | 5 | 453.84 | 1e-08 | 440.00 | 1e-08 | 797.50 | 1e-08 | 0.969 | **1.757** |
| F2 | 2 | 426.44 | 1e-08 | 434.06 | 1e-08 | 660.375 | 1e-08 | **1.018** | **1.549** |
| F2 | 3 | 878.13 | 1e-08 | 678.59 | 1e-08 | 1160.03 | 1e-08 | 0.773 | **1.321** |
| F2 | 5 | 1685.00 | 1e-08 | 1364.50 | 1e-08 | 2314.75 | 1e-08 | 0.810 | **1.374** |
| F3 | 2 | 3213.44 | 1e-08 | 1795.73 | 0.2912 | 2742.35 | 0.3582 | 0.559 | 0.853 |
| F3 | 3 | 12877.64 | 0.0679 | 5380.40 | 0.8374 | 12346.00 | 1.062 | 0.418 | 0.959 |
| F3 | 5 | 47132.73 | 0.308 | 29318.80 | 2.067 | 76816.00 | 5.074 | 0.622 | **1.630** |
| F4 | 2 | 7054.41 | 0.0509 | 4089.00 | 0.8781 | 11877.60 | 1.464 | 0.580 | **1.684** |
| F4 | 3 | 62078.69 | 0.887 | 17803.20 | 3.561 | 93793.00 | 4.469 | 0.287 | **1.511** |
| F4 | 5 | 1553072.00 | 3.48 | 155980.00 | 8.998 | N/A | 5.905 | 0.100 | **1.697** |
| F5 | 2 | 505.25 | 1e-08 | 466.59 | 1e-08 | 605.81 | 1e-08 | 0.923 | **1.199** |
| F5 | 3 | 809.13 | 1e-08 | 736.66 | 1e-08 | 940.69 | 1e-08 | 0.910 | **1.163** |
| F5 | 5 | 1772.66 | 1e-08 | 1324.28 | 1e-08 | 1731.00 | 1e-08 | 0.747 | 0.977 |
| F6 | 2 | 334.72 | 1e-08 | 315.06 | 1e-08 | 428.44 | 1e-08 | 0.941 | **1.280** |
| F6 | 3 | 566.69 | 1e-08 | 573.38 | 1e-08 | 805.00 | 1e-08 | **1.012** | **1.421** |
| F6 | 5 | 1099.38 | 1e-08 | 1141.78 | 1e-08 | 1574.00 | 1e-08 | 1.039 | **1.432** |
| F7 | 2 | 268.63 | 1e-08 | 266.00 | 1e-08 | 438.75 | 1e-08 | 0.990 | **1.633** |
| F7 | 3 | 564.66 | 1e-08 | 540.72 | 1e-08 | 886.51 | 0.002689 | 0.958 | **1.570** |
| F7 | 5 | 1270.25 | 1e-08 | 1613.56 | 1e-08 | 2587.50 | 1e-08 | **1.270** | **2.037** |
| F8 | 2 | 506.09 | 1e-08 | 422.31 | 1e-08 | 574.13 | 1e-08 | 0.834 | **1.134** |
| F8 | 3 | 1041.82 | 1e-08 | 827.44 | 1e-08 | 1034.69 | 1e-08 | 0.794 | 0.993 |
| F8 | 5 | 2406.09 | 1e-08 | 1705.62 | 1e-08 | 2705.43 | 0.4914 | 0.709 | **1.124** |
| F9 | 2 | 398.00 | 1e-08 | 423.06 | 1e-08 | 546.94 | 1e-08 | **1.063** | **1.374** |
| F9 | 3 | 959.75 | 1e-08 | 764.25 | 1e-08 | 1183.66 | 1e-08 | 0.796 | **1.233** |
| F9 | 5 | 2307.69 | 1e-08 | 1710.22 | 1e-08 | 2607.23 | 0.000213 | 0.741 | **1.130** |
| F10 | 2 | 502.38 | 1e-08 | 441.69 | 1e-08 | 662.06 | 1e-08 | 0.879 | **1.318** |
| F10 | 3 | 956.50 | 1e-08 | 693.50 | 1e-08 | 1146.25 | 1e-08 | 0.725 | **1.198** |
| F10 | 5 | 1915.88 | 1e-08 | 1458.25 | 1e-08 | 2228.00 | 1e-08 | 0.761 | **1.163** |
| F11 | 2 | 448.66 | 1e-08 | 453.19 | 1e-08 | 629.63 | 1e-08 | **1.010** | **1.403** |
| F11 | 3 | 864.66 | 1e-08 | 734.97 | 1e-08 | 1334.38 | 1e-08 | 0.850 | **1.543** |
| F11 | 5 | 1272.06 | 1e-08 | 1518.75 | 1e-08 | 2493.00 | 1e-08 | **1.194** | **1.960** |
| F12 | 2 | 1126 | 1e-08 | 776.07 | 1e-08 | 1360.85 | 0.1986 | 0.689 | **1.209** |
| F12 | 3 | 2672.03 | 1e-08 | 2084.69 | 0.02808 | 3426.50 | 390.5 | 0.780 | **1.282** |
| F12 | 5 | 6082.81 | 1e-08 | 4017.19 | 1.03e-05 | 6489.26 | 0.1195 | 0.660 | **1.067** |

Table IV: *F13 - F24 from the MIES Compared to the best from GA and best from Commen Variant, both from [1]. In the last two coloms, if the value is greater then* 1 *then there is an improvement. Note: All experiments from the MIES only had 10 runs while both other options had 30 tries.*

| F-ID | N | ERT-MIES | FCE-MIES | ERT-GA | FCE-GA | ERT-CV | FCE-CV | GA/MIES | CV/MIES |
|---|---|---|---|---|---|---|---|---|---|
| F13 | 2 | 699.25 | 1e-08 | 518.50 | 1e-08 | 971.81 | 1e-08 | 0.742 | **1.390** |
| F13 | 3 | 1400.92 | 1e-08 | 883.34 | 1e-08 | 1704.28 | 1e-08 | 0.631 | **1.217** |
| F13 | 5 | 3371.44 | 1e-08 | 2967.23 | 1e-08 | 3830.93 | 3.862e-08 | 0.880 | **1.136** |
| F14 | 2 | 571.19 | 1e-08 | 410.78 | 1e-08 | 618.94 | 1e-08 | 0.719 | **1.084** |
| F14 | 3 | 893.75 | 1e-08 | 630.75 | 1e-08 | 1196.12 | 1e-08 | 0.706 | **1.338** |
| F14 | 5 | 1745.81 | 1e-08 | 1327.00 | 1e-08 | 2506.75 | 1e-08 | 0.760 | **1.436** |
| F15 | 2 | 2813.91 | 1e-08 | 1881.09 | 0.2853 | 2864.47 | 0.5562 | 0.668 | **1.018** |
| F15 | 3 | 8473.56 | 1e-08 | 3882.94 | 0.6265 | 7889.00 | 1.29 | 0.458 | 0.931 |
| F15 | 5 | 21929.10 | 0.0622 | 17811.00 | 1.492 | 51880.00 | 2.363 | 0.812 | **2.366** |
| F16 | 2 | 1171.25 | 1e-08 | 740.37 | 0.001413 | 897.80 | 0.003917 | 0.632 | 0.767 |
| F16 | 3 | 2091.88 | 1e-08 | 1490.07 | 0.03927 | 3739.84 | 0.06716 | 0.712 | **1.788** |
| F16 | 5 | 6479.91 | 1e-08 | 4970.18 | 0.0417 | 16217.80 | 0.07463 | 0.767 | **2.503** |
| F17 | 2 | 1397.16 | 1e-08 | 952.00 | 9.01e-07 | 1643.77 | 0.0001774 | 0.681 | **1.177** |
| F17 | 3 | 2346.06 | 1e-08 | 1610.94 | 2.25e-07 | 5332.69 | 0.01492 | 0.687 | **2.273** |
| F17 | 5 | 5595.91 | 1e-08 | 4393.66 | 5.07e-07 | 79964.00 | 0.01337 | 0.785 | **14.290** |
| F18 | 2 | 1723.22 | 1e-08 | 2210.00 | 1.14e-01 | 7734.00 | 0.3329 | **1.282** | **4.488** |
| F18 | 3 | 3904.69 | 1e-08 | 4807.50 | 1.48e-01 | 22480.50 | 0.1571 | **1.231** | **5.757** |
| F18 | 5 | 9190.69 | 1e-08 | 22228.60 | 1.41e-02 | N/A | 0.1211 | **2.419** | **12110000** |
| F19 | 2 | 3460.33 | 0.000982 | 1833.39 | 4.91e-03 | 3498.86 | 0.01915 | 0.530 | **1.011** |
| F19 | 3 | 15316.14 | 0.00144 | 12702.00 | 2.47e-01 | 22671.20 | 0.2385 | 0.829 | **1.480** |
| F19 | 5 | 71535.75 | 0.0579 | 77184.00 | 1.95e-01 | N/A | 0.405 | 1.079 | **6.995** |
| F20 | 2 | 2330.81 | 1e-08 | 1571.65 | 2.31e-01 | 3022.40 | 0.2513 | 0.674 | **1.297** |
| F20 | 3 | 7668.81 | 0.01 | 7379.73 | 5.70e-01 | 29992.70 | 0.8192 | 0.962 | 3.911 |
| F20 | 5 | 37833.27 | 0.0449 | 50956.70 | 9.24e-01 | 156448.00 | 1.313 | **1.347** | **4.135** |
| F21 | 2 | 590.19 | 1e-08 | 352.97 | 1e-08 | 459.75 | 1e-08 | 0.598 | 0.779 |
| F21 | 3 | 1842.44 | 1e-08 | 1265.62 | 9.91e-02 | 2074.62 | 0.1948 | 0.687 | **1.126** |
| F21 | 5 | 7702.25 | 1e-08 | 4443.60 | 6.98e-01 | 6536.50 | 0.9468 | 0.577 | 0.849 |
| F22 | 2 | 434.44 | 1e-08 | 360.00 | 1e-08 | 431.63 | 1e-08 | 0.829 | 0.994 |
| F22 | 3 | 1565.59 | 1e-08 | 1136.71 | 5.94e-02 | 1609.25 | 0.08648 | 0.726 | **1.028** |
| F22 | 5 | 7979.37 | 0.0123 | 3573.12 | 2.99e-01 | 5198.74 | 1.097 | 0.448 | 0.652 |
| F23 | 2 | 1664.84 | 1e-08 | 1528.80 | 1.04e-01 | 2652.00 | 0.2817 | 0.918 | **1.593** |
| F23 | 3 | 8568.59 | 1e-08 | 3794.21 | 2.22e-01 | 7371.58 | 0.4927 | 0.443 | 0.860 |
| F23 | 5 | 22807.11 | 0.0155 | 21265.10 | 2.67e-01 | 52277.30 | 0.5859 | 0.932 | 2.292 |
| F24 | 2 | 4016.00 | 0.0348 | 4411.54 | 1.501 | 63342.00 | 2.09 | **1.098** | **15.772** |
| F24 | 3 | 50193.29 | 2.44 | 30994.00 | 6.622 | N/A | 4.007 | 0.617 | **1.642** |
| F24 | 5 | 1557063.00 | 19.7 | N/A | 5.931 | N/A | 7.855 | 0.301 | 0.399 |

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